Find The General Solution.(a) Y" +4y' + 4y = E-x Cos X(b) (3D2+27I)y = 3 Cos X + Cos 3x(c) (D + 2D +3/4I)y

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Understanding how to find the general solution of differential equations is fundamental in advanced mathematics, physics, engineering, and many applied sciences. Whether dealing with ordinary differential equations (ODEs) with constant coefficients, nonhomogeneous equations, or higher-order equations, mastering the techniques to derive the general solutions enables us to model and analyze complex systems effectively. In this article, we will explore three specific differential equations, each illustrating a different approach to solving linear differential equations with constant and variable coefficients. We will systematically break down the steps involved, provide detailed solutions, and highlight key concepts and methods for solving such equations.

Understanding the Problem Statement

The initial problem involves three differential equations, labeled (a), (b), and (c):

1. Equation (a): $Y'' + 4Y' + 4Y = E - x \cos x$
2. Equation (b): $(3D^2 + 27I) y = 3 \cos x + \cos 3x$
3. Equation (c): $(D + 2D + 3/4 I) y$

Let's clarify each:

- Equation (a): A second-order linear nonhomogeneous differential equation with constant coefficients, involving a polynomial times a trigonometric function as the nonhomogeneous part.
- Equation (b): An operator form involving D, the differentiation operator, with a right-hand side composed of cosine functions.
- Equation (c): An operator form that appears to be a third-order differential equation with constant coefficients.

Our goal is to find the general solution for each of these equations, which involves:

- Solving the homogeneous part
- Finding a particular solution to the nonhomogeneous part
- Combining both to get the overall general solution

General Solution of Differential Equations: Key Concepts

Before diving into each specific problem, it is essential to understand the main techniques involved:

1. Homogeneous Solution (y_h)

- Solve the associated homogeneous equation (set RHS to zero).
- Find the roots of the characteristic equation.
- Write the general homogeneous solution based on root types (real, repeated, complex conjugates).

2. Particular Solution (y_p)

- Use methods such as undetermined coefficients or variation of parameters.
- Tailor the approach based on the form of the nonhomogeneous term.

3. General Solution (y)

- Sum of homogeneous and particular solutions:

$$
y = y_h + y_p
$$

Solution to Equation (a): $Y'' + 4Y' + 4Y = E - x \cos x$

Step 1: Homogeneous Equation

Solve:

$$
Y'' + 4Y' + 4Y = 0
$$

- Write the characteristic equation:

$$
r^2 + 4r + 4 = 0
$$

- Factor or use quadratic formula:

$$
(r + 2)^2 = 0

\]
- Roots:
\[
r = -2 \quad \text{(repeated roots)}
\]

- Homogeneous solution:
\[
Y_h = (A + Bx) e^{-2x}
\]

Step 2: Particular Solution

The RHS is \(E - x \cos x \). Since the RHS combines a constant and a product of polynomial and trig function, we split the particular solution into two parts:

- For \(E \), a constant, try \(Y_{p1} = C \)
- For \(-x \cos x \), which is more complex, we use the method of undetermined coefficients, considering the form of the nonhomogeneous term.

Because \(x \cos x \) is involved, and the associated homogeneous solution involves exponentials, the particular solution can be guessed as:

\[
Y_{p2} = x (P \cos x + Q \sin x) + R \cos x + S \sin x
\]

But since the RHS is \(-x \cos x \), it's better to consider:

\[
Y_p = x (P \cos x + Q \sin x)
\]

Adjusting for the polynomial factor, the full particular solution:

\[
Y_p = C + x (P \cos x + Q \sin x)
\]

Plugging into the differential equation and solving for \(P, Q, C \) is detailed but straightforward with systematic derivatives.

Note: For brevity, the detailed substitution process involves differentiating \(Y_p \), substituting into the differential equation, and equating coefficients for the terms involving \(\cos x \), \(\sin x \), and constants to solve for the unknowns.

Step 3: Final Solution

The general solution is:

$$
Y(x) = Y_h + Y_{p1} + Y_{p2}
$$

where each part is explicitly determined through algebraic calculations.

Solution to Equation (b): $(3D^2 + 27I) y = 3 \cos x + \cos 3x$

Step 1: Rewrite the Differential Operator Equation

The operator form:

$$
(3D^2 + 27) y = 3 \cos x + \cos 3x
$$

can be simplified as:

$$
3 \frac{d^2 y}{dx^2} + 27 y = 3 \cos x + \cos 3x
$$

or

$$
\frac{d^2 y}{dx^2} + 9 y = \cos x + \frac{1}{3} \cos 3x
$$

Step 2: Homogeneous Solution

Solve:

$$
\frac{d^2 y}{dx^2} + 9 y = 0
$$

- Characteristic equation:

$$
r^2 + 9 = 0
$$

- Roots:

$$
r = \pm 3i
$$

- Homogeneous solution:

$$
y_h = C_1 \cos 3x + C_2 \sin 3x
$$

Step 3: Particular Solution

The RHS consists of $\cos x$ and $\cos 3x$.

- For $\cos x$, guess:

$$
y_{p1} = A \cos x + B \sin x
$$

- For $\cos 3x$, guess:

$$
y_{p2} = D \cos 3x + E \sin 3x
$$

However, note that $\cos 3x$ is part of the homogeneous solution (since it appears in y_h), so for y_{p2}, we need to multiply by x to account for resonance:

$$
y_{p2} = x (D \cos 3x + E \sin 3x)
$$

Plugging these guesses into the differential equation and solving for coefficients yields the particular solution.

Step 4: General Solution

$$
y(x) = y_h + y_{p1} + y_{p2}
$$

This combines the homogeneous solution and particular solutions for each RHS term.

Solution to Equation (c): $(D + 2D + 3/4 I) y$

This expression appears to be an operator form, possibly intended as:

$$
(D + 2D + \frac{3}{4} I) y = \text{(some RHS)}
$$

or possibly a typo or shorthand notation for a differential operator. Assuming it is:

$$
(D^3 + 2 D^2 + \frac{3}{4} D) y = \text{RHS}
$$

which simplifies to:

$$
D^3 y + 2 D^2 y + \frac{3}{4} D y = \text{RHS}
$$

or

$$
\frac{d^3 y}{dx^3} + 2 \frac{d^2 y}{dx^2} + \frac{3}{4} \frac{dy}{dx} = \text{RHS}
$$

Assuming RHS is known, the method involves:

- Solving the homogeneous equation:
$$
\frac{d^3 y}{dx^3} + 2 \frac{d^2 y}{dx^2} + \frac{3}{4} \frac{dy}{dx} = 0
$$
- Find characteristic roots, which involves solving the characteristic polynomial:
$$
r^3 + 2 r^2 + \frac{3}{4} r = 0
$$
- Factor:
$$
r (r^2 + 2 r + \frac{3}{4}) = 0
$$
- Roots:
$$
r = 0
$$

Frequently Asked Questions

How do you find the general solution of the differential equation $Y'' + 4Y' + 4Y = e^{-x} \cos x$?

First, solve the homogeneous part: $Y'' + 4Y' + 4Y = 0$, which has characteristic equation $(r + 2)^2 = 0$, so the homogeneous solution is $Y_h = (A + Bx) e^{-2x}$. Then, find a particular solution Y_p using method of undetermined coefficients or variation of parameters for the RHS $e^{-x} \cos x$. The general solution is $Y = Y_h + Y_p$.

What is the approach to solve $(3D^2 + 27I) y = 3 \cos x + \cos 3x$?

Rewrite the differential operator as $(3 D^2 + 27 I) y = 3 \cos x + \cos 3x$. Solve the homogeneous equation $(3 D^2 + 27 I) y = 0$, then find particular solutions for each RHS term by assuming solutions involving $\cos x$ and $\cos 3x$, respectively, and applying the method of undetermined coefficients.

How do you find the general solution of (D^2 + 2D + 3/4 I) y?

Identify the characteristic equation: $r^2 + 2r + 3/4 = 0$, which has roots $r = -1 \pm i\sqrt{(1/4)}$. The homogeneous solution is $y_h = e^{-x}(C_1 \cos(0.5 x) + C_2 \sin(0.5 x))$. Since no RHS is given, this is the general homogeneous solution.

What is the significance of the auxiliary equation in solving these differential equations?

The auxiliary (characteristic) equation determines the form of the homogeneous solution by solving for roots, which indicate whether solutions involve exponentials, sines, cosines, or polynomials, depending on whether roots are real, repeated, or complex.

When solving nonhomogeneous equations like Y'' + 4Y' + 4Y = e^{-x} cos x, how do you choose the particular solution form?

Since the RHS involves $e^{-x} \cos x$, try a particular solution of the form $Y_p = e^{-x} (A \cos x + B \sin x)$. Adjust the form if it overlaps with the homogeneous solution, possibly multiplying by x.

How do you handle differential equations with higher-order derivatives, such as (D + 2D + 3/4 I) y?

Simplify the operator to standard form, find the characteristic roots, and then write the general homogeneous solution based on those roots. For nonhomogeneous cases, use particular solutions matching the RHS function.

What method is suitable for solving (3D^2 + 27 I) y = 3 cos x + cos 3x?

Use the method of undetermined coefficients to find particular solutions for each RHS term, and solve the homogeneous equation to find the complementary solution. Superimpose the two to get the general solution.

How do you verify the solution of a differential equation involving complex roots?

Substitute the general solution into the original differential equation to check if it satisfies the equation. For solutions involving exponentials and trigonometric functions, use derivatives accordingly, and verify equality.

What are common challenges in solving these types of

differential equations, and how can they be addressed?

Common challenges include handling repeated roots, resonance cases, and particular solutions overlapping with homogeneous solutions. These are addressed by modifying the trial solutions (multiplying by x), using variation of parameters, or complex root methods.

Additional Resources

Find The General Solution.(a) Y" +4y' + 4y = E-x Cos X(b) (3D2+27I)y = 3 Cos X + Cos 3x(c) (D + 2D +3/4I)y

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Introduction

Differential equations are fundamental in modeling a wide array of phenomena across physics, engineering, biology, and economics. They describe how a quantity changes with respect to another—typically time or space—and solving them provides insights into system behavior. Among the most common types are linear differential equations with constant coefficients, which often appear in oscillatory systems, electrical circuits, and mechanical vibrations.

This article delves into solving three distinct differential equations, each presenting unique challenges and requiring specific methods. These are:

- (a) $y'' + 4y' + 4y = e^{-x} \cos x$
- (b) $(3D^2 + 27I) y = 3 \cos x + \cos 3x$
- (c) $(D^2 + 2D + \frac{3}{4}I) y = 0$

Here, D symbolizes differentiation with respect to x, i.e., $D y = y'$. Our goal is to find the general solution for each equation, which combines the complementary (homogeneous) solution with a particular solution tailored to the nonhomogeneous part.

Understanding the Framework

Before diving into solutions, it’s important to clarify the general approach:

1. Identify the type of differential equation—homogeneous or nonhomogeneous.
2. Solve the homogeneous equation to find the complementary solution y_c.
3. Find a particular solution y_p that accounts for the nonhomogeneous part.
4. Combine these solutions to obtain the general solution $y = y_c + y_p$.

Part (a): Solving $y'' + 4y' + 4y = e^{-x} \cos x$

Homogeneous Equation

The first step is to analyze the associated homogeneous equation:

$$
y'' + 4y' + 4y = 0
$$

The characteristic equation:

$$
r^2 + 4r + 4 = 0
$$

Factoring:

$$
(r + 2)^2 = 0
$$

This indicates a repeated root $r = -2$. Therefore, the complementary solution:

$$
y_c = (A + Bx) e^{-2x}
$$

where A and B are arbitrary constants.

Nonhomogeneous Part

The right-hand side $e^{-x} \cos x$ suggests that the particular solution may involve terms similar to the nonhomogeneous function but considering the method of undetermined coefficients or variation of parameters due to the exponential and trigonometric combination.

Given the complexity, method of variation of parameters is more appropriate here.

Step-by-Step Solution

1. Find the fundamental solutions

From the homogeneous solution, the fundamental set:

$$
y_1 = e^{-2x}
$$

$$
y_2 = x e^{-2x}
$$

2. Apply variation of parameters

The particular solution y_p is:

$$
y_p = u_1(x) y_1 + u_2(x) y_2
$$

where u_1' and u_2' satisfy:

$$
\begin{cases}
u_1' y_1 + u_2' y_2 = 0 \\
u_1' y_1' + u_2' y_2' = \text{Right-hand side}
\end{cases}
$$

Calculating the Wronskian W:

$$
W = y_1 y_2' - y_1' y_2
$$

which simplifies to:

$$
W = e^{-2x} \cdot e^{-2x} = e^{-4x}
$$

(Details involve derivatives of y_1 and y_2). The explicit expressions for u_1' and u_2' involve integrating functions derived from the right side, which, due to the exponential and cosine, leads to integrals involving $e^{ax} \cos bx$.

Final expression for y_p

The particular solution involves integrals of the form:

$$
u_1 = - \int \frac{y_2 \cdot RHS}{W} dx, \quad u_2 = \int \frac{y_1 \cdot RHS}{W} dx
$$

which, after evaluation, involve standard integrals for exponential-trigonometric functions.

Summary of Part (a):

- Complementary solution:

$$
y_c = (A + Bx) e^{-2x}
$$

- Particular solution:

$$
y_p = \text{obtained via variation of parameters, involving integrals of } e^{ax} \cos bx
$$

- General solution:

$$
\boxed{
y(x) = (A + Bx) e^{-2x} + y_p
}
$$

Part (b): Solving $(3D^2 + 27I) y = 3 \cos x + \cos 3x$

Understanding the Operator

Expressed explicitly:

$$
3 y'' + 27 y = 3 \cos x + \cos 3x
$$

Dividing through by 3:

$$
y'' + 9 y = \cos x + \frac{1}{3} \cos 3x
$$

This is a second-order linear nonhomogeneous differential equation.

Homogeneous Equation

$$
y'' + 9 y = 0
$$

Characteristic equation:

$$
r^2 + 9 = 0 \Rightarrow r = \pm 3i
$$

Complementary solution:

\[

y_c = C_1 \cos 3x + C_2 \sin 3x
\]

Particular Solution

The right side involves \(\cos x \) and \(\cos 3x \). The method of undetermined coefficients is suitable here, considering the form of the RHS.

1. For \(\cos x \):

Since the homogeneous solutions involve \(\cos 3x \) and \(\sin 3x \), which are different from \(\cos x \), no resonance occurs.

Guess:

\[
y_{p1} = A \cos x + B \sin x
\]

Plug into the LHS:

\[
y'' + 9 y
\]

Compute derivatives:

\[
y_{p1}'' = -A \cos x - B \sin x
\]

Substitute:

\[
(-A \cos x - B \sin x) + 9(A \cos x + B \sin x) = \cos x
\]

Simplify:

\[
(-A + 9A) \cos x + (-B + 9B) \sin x = \cos x
\]
\[
8A \cos x + 8B \sin x = \cos x
\]

Matching coefficients:

\[

$$
8A = 1 \Rightarrow A = \frac{1}{8}
$$
$$
8B = 0 \Rightarrow B= 0
$$

---

2. For $\cos 3x$:

Guess:

$$
y_{p2} = D \cos 3x + E \sin 3x
$$

Calculate derivatives:

$$
y_{p2}'' = -9 D \cos 3x - 9 E \sin 3x
$$

Substitute into the LHS:

$$
-9 D \cos 3x - 9 E \sin 3x + 9(D \cos 3x + E \sin 3x) = \frac{1}{3} \cos 3x
$$

Simplify:

$$
(-9 D + 9 D) \cos 3x + (-9 E + 9 E) \sin 3x = \frac{1}{3} \cos 3x
$$

which simplifies further to:

$$
0 \cdot \cos 3x + 0 \cdot \sin 3x = \frac{1}{3} \cos 3x
$$

This indicates resonance; the RHS is part of the homogeneous solution. To handle this, we multiply the trial solution by x:

$$
y_{p2} = x (D \cos 3x + E \sin 3x)
$$

Applying the method of undetermined coefficients with this trial form involves more detailed derivatives and algebra, but generally, the particular solution will involve terms like $x \sin 3x$ and $x \cos 3x$.

Summary of Part (b):

- Complementary solution:

\[
y_c = C_1 \cos 3x + C_2 \sin 3x
\]

- Particular solution:

\[
y_p = \frac{1}{8} \cos x + \text{terms involving } x \sin 3

Find The General Solution A Y 4y 4y E X Cos X B 3d2 27i Y 3 Cos X Cos 3x C D 2d 3 4i Y

Find The General Solution A Y 4y 4y E X Cos X B 3d2 27i Y 3 Cos X Cos 3x C D 2d 3 4i Y

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