Find The Equation Of The Line That Is Perpendicular To The Given Line And Passes Through The Given Point.y

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When working with lines in coordinate geometry, a common problem involves determining the equation of a line that meets specific conditions. One such problem is to find the equation of a line that is perpendicular to a given line and passes through a particular point. This task is fundamental in various fields such as engineering, physics, and mathematics, and mastering it enhances your understanding of slopes, equations, and geometric relationships.

In this comprehensive guide, we will explore the steps and concepts necessary to find the equation of a perpendicular line passing through a given point. We will cover the essential theory, practical methods, and examples to ensure you can confidently approach and solve this problem.

Understanding the Basics: Lines, Slopes, and Perpendicularity

Before delving into the solution process, it's crucial to understand some foundational concepts related to lines in coordinate geometry.

1. The Equation of a Line

The most common form to represent a line in a coordinate plane is the slope-intercept form:

$$ y = mx + b $$

where:
- m is the slope of the line,
- b is the y-intercept (the point where the line crosses the y-axis).

2. Slope of a Line

The slope (m) indicates the steepness and direction of the line. It is calculated as:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

for two points (x_1, y_1) and (x_2, y_2) on the line.

3. Perpendicular Lines and Their Slopes

Two lines are perpendicular if they intersect at right angles (90 degrees). The slopes of perpendicular lines are negative reciprocals of each other:

$$m_1 \times m_2 = -1$$

or equivalently:

$$m_2 = -\frac{1}{m_1}$$

where:
- m_1 is the slope of the original line,
- m_2 is the slope of the perpendicular line.

Step-by-Step Process to Find the Perpendicular Line Equation

The process involves several key steps, which are outlined below:

1. Determine the slope of the given line

- If the equation of the given line is in slope-intercept form $y = mx + b$, identify m directly.
- If the equation is in standard form $Ax + By = C$, convert it to slope-intercept form or directly compute the slope as:

$$m = -\frac{A}{B}$$

- If the line is given through two points, calculate the slope using the two points.

2. Calculate the slope of the perpendicular line

- Use the negative reciprocal of the original line’s slope:

$$ m_{perp} = -\frac{1}{m} $$

- Remember: if $m = 0$ (horizontal line), the perpendicular line will be vertical with an undefined slope.

3. Use the point-slope form to find the line's equation

Once you have the slope of the perpendicular line and the point it passes through (say, (x_0, y_0)), use the point-slope form:

$$ y - y_0 = m_{perp} (x - x_0) $$

This form is convenient for deriving the equation directly.

4. Simplify to slope-intercept or standard form

- Expand and simplify the equation into the preferred form for clarity or specific application needs.

Practical Examples of Finding the Perpendicular Line Equation

To solidify understanding, consider the following real-world examples.

Example 1: Find the perpendicular line passing through a point when the original line is given in slope-intercept form

Suppose the original line is:

$$ y = 2x + 3 $$

and the point the perpendicular line passes through is $(4, 1)$.

Step 1: Find the slope of the original line:

$$ m = 2 $$

Step 2: Find the slope of the perpendicular line:

$$ m_{perp} = -\frac{1}{2} $$

Step 3: Use point-slope form with $(x_0, y_0) = (4, 1)$:

$$ y - 1 = -\frac{1}{2} (x - 4) $$

Step 4: Simplify:

$$ y - 1 = -\frac{1}{2}x + 2 $$

$$ y = -\frac{1}{2}x + 3 $$

This is the equation of the line perpendicular to $y = 2x + 3$ passing through $(4, 1)$.

Example 2: When the original line is in standard form and passing through a point

Given line:

$$ 3x - 4y = 12 $$

and point $(2, -1)$.

Step 1: Find the slope of the original line:

$$ m = -\frac{A}{B} = -\frac{3}{-4} = \frac{3}{4} $$

Step 2: Find the perpendicular slope:

$$ m_{perp} = -\frac{1}{m} = -\frac{1}{\frac{3}{4}} = -\frac{4}{3} $$

Step 3: Use point-slope form:

$$ y - (-1) = -\frac{4}{3} (x - 2) $$

$$ y + 1 = -\frac{4}{3}x + \frac{8}{3} $$

Step 4: Simplify:

$$ y = -\frac{4}{3}x + \frac{8}{3} - 1 $$

$$ y = -\frac{4}{3}x + \frac{8}{3} - \frac{3}{3} $$

$$ y = -\frac{4}{3}x + \frac{5}{3} $$

This is the required perpendicular line passing through $ (2, -1) $.

Special Cases and Considerations

While following the steps above, you might encounter some special cases:

1. Original line is horizontal

- Slope $ m = 0 $
- Perpendicular line is vertical, which cannot be expressed as a function $ y = mx + b $, but can be written as:

$$ x = x_0 $$

where $ x_0 $ is the x-coordinate of the given point.

2. Original line is vertical

- Slope is undefined.
- The perpendicular line is horizontal, with equation:

$$ y = y_0 $$

passing through the given point (x_0, y_0).

3. The given line is already perpendicular to the x- or y-axis

- Handle these cases carefully by recognizing the nature of the slopes and the line orientations.

Additional Tips for Accurate Line Equation Calculation

- Always verify the slope before computing the negative reciprocal.
- When simplifying fractions, reduce to simplest form.
- Double-check the point substitution to ensure the line passes through the specified point.
- Use graphing tools or software for visual confirmation when necessary.

Conclusion

Finding the equation of a line perpendicular to a given line and passing through a specific point is a fundamental skill in coordinate geometry. By understanding the relationship between slopes, utilizing the point-slope form, and carefully simplifying, you can solve such problems efficiently. Whether the original line is given in slope-intercept form, standard form, or through points, the method remains consistent—calculate the original slope, find its negative reciprocal, and then use the point-slope form to derive the new line's equation.

Mastering this process not only enhances your analytical skills but also prepares you for more complex geometric problems involving angles, distances, and transformations. Practice with diverse examples to build confidence and ensure accuracy in your solutions.

Keywords for SEO Optimization:
Find Equation of Perpendicular Line, Perpendicular Line Equation, Slope of Line, Negative Reciprocal, Coordinate Geometry, Line Passing Through Point, Line Equation Standard Form, Slope-Intercept Form, Geometry Problems, Mathematical Solutions

Frequently Asked Questions

How do I find the equation of a line perpendicular to a given line and passing through a specific point?

First, determine the slope of the given line. Then, find the negative reciprocal of that slope to get the perpendicular slope. Finally, use the point-slope form with the given point to write the equation.

What is the step-by-step process to find a perpendicular line passing through a point?

Identify the slope of the original line, invert and change its sign to get the perpendicular slope, then substitute this slope and the given point into the point-slope form to find the equation.

How do I find the slope of a given line from its equation?

For an equation in standard form $Ax + By = C$, solve for y to get slope-intercept form $y = mx + b$. The coefficient of x (after solving) is the slope m.

What is the point-slope form of a line equation?

The point-slope form is written as $y - y_1 = m(x - x_1)$, where (x_1, y_1) is a point on the line and m is the slope.

Can you provide an example of finding a perpendicular line passing through a point?

Suppose the given line is $y = 2x + 3$, and the point is (4, 1). The slope of the given line is 2. The perpendicular slope is -1/2. Using point-slope form: $y - 1 = -1/2(x - 4)$. Simplify to get the equation.

What if the given line is vertical or horizontal? How do I find the perpendicular line?

If the line is vertical ($x = k$), the perpendicular line is horizontal ($y = k_2$). If the line is horizontal ($y = c$), the perpendicular line is vertical ($x = c_2$).

How do I convert the equation of the perpendicular line into slope-intercept form?

Start with the point-slope form, then solve for y by distributing and isolating y on one side to write the equation as $y = mx + b$.

What common mistakes should I avoid when finding perpendicular lines through a point?

Avoid mixing up slopes, forgetting to invert the slope for perpendicularity, and not substituting the correct point into the equation. Also, ensure proper algebraic manipulation when simplifying.

How does the concept of perpendicular lines relate to their slopes?

Perpendicular lines have slopes that are negative reciprocals of each other. If one line has slope m, the perpendicular line's slope is -1/m.

Additional Resources

Find The Equation Of The Line That Is Perpendicular To The Given Line And Passes Through The Given Point: A Comprehensive Guide

In the realm of coordinate geometry, the problem of finding a line that is perpendicular to a given line and passes through a specific point is a fundamental yet nuanced task. It combines understanding of slopes, equations of lines, and the geometric relationships that govern perpendicularity. This comprehensive guide aims to delve deeply into this process, providing clarity, step-by-step procedures, key concepts, and illustrative examples.

Understanding the Core Concepts

Before tackling the problem, it’s essential to grasp the foundational principles involved:

1. Equations of Lines in the Coordinate Plane

- Slope-Intercept Form ($y = mx + b$):
The most common way to express a line, where
- m is the slope, indicating the steepness and direction of the line
- b is the y-intercept, the point where the line crosses the y-axis

- Point-Slope Form ($y - y_1 = m(x - x_1)$):
Used when a point \((x_1, y_1)\) on the line is known, along with the line’s slope.

- Standard Form (Ax + By + C = 0):
An alternative form often used in algebraic manipulations.

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2. Determining the Slope of the Given Line

- If the equation of the given line is known, identify its slope directly from the form.

- For example, if the line is given as $y = 2x + 3$, the slope $m_1 = 2$.

- If the line is in standard form, e.g., $3x - 4y + 7 = 0$, then rearranged to slope-intercept form to find slope or directly use the coefficients.

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3. Perpendicular Lines and Their Slopes

- Key Principle:
The slope of a line perpendicular to another is the negative reciprocal of the original line’s slope.

- Mathematically:
If the original line has slope m_1, then the perpendicular line has slope m_2, where:

$$
m_2 = -\frac{1}{m_1}
$$

- Special cases:
- If $m_1 = 0$ (horizontal line), then the perpendicular line is vertical, with an undefined slope.
- If the original line is vertical (slope undefined), the perpendicular line is horizontal (slope zero).

Step-by-Step Approach to Find the Equation of the Perpendicular Line

The process can be systematically broken down into clear steps:

Step 1: Identify the Equation and Slope of the Given Line

- Extract the slope m_1 from the given line.

- Example: Given line $y = 3x + 4$, then $m_1 = 3$.

- Special case: If given in standard form, e.g., $2x - y + 5 = 0$, rearranged as $y = 2x + 5$, so $m_1 = 2$.

Step 2: Calculate the Slope of the Perpendicular Line

- Use the negative reciprocal relationship:

$$
m_2 = -\frac{1}{m_1}
$$

- Example: For $m_1 = 3$, then $m_2 = -\frac{1}{3}$.

- Special cases:
- If $m_1 = 0$, then m_2 is undefined (vertical line).
- If the original line is vertical (m_1 undefined), then the perpendicular line has a slope of 0 (horizontal).

Step 3: Use the Given Point to Write the Equation of the Perpendicular Line

- Suppose the point through which the line passes is (x_0, y_0).

- Use the point-slope form:

\[

$$
y - y_0 = m_2 (x - x_0)
$$

- Example: Given point $(2, 5)$ and the perpendicular slope $m_2 = -\frac{1}{3}$:

$$
y - 5 = -\frac{1}{3} (x - 2)
$$

Step 4: Simplify to the Desired Form

- Convert the point-slope form to slope-intercept form or standard form as needed.

- Example continuation:

$$
y - 5 = -\frac{1}{3}x + \frac{2}{3}
$$

$$
y = -\frac{1}{3}x + \frac{2}{3} + 5
$$

$$
y = -\frac{1}{3}x + \frac{2}{3} + \frac{15}{3} = -\frac{1}{3}x + \frac{17}{3}
$$

- The final equation:

$$
y = -\frac{1}{3}x + \frac{17}{3}
$$

Special and Edge Cases

Certain scenarios require particular attention:

1. When the Given Line is Horizontal (m = 0)

- The perpendicular line is vertical, with an equation of the form:

$$
x = x_0
$$

- To find this line:
- Use the given point's x-coordinate as the equation.

- Example: Given line $y = 4$ (horizontal), passing through $(3, 2)$:

$$
x = 3
$$

---

2. When the Given Line is Vertical (slope undefined)

- The perpendicular line is horizontal:

$$
y = y_0
$$

- Example: Given line $x = 5$, passing through $(2, -1)$:

$$
y = -1
$$

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3. When the Slope of the Given Line is Rational or Integer

- Calculations involve simple reciprocals, but always be cautious with negative signs.

Practical Examples and Illustrations

Let's work through several illustrative examples to reinforce understanding.

Example 1: Line with a Known Equation

- Given: Line $y = -2x + 3$, point $(4, 1)$.

- Find: Equation of the line perpendicular to the given line passing through $(4, 1)$.

- Solution:

1. Slope of given line: $m_1 = -2$.

2. Slope of perpendicular line:

$$
m_2 = -\frac{1}{-2} = \frac{1}{2}
$$

3. Use point-slope form with $(x_0, y_0) = (4, 1)$:

$$
y - 1 = \frac{1}{2}(x - 4)
$$

4. Simplify:

$$
y - 1 = \frac{1}{2}x - 2
$$

$$
y = \frac{1}{2}x - 2 + 1 = \frac{1}{2}x - 1
$$

- Final equation:

$$
y = \frac{1}{2}x - 1
$$

Example 2: Vertical Original Line

- Given: Line $x = 7$, point $(3, 5)$.

- Find: Perpendicular line passing through $(3, 5)$.

- Solution:

- Since the original line is vertical, the perpendicular line is horizontal.

$$
y = y_0 = 5
$$

- Answer:

$$
y = 5
$$

Applications and Real-World Relevance

Understanding how to find perpendicular lines is crucial in many practical applications:

- Engineering and Design:
Ensuring components are perpendicular for structural stability.

- Navigation and Robotics:
Calculating paths that are orthogonal to existing trajectories.

- Physics:
Analyzing force components and directions.

- Computer Graphics:
Aligning objects at right angles.

- Mathematical Modeling:
Establishing coordinate relationships with perpendicular constraints.

Common Mistakes and How to Avoid Them

While solving these problems, students often encounter pitfalls. Awareness of these can streamline the process:

- Incorrectly identifying the slope:
Always verify the form of the line and accurately extract the slope.

- Ignoring special cases:

[Find The Equation Of The Line That Is Perpendicular To The Given Line And Passes Through The Given Point Y]

Find The Equation Of The Line That Is Perpendicular To The Given Line And Passes Through The Given Point Y

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