Determine Whether The Equation Is Exact. If It Is Exact, Find The Solution. 4 2eycosy + 27-1 = C 4 2eycosy

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When solving differential equations, especially first-order equations, determining whether an equation is exact is a fundamental step that simplifies the process of finding solutions. The given equation appears complex at first glance:

$$ 4 \, 2 e^{y} \cos y + 27 - 1 = C \, 4 \, 2 e^{y} \cos y $$

This article will guide you through the process of analyzing whether this differential equation is exact, and if so, how to find its general solution. We will explore the concepts of exact differential equations, how to test for exactness, and the methods to solve them systematically.

Understanding Exact Differential Equations

Before diving into the specific equation, it's essential to understand what constitutes an exact differential equation.

What Is an Exact Differential Equation?

An ordinary differential equation (ODE) of the form:

$$ M(x, y) \, dx + N(x, y) \, dy = 0 $$

is said to be exact if there exists a potential function $\Psi(x, y)$ such that:

$$ \frac{\partial \Psi}{\partial x} = M(x, y) \quad \text{and} \quad \frac{\partial \Psi}{\partial y} = N(x, y) $$

This means that the differential form $M \, dx + N \, dy$ is an exact differential of Ψ. The key property of exact equations is that they can be integrated directly once the exactness condition is verified.

Conditions for Exactness

Given $M(x, y)$ and $N(x, y)$, the equation is exact if:

$$ \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} $$

If this condition holds, the equation is exact, and the solution involves finding a potential function Ψ.

Rearranging the Given Equation

Let's analyze the provided equation:

$$ 4 \, 2 e^{y} \cos y + 27 - 1 = C \, 4 \, 2 e^{y} \cos y $$

First, simplify the constants:

- $4 \times 2 e^{y} \cos y = 8 e^{y} \cos y$

- $27 - 1 = 26$

So, the equation becomes:

$$ 8 e^{y} \cos y + 26 = C \times 8 e^{y} \cos y $$

Assuming C is an arbitrary constant, this suggests the original relationship is more of an implicit relation involving y, or perhaps a solution form for a differential equation.

However, the form resembles an implicit solution to a differential equation involving the variables x and y. To analyze whether the differential equation is exact, we need to express it in differential form.

Formulating the Differential Equation

Suppose the original differential equation is:

$$ \frac{dy}{dx} = \text{some function involving } y $$

or, more generally, an implicit form involving derivatives.

Given the expression:

$$ 8 e^{y} \cos y + 26 = C \times 8 e^{y} \cos y $$

we might interpret this as an implicit solution to an ODE, possibly derived from integrating factors or substitution.

Alternatively, perhaps the original differential equation is:

$$ \frac{dy}{dx} = f(x, y) $$

and the expression is an implicit solution function involving y.

Since the problem states, "Determine whether the equation is exact," and then "if it is, find the solution," it suggests that the initial step is to identify the differential form.

Identifying the Differential Equation and Its Form

Given the structure, perhaps the differential equation is:

$$ M(x, y) \, dx + N(x, y) \, dy = 0 $$

which, after some rearrangement, leads to the implicit relation:

$$ 8 e^{y} \cos y + 26 = C \times 8 e^{y} \cos y $$

Alternatively, the differential equation might be:

$$ \frac{dy}{dx} = \frac{f(x, y)}{g(x, y)} $$

But due to the structure involving exponential and trigonometric functions, perhaps the differential equation has the form:

$$\frac{dy}{dx} = \text{some function involving } y$$

Testing for Exactness

To test whether a differential equation is exact, we need to express it explicitly in the form:

$$M(x, y) \, dx + N(x, y) \, dy = 0$$

Suppose we consider the differential form:

$$M(x, y) \, dx + N(x, y) \, dy = 0$$

and that the differential equation derived from the original expression can be represented as such.

Constructing M and N

Let's consider the possibility that the original differential equation is:

$$\frac{dy}{dx} = \frac{8 e^{y} \cos y + 26}{\text{some denominator}}$$

Alternatively, perhaps the differential form is:

$$ (8 e^{y} \cos y + 26) \, dx + \text{something} \, dy = 0 $$

But since the initial expression involves constants and exponential/trigonometric functions, perhaps the key is to see if the differential form:

$$ M(x, y) = 8 e^{y} \cos y + 26 $$

$$ N(x, y) = \text{some function} $$

are related such that:

$$ \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} $$

Calculating Partial Derivatives

Suppose:

$$ M(y) = 8 e^{y} \cos y + 26 $$

$$ N(y) = \text{related expression} $$

Calculate:

$$ \frac{\partial M}{\partial y} $$

- For the term $8 e^{y} \cos y$:

Using the product rule:

$$ \frac{\partial}{\partial y} (e^{y} \cos y) = e^{y} \cos y + e^{y} (-\sin y) = e^{y} (\cos y - \sin y) $$

Therefore,

$$\frac{\partial M}{\partial y} = 8 \times e^{y} (\cos y - \sin y)$$

The derivative of constant 26 with respect to y is zero.

Similarly, to verify exactness, we need:

$$\frac{\partial N}{\partial x} = \frac{\partial M}{\partial y} = 8 e^{y} (\cos y - \sin y)$$

If N is independent of x, then $\partial N / \partial x = 0$, and the differential equation is not exact.

Conclusion on Exactness

Based on the derivatives calculated, if the corresponding $N(x, y)$ does not satisfy the symmetry condition $\partial M/\partial y = \partial N/\partial x$, then the differential equation is not exact as is.

Alternatively, the structure of the original expression suggests the need for an integrating factor, possibly depending on y, to make the differential equation exact.

Finding the Solution if the Equation Is Exact

If the differential equation is verified to be exact, the next step is to find the potential function \(\Psi(x,

y) \) such that:

$$ \frac{\partial \Psi}{\partial x} = M(x, y) $$

$$ \frac{\partial \Psi}{\partial y} = N(x, y) $$

The general method involves:

1. Integrating $M(x, y)$ with respect to x to find $\Psi(x, y)$.
2. Differentiating Ψ with respect to y and equating to $N(x, y)$.
3. Solving for the arbitrary function of y introduced during integration.

Practical Steps to Solve the Differential Equation

Given the potential complexity of the functions involved, here are practical steps to proceed:

- Identify $M(x, y)$ and $N(x, y)$ from the differential form.
- Compute $\partial M / \partial y$ and $\partial N / \partial x$ to verify exactness.
- If exact, integrate M w.r.t. x (or N w.r.t. y) to find Ψ.
- Determine the general solution by setting $\Psi(x, y) = \text{constant}$.

Summary and Final Thoughts

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Frequently Asked Questions

How can I determine if the differential equation 4 2eycosy + 27 - 1 = C 4 2eycosy is exact?

First, rewrite the equation in the form M(x,y)dx + N(x,y)dy = 0. Then, check if the partial derivative of M with respect to y equals the partial derivative of N with respect to x. If they are equal, the equation is exact.

What steps should I follow to solve an exact differential equation like 4 2eycosy + 27 - 1 = C 4 2eycosy?

Identify M(x,y) and N(x,y) from the differential form, verify if the equation is exact, then find the potential function by integrating M with respect to x and N with respect to y, and finally determine the solution by equating the potential functions.

In the equation 4 2eycosy + 27 - 1 = C 4 2eycosy, what does the constant C represent?

The constant C represents the constant of integration that arises when solving the exact differential equation, indicating the family of solutions.

Can the given equation be simplified before checking if it's exact? If

so, how?

Yes, simplify the equation algebraically to its standard differential form, ensuring all terms are expressed as functions of x and y with differential operators arranged as M(x,y)dx + N(x,y)dy = 0, making it easier to verify exactness.

If the differential equation is not exact, what method can I use to solve it?

If it's not exact, consider finding an integrating factor—often a function of x or y—that makes the equation exact, or use other methods such as substitution or integrating factors tailored to the specific form of the equation.

Additional Resources

Exact Differential Equation: An In-Depth Analysis and Solution Approach

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Understanding the Nature of Differential Equations

Differential equations are fundamental tools in mathematics and science, describing how quantities change over time or space. They are equations involving derivatives of unknown functions, and solving them allows us to model real-world phenomena, from physics to biology. Among the various types, exact differential equations occupy a special place due to their elegant solvability.

In this article, we will explore the process of determining whether the given differential equation is exact, and if so, how to find its solution. Our focus will be on a specific equation:

$$
4e^{2y} \cos y\, dx + (27 - 1) dy = C
$$

which appears to be a misstatement or typo; we will correct and interpret it to reflect a typical form suitable for analysis.

Deciphering the Given Equation: Clarification and Restatement

Before delving into the criteria for exactness, it is essential to interpret the provided equation correctly. The original expression:

$$
4 2ey \cos y + 27 - 1 = C 4 2ey \cos y
$$

appears to contain typographical errors or formatting issues. Likely, the intended differential equation resembles:

$$
4 e^{2y} \cos y\, dx + (27 - 1) dy = 0
$$

which simplifies to:

$$
4 e^{2y} \cos y\, dx + 26\, dy = 0
$$

or perhaps, the original problem is to analyze an equation of the form:

$$
M(x, y) dx + N(x, y) dy = 0
$$

with:

$$
M(x, y) = 4 e^{2y} \cos y
$$

$$
N(x, y) = 26
$$

Given that, our goal is to determine whether this equation is exact.

Defining Exact Differential Equations

An ordinary differential equation (ODE) of the form:

$$
M(x, y) dx + N(x, y) dy = 0
$$

is called exact if there exists a function $\Psi(x, y)$ such that:

\[

$$
\frac{\partial \Psi}{\partial x} = M(x, y)
$$

$$
\frac{\partial \Psi}{\partial y} = N(x, y)
$$

This condition implies that the differential form $M dx + N dy$ is an exact differential of some potential function $\Psi(x, y)$.

Criterion for Exactness:

$$
\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}
$$

If this condition holds, then the differential equation is exact, and its solution can be found by integrating M with respect to x and N with respect to y.

Step 1: Verify Exactness of the Equation

Given the proposed functions:

$$
M(x, y) = 4 e^{2y} \cos y
$$

$$
N(x, y) = 26
$$

Note that N is a constant, independent of x and y.

Calculate $\partial M / \partial y$:

$$
\frac{\partial M}{\partial y} = 4 \frac{\partial}{\partial y} \left(e^{2y} \cos y \right)
$$

Applying the product rule:

$$
\frac{\partial}{\partial y} \left(e^{2y} \cos y \right) = e^{2y} \cdot 2 \cos y + e^{2y} \cdot (-\sin y) = e^{2y} (2 \cos y - \sin y)
$$

Therefore:

$$
\frac{\partial M}{\partial y} = 4 \times e^{2y} (2 \cos y - \sin y) = 4 e^{2y} (2 \cos y - \sin y)
$$

Calculate $\partial N / \partial x$:

Since $N = 26$ (a constant),

$$
\frac{\partial N}{\partial x} = 0
$$

Compare the derivatives:

$$
\frac{\partial M}{\partial y} = 4 e^{2y} (2 \cos y - \sin y)
$$

$$
\frac{\partial N}{\partial x} = 0
$$

Since these are not equal, the differential equation is not exact in its current form.

Step 2: Is There a Way to Make It Exact? — Using an Integrating Factor

When a differential equation isn't exact, it might be made exact by multiplying through by an integrating factor $\mu(x, y)$, which can depend on x, y, or both.

Common strategies:

- Integrating factor depending on x: $\mu(x)$
- Integrating factor depending on y: $\mu(y)$
- Integrating factor depending on both x and y: more complex, less common

Criteria for integrating factors:

- For $\mu(y)$:

$$
\frac{\partial}{\partial y} \left(\frac{1}{\mu(y)} M \right) = \frac{\partial}{\partial x} \left(\frac{1}{\mu(y)} N
$$

\right)

\]

- For \(\mu(x)\):

\[

\frac{\partial}{\partial y} \left(\frac{1}{\mu(x)} M \right) = \frac{\partial}{\partial x} \left(\frac{1}{\mu(x)} N \right)

\]

Given the form, testing for an integrating factor depending solely on \(y\) seems promising, because \(N\) is constant.

Step 3: Applying an Integrating Factor \(\mu(y)\)

Assuming \(\mu(y)\) exists such that:

\[

\tilde{M}(x, y) = \mu(y) M(x, y)

\]

\[

\tilde{N}(x, y) = \mu(y) N(x, y)

\]

and the new differential equation:

\[

\tilde{M}(x, y) dx + \tilde{N}(x, y) dy = 0

\]

is exact, i.e.,

$$
\frac{\partial \tilde{M}}{\partial y} = \frac{\partial \tilde{N}}{\partial x}
$$

Calculating:

$$
\frac{\partial \tilde{M}}{\partial y} = \frac{\partial}{\partial y} \left(\mu(y) M \right) = \mu'(y) M + \mu(y) \frac{\partial M}{\partial y}
$$

$$
\frac{\partial \tilde{N}}{\partial x} = \frac{\partial}{\partial x} \left(\mu(y) N \right) = 0
$$

since $\mu(y)$ depends only on y and $N = 26$ is constant.

Thus, the exactness condition becomes:

$$
\mu'(y) M + \mu(y) \frac{\partial M}{\partial y} = 0
$$

or,

$$
\frac{\mu'(y)}{\mu(y)} = - \frac{\partial M / \partial y}{M}
$$

\]

Calculating:

\[
\frac{\partial M}{\partial y} = 4 e^{2y} (2 \cos y - \sin y)
\]
\[
M = 4 e^{2y} \cos y
\]

Therefore:

\[
\frac{\partial M / \partial y}{M} = \frac{4 e^{2y} (2 \cos y - \sin y)}{4 e^{2y} \cos y} = \frac{2 \cos y - \sin y}{\cos y}
\]

Simplify numerator:

\[
\frac{2 \cos y - \sin y}{\cos y} = 2 - \tan y
\]

Hence,

\[
\frac{\mu'(y)}{\mu(y)} = - (2 - \tan y) = -2 + \tan y
\]

This is a separable ODE for \(\mu(y)\):

\[

\frac{d \mu}{dy} = \left(-2 + \tan y \right) \mu(y)

\]

which can be written as:

\[

\frac{1}{\mu(y)} d \mu = \left(-2 + \tan y \right) dy

\]

Integrate both sides:

\[

\int \frac{1}{\mu} d \mu = \int (-2 + \tan y) dy

\]

\[

\ln |\mu| = -2 y + \int \tan y\, dy + C

\]

Recall:

\[

\int \

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