Find A3, A4, And A5.a = -4a2 = 5an = -an-1- 2an - 2Write Your Answers As Integers Or Fractions In Simplest

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Understanding recursive sequences is fundamental in algebra and mathematical analysis. The sequence defined by the relation $a_n = -a_{n-1} - 2a_n$ and the initial conditions provided can seem complex at first glance. However, by carefully analyzing the recurrence relation and applying systematic methods, we can determine the specific terms $a_3, a_4,$ and a_5. This article aims to guide you through the process step-by-step, providing clarity on solving such recurrence relations and illustrating how to write the answers as simplified fractions or integers.

Understanding the Recurrence Relation

Deciphering the Given Expression

The initial statement appears somewhat ambiguous. It reads:
Find A3, A4, And A5.a = -4a2 = 5an = -an-1- 2an - 2Write Your Answers As Integers Or Fractions In Simplest.

Interpreting this, it is likely that the sequence is defined by a recurrence relation involving previous terms, specifically:

$$a_n = -a_{n-1} - 2a_{n-2}$$

with initial terms a_0 and a_1 given or to be determined.

Alternatively, the phrase might suggest that the sequence satisfies a relation similar to:

$$a_n = -a_{n-1} - 2a_{n-2}$$

which is a common linear recurrence.

Given the context and typical sequence problems, this is a reasonable assumption.

Step 1: Clarify the Initial Conditions

Before solving for specific terms, we need initial terms a_0 and a_1. If these are not provided explicitly, they are often assumed to be arbitrary or are part of the problem to find general solutions.

Suppose the problem provides:

$$a_0 = 1, \quad a_1 = 2$$

If initial conditions differ, they can be substituted similarly.

Step 2: Write the Recurrence Relation

Based on the interpretation, the recurrence relation is:

$$
a_n = -a_{n-1} - 2a_{n-2}
$$

This is a second-order linear recurrence relation, which can be solved systematically.

Step 3: Find the Characteristic Equation

To solve the recurrence relation, we assume solutions of the form:

$$
a_n = r^n
$$

Substituting into the recurrence:

$$
r^n = - r^{n-1} - 2 r^{n-2}
$$

Dividing both sides by r^{n-2} (assuming $r \neq 0$):

$$
r^2 = - r - 2
$$

Rearranged:

$$
r^2 + r + 2 = 0
$$

This quadratic equation is the characteristic equation.

Step 4: Solve the Characteristic Equation

Solve for r:

$$
r^2 + r + 2 = 0
$$

Using the quadratic formula:

$$
r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

where $a=1, b=1, c=2$:

$$
r = \frac{-1 \pm \sqrt{1^2 - 4 \times 1 \times 2}}{2} = \frac{-1 \pm \sqrt{1 - 8}}{2} = \frac{-1 \pm \sqrt{-7}}{2}
$$

Since the discriminant is negative:

$$
\sqrt{-7} = i \sqrt{7}
$$

the roots are complex conjugates:

$$
r = \frac{-1 \pm i \sqrt{7}}{2}
$$

Expressed explicitly:

$$
r_1 = \frac{-1 + i \sqrt{7}}{2}
$$

$$
r_2 = \frac{-1 - i \sqrt{7}}{2}
$$

Step 5: Write the General Solution

For complex roots of the form:

$$
r = \alpha \pm i \beta
$$

the general solution to the recurrence is:

$$
a_n = \gamma_1 r_1^n + \gamma_2 r_2^n
$$

which can be expressed using Euler's formula as:

$$
a_n = 2^n \left[A \cos \left(\frac{\sqrt{7}}{2} n \right) + B \sin \left(\frac{\sqrt{7}}{2} n \right) \right]
$$

where A and B are constants determined by initial conditions.

Step 6: Determine Constants Using Initial Conditions

Suppose initial conditions are:

$$
a_0 = 1, \quad a_1 = 2
$$

At $n=0$:

$$
a_0 = A \times 1 + B \times 0 = A = 1
$$

At $n=1$:

$$
a_1 = 2^1 \left[A \cos \left(\frac{\sqrt{7}}{2} \times 1 \right) + B \sin \left(\frac{\sqrt{7}}{2} \times 1 \right) \right]
$$

$$
2 = 2 \left[1 \times \cos \left(\frac{\sqrt{7}}{2} \right) + B \times \sin \left(\frac{\sqrt{7}}{2} \right) \right]
$$

Divide both sides by 2:

\[

$$
1 = \cos \left(\frac{\sqrt{7}}{2} \right) + B \sin \left(\frac{\sqrt{7}}{2} \right)
$$

Solve for B:

$$
B = \frac{1 - \cos \left(\frac{\sqrt{7}}{2} \right)}{\sin \left(\frac{\sqrt{7}}{2} \right)}
$$

This provides the explicit form of the sequence terms.

Calculating Specific Terms $a_3, a_4,$ and a_5

Using the explicit formula, we can compute these terms:

$$
a_n = 2^n \left[\cos \left(\frac{\sqrt{7}}{2} n \right) + B \sin \left(\frac{\sqrt{7}}{2} n \right) \right]
$$

Computing a_3

$$
a_3 = 2^3 \left[\cos \left(\frac{\sqrt{7}}{2} \times 3 \right) + B \sin \left(\frac{\sqrt{7}}{2} \times 3 \right) \right]
$$

$$
a_3 = 8 \left[\cos \left(\frac{3 \sqrt{7}}{2} \right) + B \sin \left(\frac{3 \sqrt{7}}{2} \right) \right]
$$

Given the values of B and the trigonometric functions, this expression can be evaluated numerically or left in exact form.

Computing a_4

Similarly:

$$
a_4 = 2^4 \left[\cos \left(2 \sqrt{7} \right) + B \sin \left(2 \sqrt{7} \right) \right] = 16 \left[\cos (2 \sqrt{7}) + B \sin (2 \sqrt{7}) \right]
$$

Computing a_5

$$
a_5 = 2^5 \left[\cos \left(\frac{5 \sqrt{7}}{2} \right) + B \sin \left(\frac{5 \sqrt{7}}{2} \right) \right] = 32 \left[\cos \left(\frac{5 \sqrt{7}}{2} \right) + B \sin \left(\frac{5 \sqrt{7}}{2} \right) \right]
$$

Alternative Approach: Numerical Approximation

If the exact form is less critical, numerical approximations can be made:

- Calculate approximate values of $\cos \left(\frac{\sqrt{7}}{2} n \right)$ and $\sin \left(\frac{\sqrt{7}}{2} n \right)$.
- Substitute the value of B.
- Multiply by 2^n.

This method yields approximate decimal values for \(a_3, a

Frequently Asked Questions

Given the recurrence relation $a_n = -a_{n-1} - 2a_{n-2}$ with initial values $a_3 = -4$, find a_4.

$a_4 = 2$

Using the recurrence $a_n = 5a_{n-1}$ with $a_3 = -4$, what is a_5?

-500

Given the recurrence $a_n = -a_{n-1} - 2a_{n-2}$ and $a_3 = -4$, $a_4 = 10$, find a_5.

-22

If $a_3 = -4$ and $a_4 = 2$, find a_5 using the recurrence $a_n = -a_{n-1} - 2a_{n-2}$.

-6

Given the recurrence relation $a_n = 5a_{n-1}$ and $a_3 = -4$, compute a_4.

-20

With $a_3 = -4$ and $a_4 = 2$, find a_5 using the recurrence $a_n = -a_{n-1} - 2a_{n-2}$.

-6

Given $a_3 = -4$, $a_4 = 10$, and the recurrence $a_n = -a_{n-1} - 2a_{n-2}$, what is a_5?

-22

If $a_3 = -4$ and $a_4 = 2$, find a_5 when the recurrence is $a_n = 5a_{n-1}$.

-100

Determine a_4 and a_5 for the recurrence $a_n = -a_{n-1} - 2a_{n-2}$ with initial $a_3 = -4$, assuming $a_4 = 10$.

$a_4 = 10$, $a_5 = -22$

Additional Resources

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Understanding and solving recurrence relations is a fundamental aspect of discrete mathematics and algebra, especially when dealing with sequences. The problem at hand involves a sequence $\{a_n\}$ that is defined by a complex recurrence relation, and our goal is to find specific terms: A_3, A_4, and A_5. The relation appears to be given in a somewhat condensed and intertwined manner, which we will interpret carefully to derive a clear solution. This article will guide you through the process step-by-step, presenting the necessary concepts, mathematical techniques, and detailed calculations to arrive at the desired terms.

Deciphering the Given Relation

Before jumping into calculations, it is essential to interpret the notation and the relation correctly. The sequence is described by the relation:

$$a = -4a_2 = 5a_n = -a_{n-1} - 2a_n - 2$$

This notation suggests that the sequence $\{a_n\}$ is related through multiple equalities, possibly indicating proportional relationships or recursive formulas. However, in standard practice, such an expression can be confusing, so we need to clarify and rewrite it in a more conventional form.

Step 1: Clarify the Relation

Typically, recurrence relations are written as:

$$a_n = \text{function of previous terms}$$

Given the expression, it seems to involve the following components:

- a (possibly a constant or initial term)
- a_2 (the second term)
- a_n (the general term)
- a_{n-1} (the previous term)

The relation appears to be:

$$a = -4a_2 = 5a_n = -a_{n-1} - 2a_n - 2$$

which suggests a chain of equalities linking these quantities.

Step 2: Interpret the Chain

Assuming the intended meaning is:

$$a = -4a_2$$
$$a = 5a_n$$

$$ a = -a_{n-1} - 2a_n - 2 $$

This implies that:

$$ -4a_2 = 5a_n = -a_{n-1} - 2a_n - 2 $$

To proceed, we can interpret the sequence as satisfying the following relations:

- $a = -4a_2$ (connecting some constant a to a_2)
- $a = 5a_n$ (relating the same constant to the n-th term)
- $a = -a_{n-1} - 2a_n - 2$ (a recurrence relation involving a_{n-1} and a_n)

Given this, our primary focus is to find the explicit terms a_3, a_4, and a_5.

Establishing the Initial Conditions and Relations

To find specific terms, we need initial conditions or relations between terms that allow us to build subsequent terms.

Step 1: Express a_n in terms of a

From $a = 5a_n$, we have:

$$ a_n = \frac{a}{5} $$

Similarly, from $a = -4a_2$:

$$ a_2 = -\frac{a}{4} $$

Now, since a seems to be a common constant linking these quantities, we can choose a convenient value for a to simplify calculations, or express all terms relative to a.

Step 2: Derive the recurrence relation involving a_{n} and a_{n-1}

From the third equality:

$$ a = -a_{n-1} - 2a_n - 2 $$

Substituting $a_n = \frac{a}{5}$, we get:

$$ a = -a_{n-1} - 2 \left(\frac{a}{5} \right) - 2 $$

$$ a = -a_{n-1} - \frac{2a}{5} - 2 $$

Rearranged to solve for a_{n-1}:

$$ -a_{n-1} = a - \frac{2a}{5} - 2 $$

$$ -a_{n-1} = \left(1 - \frac{2}{5} \right)a - 2 $$

$$ -a_{n-1} = \frac{3}{5}a - 2 $$

$$ a_{n-1} = - \frac{3}{5}a + 2 $$

This expression links a_{n-1} to a.

Expressing Terms of the Sequence

Now, we need to find the specific terms a_3, a_4, and a_5. To do this systematically, consider the recurrence relation derived from the pattern.

Step 1: Identify the recurrence pattern

Given the relation involving a_{n-1} and a_n:

$$ a_{n-1} = - \frac{3}{5}a + 2 $$
$$ a_n = \frac{a}{5} $$

But this suggests that for each n, the sequence might be linear or at least follow a pattern that allows expressing higher terms recursively.

Step 2: Find initial terms

Let's assume that the sequence starts at a_1, but since the initial relation involves a_2, and we have an expression for a_2:

$$ a_2 = -\frac{a}{4} $$

Similarly, a_1 could be related to a as well. However, the problem doesn't specify a_1, so to proceed, we can assign a an arbitrary value for simplicity, or set it to 1 to find ratios.

Suppose we choose $a = 1$, then:

$$ a_n = \frac{1}{5} $$
$$ a_2 = - \frac{1}{4} $$
$$ a_{n-1} = - \frac{3}{5} \times 1 + 2 = - \frac{3}{5} + 2 = \frac{7}{5} $$

This gives us initial values:

- $a_2 = -\frac{1}{4}$
- $a_{n-1} = \frac{7}{5}$

But these are inconsistent unless we specify the exact n. Alternatively, consider that the sequence follows a linear recurrence relation of the form:

$$ a_n = p a_{n-1} + q $$

which is typical for sequences with constant coefficients.

Formulating and Solving the Recurrence Relation

Given the relation:

$$ a_{n-1} = - \frac{3}{5}a + 2 $$

and knowing $a_n = \frac{a}{5}$, the sequence appears to stabilize or follow a pattern.

Step 1: Derive a recurrence relation

Suppose the sequence satisfies:

$$ a_{n+1} = r a_n + s $$

Our task is to identify r and s.

From previous steps, the relation between a_{n} and a_{n-1} can be expressed as:

$$ a_{n} = p a_{n-1} + q $$

Given the earlier derivation:

$$ a_{n-1} = - \frac{3}{5}a + 2 $$

and

$$ a_{n} = \frac{a}{5} $$

If we accept that the sequence is linear, then the general form must be consistent for all n.

Step 2: Assume an explicit formula for a_n

Suppose the sequence satisfies:

$$ a_n = A r^{n-1} + K $$

where A and K are constants to be determined, and r is the common ratio.

Given the sequence's complexity, it might be more straightforward to assume a linear recurrence:

$$ a_{n} = p a_{n-1} + q $$

and attempt to find p and q.

Step 3: Use known terms to find p and q

Assuming initial terms:

- $a_2 = -\frac{a}{4}$
- $a_3 = ?$
- $a_4 = ?$
- $a_5 = ?$

We need to find these.

Calculating Specific Terms: Step-by-Step

Given the earlier relations, let's proceed with specific assumptions to compute a_3, a_4, and a_5.

Step 1: Choose a for simplicity

Let's set $a=1$. Then:

\[a_n = \frac{1

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