Details Cos(52)dz Using Trapezoidal And Simpson's Rule With N = 4, We Can Estimate The Error In Estimating

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Numerical integration is a fundamental technique in calculus used to approximate the definite integral of a function, especially when an analytical solution is difficult or impossible to obtain. In this article, we explore the process of estimating the integral of cos(z) over a specific interval using two popular methods: the Trapezoidal Rule and Simpson's Rule, with a focus on N=4 subdivisions. Additionally, we delve into how to estimate the error associated with these numerical methods, providing insight into the accuracy and reliability of the approximations.

Understanding Numerical Integration and Its Significance

Numerical integration serves as a practical tool in various scientific and engineering applications. When integrating functions like cos(z), especially over complex intervals, exact solutions may be cumbersome. Therefore, approximation methods become invaluable.

What is the Integral of Cos(z)?

The indefinite integral of cos(z) is sin(z) + C, where C is the constant of integration. However, when evaluating a definite integral over a specific interval [a, b], numerical methods approximate the area under the curve between these points.

Why Use Trapezoidal and Simpson's Rules?

- Trapezoidal Rule approximates the area under the curve by dividing it into trapezoids, providing a straightforward approach.
- Simpson's Rule uses quadratic polynomials to approximate the function, often yielding higher accuracy with fewer subdivisions.

Setting Up the Problem: Integrating Cos(z) with N=4

Suppose we want to evaluate the integral:

$$\int_{a}^{b} \cos(z) \, dz$$

for specific limits a and b. For illustration, let's choose:

$$a = 0, \quad b = 52$$

This means we want to approximate:

$$\int_{0}^{52} \cos(z) \, dz$$

using N=4 subdivisions.

Calculating Step Size (h)

The step size h, which determines the width of each subinterval, is calculated as:

$$h = \frac{b - a}{N}$$

For our case:

$$h = \frac{52 - 0}{4} = 13$$

This divides the interval [0, 52] into four subintervals: [0,13], [13,26], [26,39], and [39,52].

Applying the Trapezoidal Rule

The Trapezoidal Rule estimates the integral as:

$$T_N = \frac{h}{2} \left[f(z_0) + 2 \sum_{i=1}^{N-1} f(z_i) + f(z_N) \right]$$

where:
- $z_0 = a$
- $z_N = b$
- $z_i = a + i \times h$

Step-by-Step Calculation

1. Identify the points:

i	z_i	$\cos(z_i)$
0	0	$\cos(0) = 1$
1	13	$\cos(13)$
2	26	$\cos(26)$
3	39	$\cos(39)$
4	52	$\cos(52)$

2. Compute $\cos(z_i)$:

Using a calculator or computational tool:

- $\cos(0) = 1$
- $\cos(13) \approx 0.4848$
- $\cos(26) \approx -0.9074$
- $\cos(39) \approx 0.7557$
- $\cos(52) \approx -0.3746$

3. Calculate T_4:

$$
T_4 = \frac{13}{2} [1 + 2(0.4848 + (-0.9074) + 0.7557) + (-0.3746)]
$$

Sum the middle terms:

$$
2(0.4848 - 0.9074 + 0.7557) = 2(0.4848 - 0.9074 + 0.7557) = 2(0.3329) = 0.6658
$$

Now, substitute back:

$$
T_4 = \frac{13}{2} [1 + 0.6658 - 0.3746] = 6.5 \times (1 + 0.6658 - 0.3746)
$$

Simplify inside brackets:

\[

1 + 0.6658 - 0.3746 = 1.2912
\]

Finally:

\[
T_4 = 6.5 \times 1.2912 \approx 8.393
\]

Approximate value of the integral using Trapezoidal Rule: \(\boxed{8.393}\).

Applying Simpson's Rule

Simpson's Rule provides a higher degree of accuracy with the same number of subintervals when N is even:

\[S_N = \frac{h}{3} \left[f(z_0) + 4 \sum_{\text{odd } i} f(z_i) + 2 \sum_{\text{even } i, i \neq 0, N} f(z_i) + f(z_N) \right] \]

For N=4, the points are the same as above.

Step-by-Step Calculation

1. Identify the points:

i	\(z_i \)	\(\cos(z_i) \)
0	0	1
1	13	0.4848
2	26	-0.9074
3	39	0.7557
4	52	-0.3746

2. Apply Simpson's Rule:

\[
S_4 = \frac{13}{3} [1 + 4(0.4848 + 0.7557) + 2(-0.9074) + (-0.3746)]
\]

Calculate the sum of the odd terms:

$$
4 (0.4848 + 0.7557) = 4 \times 1.2405 = 4.962
$$

Sum the even terms (excluding the first and last):

$$
2 \times (-0.9074) = -1.814
$$

Now, assemble:

$$
S_4 = \frac{13}{3} [1 + 4.962 - 1.814 - 0.3746]
$$

Simplify inside brackets:

$$
1 + 4.962 - 1.814 - 0.3746 = 1 + 4.962 - 2.1886 = 3.7734
$$

Finally:

$$
S_4 = \frac{13}{3} \times 3.7734 \approx 4.333 \times 3.7734 \approx 16.353
$$

Approximate value of the integral using Simpson's Rule: $\boxed{16.353}$.

Note: The discrepancy between the two methods is expected since they approximate the same integral differently.

Estimating the Error in Numerical Integration

Accurately estimating the error involved in these numerical methods is crucial for understanding the reliability of the approximation. Both the Trapezoidal and Simpson's Rules have specific error bounds derived from the derivatives of the function being integrated.

Trapezoidal Rule Error Estimate

The error bound for the Trapezoidal Rule is:

$$ |E_T| \leq \frac{(b - a)^3}{12 N^2} \max_{z \in [a,b]} |f''(z)| $$

- $f''(z)$ is the second derivative of $f(z)$, which for $\cos(z)$ is:

$$
f''(z) = -\cos(z)
$$

- The maximum of $|\cos(z)|$ over [0, 52] is 1, since cosine oscillates between -1 and 1.

Calculate the error bound:

$$
|E_T| \leq \frac{(52 - 0)^3}{12 \times 4^2} \times 1 = \frac{52^3}{12 \times 16}
$$

$$
52^3 = 52 \times 52 \times 52 = 140608
$$

So:

$$
|E_T| \leq \frac{140608}{192} \approx 732.33
$$

This indicates the Trapezoidal approximation is within approximately 732.33 units of the true value, which is quite large relative to the estimated integral, suggesting the method's limited accuracy for this

Frequently Asked Questions

What is the primary goal of using Trapezoidal and Simpson's rules to estimate the integral of cos(52)dz?

The primary goal is to approximate the definite integral of cos(52)dz over a specified interval and to estimate the error involved in this approximation.

How does increasing the number of subintervals (N) affect the accuracy of Trapezoidal and Simpson's Rule estimates?

Increasing N generally improves the accuracy of the approximation because the methods better capture the function's behavior over smaller subintervals, reducing the approximation error.

What is the significance of using N = 4 in estimating the integral of cos(52)dz?

Using N = 4 provides a balance between computational simplicity and accuracy, allowing for manageable calculations while still improving the approximation compared to fewer subintervals.

How do you estimate the error in the Trapezoidal and Simpson's Rule approximations?

The error can be estimated using error formulas that involve the second derivative (for Trapezoidal) and the fourth derivative (for Simpson's Rule) of the integrand, evaluated over the interval.

What is the role of the second and fourth derivatives of cos(52) in error estimation?

These derivatives determine the magnitude of the error terms; the second derivative affects the Trapezoidal rule error, while the fourth derivative influences the Simpson's rule error estimation.

Can Simpson's Rule provide a more accurate estimate than the Trapezoidal Rule for cos(52)dz?

Yes, generally Simpson's Rule tends to be more accurate than the Trapezoidal Rule because it uses quadratic interpolation, especially when N is even and the function is sufficiently smooth.

How do you calculate the error bounds for N = 4 when integrating cos(52)dz using these methods?

You apply the error formulas: for Trapezoidal, error ≤ (b - a)^3 / (12 N^2) max|f''(z)|; for Simpson's, error ≤ (b - a)^5 / (180 N^4) max|f''''(z)|, evaluating the derivatives over the interval.

Why is it important to estimate the error in numerical integration methods like Trapezoidal and Simpson's Rule?

Estimating the error helps determine the reliability of the approximation and whether the chosen N provides sufficient accuracy for the problem at hand.

Additional Resources

Numerical Integration Techniques: Estimating ∫ Cos(52) dz Using Trapezoidal and Simpson's Rule with N=4

Introduction to Numerical Integration and Its Significance

Numerical integration is a fundamental technique in applied mathematics, physics, engineering, and computational sciences. It provides approximate solutions to definite integrals when an analytical solution is either difficult or impossible to obtain. This process involves summing the areas of simple geometric shapes—like trapezoids or parabolic segments—to estimate the total area under a curve.

In practical scenarios, especially involving complex functions or large datasets, exact integration methods such as antiderivatives are not feasible. Instead, numerical methods like the Trapezoidal Rule and Simpson's Rule become indispensable tools. These methods balance computational efficiency and accuracy, providing reliable estimates with quantifiable errors.

This article explores the application of these methods to estimate the integral of the cosine function over a specified interval, specifically focusing on the integral of cos(52) with respect to z, using N=4 subintervals. We will also analyze how to estimate the errors associated with these approximations, offering a comprehensive understanding suitable for students, educators, and professionals alike.

Understanding the Problem: The Integral of Cos(52) dz

Before diving into numerical methods, it is crucial to interpret the integral:

$$
\int_{a}^{b} \cos(52) \, dz
$$

Notice that $\cos(52)$ is a constant with respect to z. This simplifies the integral significantly because integrating a constant over an interval is straightforward:

$$
\int_{a}^{b} \cos(52) \, dz = \cos(52) \times (b - a)
$$

However, to illustrate the application of Trapezoidal and Simpson's Rule, let's assume the integral is over a specific interval, say from $z=a=0$ to $z=b=4$. This choice allows us to demonstrate how numerical methods approximate the exact value and how errors are estimated.

Exact value of the integral:

$$
\text{Exact} = \cos(52) \times (4 - 0) = 4 \times \cos(52)
$$

Since $\cos(52^\circ)$ in degrees or radians can be evaluated, but the main focus here is on applying the numerical methods rather than the actual value.

Applying Trapezoidal Rule with N=4

Overview of the Trapezoidal Rule

The Trapezoidal Rule approximates the area under a curve by dividing the interval into N subintervals and approximating each segment by a trapezoid. The sum of the areas of these trapezoids provides the estimate of the integral.

The formula for the Trapezoidal Rule:

$$
T_N = \frac{h}{2} \left[f(a) + 2 \sum_{i=1}^{N-1} f(z_i) + f(b) \right]
$$

where:

- $h = (b - a)/N$ is the width of each subinterval.
- $z_i = a + i h$ are the division points.

Key steps:

1. Divide the interval into 4 equal parts.
2. Evaluate the function at each point.
3. Sum according to the formula.

Calculations for N=4

- Interval: $a=0$, $b=4$
- Number of subintervals: $N=4$
- Width: $h = (4 - 0)/4 = 1$

Division points:

- $z_0 = 0$
- $z_1 = 1$
- $z_2 = 2$
- $z_3 = 3$
- $z_4 = 4$

Function evaluations:

- $f(z) = \cos(52)$ (constant for all z), so

$$
f(z_i) = \cos(52) \quad \text{for all } i
$$

Applying the formula:

$$
T_4 = \frac{h}{2} \left[f(0) + 2(f(1) + f(2) + f(3)) + f(4) \right]
$$

$$
T_4 = \frac{1}{2} \left[\cos(52) + 2 \times 3 \times \cos(52) + \cos(52) \right]
$$

$$
T_4 = \frac{1}{2} \left[2 \times \cos(52) + 6 \times \cos(52) \right] = \frac{1}{2} \times 8 \times \cos(52) = 4 \times \cos(52)
$$

Result:

The Trapezoidal Rule yields exactly $4 \times \cos(52)$, which matches the analytical solution because the integrand is constant.

Applying Simpson's Rule with N=4

Overview of Simpson's Rule

Simpson's Rule offers a higher accuracy approximation by fitting parabolas through the points. It divides the interval into an even number of subintervals and applies the formula:

$$
S_N = \frac{h}{3} \left[f(a) + 4 \sum_{i=1,3,5,\dots}^{N-1} f(z_i) + 2 \sum_{i=2,4,6,\dots}^{N-2} f(z_i) + f(b) \right]
$$

Key steps:

1. Same division points as in the Trapezoidal Rule.
2. Evaluate the function at each point.
3. Sum with weights 4 or 2 accordingly.

Calculations for N=4

- $h = 1$ (same as above).
- Points:

- $z_0=0$
- $z_1=1$
- $z_2=2$
- $z_3=3$
- $z_4=4$

Function evaluations:

- $f(z_i) = \cos(52)$ for all points.

Applying Simpson's Rule:

$$
S_4 = \frac{h}{3} \left[f(0) + 4f(1) + 2f(2) + 4f(3) + f(4) \right]
$$

$$
S_4 = \frac{1}{3} \left[\cos(52) + 4 \times \cos(52) + 2 \times \cos(52) + 4 \times \cos(52) + \cos(52) \right]
$$

Sum inside brackets:

$$
\cos(52) \times (1 + 4 + 2 + 4 + 1) = \cos(52) \times 12
$$

Thus,

$$
S_4 = \frac{1}{3} \times 12 \times \cos(52) = 4 \times \cos(52)
$$

Again, the approximation equals the exact value because of the constant integrand.

Error Estimation in Numerical Integration

While in this particular case, the integrand's constancy makes the numerical methods exact, in general, the error estimates are critical for understanding the accuracy of the approximations, especially for non-constant functions.

Trapezoidal Rule Error Bound

The error E_T for the Trapezoidal Rule is bounded by:

$$
|E_T| \leq \frac{(b - a)^3}{12 N^2} \max_{z \in [a, b]} |f''(z)|
$$

- $f''(z)$ is the second derivative of the function.
- For $f(z) = \cos(kz)$, the second derivative is:

$$
f''(z) = -k^2 \cos(kz)
$$

In our case, $k=0$ (since $\cos(52)$ is constant), so:

$$
f''(z) = 0
$$

Thus, the error bound is zero, confirming the exactness.

Simpson's Rule Error Bound

The error E_S for Simpson's Rule is bounded by:

$$
|E_S| \leq \frac{(b - a)^5}{180 N^4} \max_{z \in [a, b]} |f^{(4)}(z)|
$$

- The fourth derivative of $f(z) = \cos(kz)$:

$$
f^{(4)}(z) = k^4 \cos(kz)
$$

Again, since the function is constant in our scenario, all derivatives beyond the zero order are zero, and the error bound is zero.

Implications and Practical Considerations

The above calculations demonstrate that for a constant integrand like $\cos(52)$, both the Trapezoidal and Simpson's Rule provide exact results even with small N. However, in real-world applications where

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