Find An Equation For The Plane Which Is Perpendicular To The Plane 2x- Y +3= = 6 And Pass Through 2 Points

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Understanding how to find the equation of a plane that is perpendicular to a given plane and passes through two specific points is a fundamental concept in analytic geometry. Whether you're a student preparing for exams or a professional dealing with spatial data, mastering this topic enhances your ability to analyze three-dimensional objects and solve complex geometric problems. This article provides a comprehensive step-by-step guide on how to determine such a plane, including the necessary mathematical background, detailed procedures, and practical examples.

Understanding the Basics of Planes in 3D Space

Before diving into the problem-solving process, it’s essential to review some core concepts related to planes in three-dimensional space.

Equation of a Plane

A plane in 3D space can typically be represented by the general equation:

$$ Ax + By + Cz + D = 0 $$

where:
- A, B, C are the coefficients that define the normal vector $\vec{n} = (A, B, C)$ of the plane.
- D is a constant.

Normal Vector to a Plane

The normal vector is perpendicular to the plane and is crucial for determining relationships between different planes. If you have the plane equation $Ax + By + Cz + D = 0$, then the vector $\vec{n} = (A, B, C)$ is perpendicular to every vector lying on the plane.

Perpendicular Planes

Two planes are perpendicular if their normal vectors are perpendicular, i.e., their dot product equals zero:

$$ \vec{n}_1 \cdot \vec{n}_2 = 0 $$

This property is vital when finding a plane perpendicular to a given plane.

Analyzing the Given Plane Equation

The problem provides the plane:

$$ 2x - y + 3 = 6 $$

To work with this equation effectively, rewrite it in the standard form:

$$ 2x - y + 3 - 6 = 0 $$
$$ 2x - y - 3 = 0 $$

Thus, the standard form is:

$$ 2x - y - 3 = 0 $$

From this, we identify the normal vector:

$$ \vec{n}_1 = (2, -1, 0) $$

because the coefficients of x, y, z are $2, -1, 0$ respectively.

Step 1: Find the Normal Vector of the Desired Plane

Since the new plane must be perpendicular to the given plane, their normal vectors must be perpendicular:

$$ \vec{n}_1 \cdot \vec{n}_2 = 0 $$

Let the normal vector of the new plane be:

$$ \vec{n}_2 = (A, B, C) $$

The perpendicularity condition becomes:

$$ (2, -1, 0) \cdot (A, B, C) = 0 $$
$$ 2A - B + 0 \times C = 0 $$
$$ 2A - B = 0 $$
$$ B = 2A $$

This relation constrains the components of the normal vector of the new plane.

Summary:
- The normal vector of the new plane is $\vec{n}_2 = (A, 2A, C)$, where A and C are parameters to be determined.

Step 2: Find the Equation of the New Plane Passing Through Two Points

To fully determine the plane, we need two points through which it passes, say:

$$ P_1 = (x_1, y_1, z_1) $$
$$ P_2 = (x_2, y_2, z_2) $$

The problem states "pass through 2 points," but does not specify which points. For demonstration, assume the points are:

$$ P_1 = (x_1, y_1, z_1) $$
$$ P_2 = (x_2, y_2, z_2) $$

(You can replace these with actual points from your specific problem.)

Step-by-step process:

1. Determine the vector between the points:

$$ \vec{v} = P_2 - P_1 = (x_2 - x_1, y_2 - y_1, z_2 - z_1) $$

2. Find the normal vector of the plane:

Since the plane passes through both points, the normal vector $\vec{n}_2$ must be perpendicular to the vector $\vec{v}$, because $\vec{v}$ lies in the plane.

Therefore:

$$ \vec{n}_2 \cdot \vec{v} = 0 $$

Using the earlier relation:

$$ \vec{n}_2 = (A, 2A, C) $$

We get:

$$ (A, 2A, C) \cdot (x_2 - x_1, y_2 - y_1, z_2 - z_1) = 0 $$

which simplifies to:

$$ A (x_2 - x_1) + 2A (y_2 - y_1) + C (z_2 - z_1) = 0 $$

Expressed as:

$$ A [(x_2 - x_1) + 2 (y_2 - y_1)] + C (z_2 - z_1) = 0 $$

This equation allows determining the ratio of A and C.

3. Choose specific values for A and C:

- Assign a value to A (say, $A=1$), then solve for C:

$$ [(x_2 - x_1) + 2 (y_2 - y_1)] + C (z_2 - z_1) = 0 $$

$$ C = - \frac{(x_2 - x_1) + 2 (y_2 - y_1)}{z_2 - z_1} $$

or vice versa, depending on the points.

Step 3: Write the Equation of the Plane

Once $\vec{n}_2 = (A, 2A, C)$ is known, the equation of the plane passing through point $P_1 = (x_1, y_1, z_1)$ is:

$$A(x - x_1) + 2A(y - y_1) + C(z - z_1) = 0$$

Or, explicitly:

$$A x + 2A y + C z + D = 0$$

where:

$$D = -A x_1 - 2A y_1 - C z_1$$

This gives the full plane equation.

Practical Example: Finding the Plane Perpendicular to $2x - y + 3 = 6$ Passing Through Given Points

Suppose the two points are:

$$P_1 = (1, 2, 3)$$
$$P_2 = (4, 0, -1)$$

and the given plane is:

$$2x - y + 3 = 6$$

which simplifies to:

$$2x - y - 3 = 0$$

Step 1: Calculate the vector between the points:

$$\vec{v} = (4 - 1, 0 - 2, -1 - 3) = (3, -2, -4)$$

Step 2: Find the ratio for A and C:

$$A [(3) + 2(-2)] + C(-4) = 0$$
$$A (3 - 4) + C (-4) = 0$$
$$A (-1) - 4 C = 0$$
$$-A = 4 C$$
$$C = - \frac{A}{4}$$

Choose $A=4$ for simplicity:

$$C = -1$$

Step 3: Write the normal vector:

$$\vec{n}_2 = (4, 8, -1)$$

Step 4: Find D:

$$ D = -A x_1 - 2A y_1 - C z_1 $$
$$ D = -4(1) - 8(2) - (-1)(3) $$
$$ D = -4 - 16 + 3 = -17 $$

Step 5: Final plane equation:

$$ 4(x - 1) + 8(y - 2) - 1(z - 3) = 0 $$

Expanding:

$$ 4x - 4 + 8y - 16 - z + 3 = 0 $$

Simplify:

$$ 4x + 8y - z - 17 = 0 $$

This is the equation of the plane perpendicular to the original plane and passing through the points P_1 and P_2

Frequently Asked Questions

How do I find the equation of a plane perpendicular to the plane 2x - y + 3 = 6?

First, identify the normal vector of the given plane, which is (2, -1, 0). Since the new plane is perpendicular to it, its normal vector must be perpendicular to (2, -1, 0). You can choose any vector perpendicular to this, then use a point to write the plane equation.

What is the normal vector of the plane 2x - y + 3 = 6?

The normal vector of the plane 2x - y + 3 = 6 is (2, -1, 0).

How do I determine the normal vector of a plane perpendicular to a given plane?

The normal vector of the perpendicular plane must be orthogonal to the original plane's normal vector. Find a vector that is perpendicular to (2, -1, 0) by solving their dot product equal to zero.

How do I find a point that the new plane passes through if it passes through two points?

To find the equation of a plane passing through two points, you need at least three points. If the plane passes through two points, you need a third point or a specific point to define the plane, or additional conditions such as the plane being perpendicular to another plane.

Can a plane pass through two points and be

perpendicular to a given plane?

Yes, but you need to specify the third point or the normal vector of the plane. If the plane is perpendicular to the given plane, its normal vector is perpendicular to the original plane's normal vector, allowing you to determine the plane's equation passing through the given points.

What steps are involved in finding the equation of a plane perpendicular to a given plane and passing through two points?

1. Find the normal vector of the original plane. 2. Determine the normal vector of the desired plane, which must be perpendicular to the original's normal vector. 3. Use the two points and the normal vector to form the plane equation. 4. Find the specific equation by substituting the points into the general form.

How do I write the equation of a plane with a given normal vector passing through specific points?

Use the point-normal form of a plane: $(x - x_0)N_1 + (y - y_0)N_2 + (z - z_0)N_3 = 0$, where (x_0, y_0, z_0) is a point on the plane, and $N = (N_1, N_2, N_3)$ is the normal vector. Plug in the point coordinates and the normal vector to get the plane equation.

Additional Resources

Find An Equation For The Plane Which Is Perpendicular To The Plane 2x - y + 3 = 6 And Pass Through 2 Points is a classic problem in vector and analytical geometry that tests one's understanding of planes, their equations, and how to manipulate them based on given conditions. This problem combines the concepts of plane equations, perpendicularity, and passing through specific points, making it an excellent example of the practical application of three-dimensional geometry principles. In this comprehensive review, we will explore the problem statement, underlying concepts, step-by-step solution methods, common mistakes, and tips to master such problems.

Understanding the Problem

Before diving into the solution, it's essential to understand what the problem entails:

- You are given a plane: $2x - y + 3 = 6$.
- You need to find a new plane that:
- is perpendicular to the given plane,
- passes through two specific points (although these points are not explicitly provided here, the methodology applies generally once the points are known).

The core of the problem involves understanding the properties of planes, the concept of perpendicularity in three dimensions, and how to formulate equations based on these properties.

Key Concepts and Theoretical Background

1. Equation of a Plane

A plane in three-dimensional space can be represented in the form:

$$ ax + by + cz + d = 0 $$

where (a, b, c) is the normal vector to the plane, and d is a scalar.

Given the plane $2x - y + 3 = 6$, it can be rewritten as:

$$ 2x - y + 3 - 6 = 0 \Rightarrow 2x - y - 3 = 0 $$

Thus, the normal vector to this plane is $\mathbf{n}_1 = (2, -1, 0)$.

2. Perpendicular Planes

Two planes are perpendicular if their normal vectors are perpendicular, i.e., their dot product is zero:

$$ \mathbf{n}_1 \cdot \mathbf{n}_2 = 0 $$

where $\mathbf{n}_2$ is the normal vector of the new plane.

3. Passing Through Points

A plane passing through two points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ can be found if we know a normal vector $\mathbf{n}_2$. The general form of the plane is:

$$ a(x - x_0) + b(y - y_0) + c(z - z_0) = 0 $$

where (x_0, y_0, z_0) is a point on the plane (e.g., P_1), and (a, b, c) is the normal vector.

Step-by-Step Approach to the Solution

Step 1: Identify the Normal Vector of the Given Plane

As established, the normal vector to the plane $2x - y - 3 = 0$ is:

$$ \mathbf{n}_1 = (2, -1, 0) $$

Step 2: Find the Normal Vector of the Required Plane

Since the new plane is perpendicular to the given plane, its normal vector $\mathbf{n}_2 = (a, b, c)$ must satisfy:

$$\mathbf{n}_1 \cdot \mathbf{n}_2 = 0$$

which implies:

$$2a + (-1)b + 0 \times c = 0 \Rightarrow 2a - b = 0 \Rightarrow b = 2a$$

Thus, $\mathbf{n}_2 = (a, 2a, c)$, where $a \neq 0$, and c is any real number.

To simplify, choose $a = 1$, then:

$$\mathbf{n}_2 = (1, 2, c)$$

with c being any real number.

Step 3: Use the Points to Find the Plane Equation

Suppose the two points through which the plane passes are $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$. The plane must pass through both points, so the equation must satisfy:

$$a(x_1 - x_0) + b(y_1 - y_0) + c(z_1 - z_0) = 0$$
$$a(x_2 - x_0) + b(y_2 - y_0) + c(z_2 - z_0) = 0$$

Alternatively, the plane's general form can be written once a point on the plane is chosen, say P_1, as:

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

Given the normal vector $(1, 2, c)$, the plane's equation passing through P_1 is:

$$1(x - x_1) + 2(y - y_1) + c(z - z_1) = 0$$

or,

$$(x - x_1) + 2(y - y_1) + c(z - z_1) = 0$$

Similarly, check for the second point to ensure the plane passes through it.

Handling the Variable c and Completing the Equation

Since c is arbitrary, the plane's equation depends on choosing a specific value for c. The simplest approach is to select c based on additional constraints or to keep c as a parameter.

Suppose the two points are:

- $P_1(1, 2, 3)$
- $P_2(4, 5, 6)$

Using these points:

- Equation through P_1:

$$(x - 1) + 2(y - 2) + c(z - 3) = 0$$
$$x - 1 + 2y - 4 + c z - 3c = 0$$
$$x + 2y + c z = 1 + 4 + 3c$$
$$x + 2y + c z = 5 + 3c$$

- Equation through P_2:

$$(x - 4) + 2(y - 5) + c(z - 6) = 0$$
$$x - 4 + 2y - 10 + c z - 6c = 0$$
$$x + 2y + c z = 4 + 10 + 6c$$
$$x + 2y + c z = 14 + 6c$$

For the plane to pass through both points, the right sides must be equal:

$$5 + 3c = 14 + 6c$$
$$5 + 3c = 14 + 6c$$
$$5 - 14 = 6c - 3c$$
$$-9 = 3c$$
$$c = -3$$

Substituting $c = -3$ back into the plane equation:

$$x + 2y - 3z = 5 + 3(-3) = 5 - 9 = -4$$

Thus, the required plane's equation is:

$$x + 2y - 3z = -4$$

This plane is perpendicular to the initial plane $2x - y - 3 = 0$, as their normal vectors are perpendicular:

- $\mathbf{n}_1 = (2, -1, 0)$
- $\mathbf{n}_2 = (1, 2, -3)$

Check dot product:

$$2 \times 1 + (-1) \times 2 + 0 \times (-3) = 2 - 2 + 0 = 0$$

which confirms perpendicularity.

Summary of the Methodology

- Find the normal vector of the given plane.
- Use the perpendicularity condition (dot product = 0) to determine possible normal vectors for the new plane.
- Use the given points to derive the specific plane equation.
- Ensure the plane passes through both points, leading to solving for any parameters involved.
- Confirm the perpendicularity condition via dot product of the normals.

Features and Pros/Cons of the Approach

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Features:
- Utilizes fundamental vector operations like dot product.
- Applies algebraic manipulation to determine the specific plane.
- Generalizable for any points and planes provided.

Pros:
- Systematic and straightforward once the concepts are clear.
- Allows for flexibility with parameters like \( c \).
- Can handle multiple points and constraints efficiently.

Cons:
- Requires careful algebraic calculations, prone to
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