Find The Interval Containing X=0 Where F(x)=16xtan(x) Is Increasing. Write Your Answer As An Open Interval

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Understanding where a function is increasing or decreasing is fundamental in calculus, especially when analyzing the behavior of functions around specific points. In this article, we focus on the function $F(x) = 16x \tan(x)$ and aim to determine the open interval containing $x=0$ where $F(x)$ is increasing. This involves examining the derivative $F'(x)$, identifying its sign, and interpreting the result to find the desired interval.

Analyzing the Function $F(x) = 16x \tan(x)$

Before diving into the interval determination, it is essential to understand the properties of $F(x)$. The function is a product of two functions: $16x$ and $\tan(x)$. Both are differentiable where $\tan(x)$ is defined, i.e., on intervals excluding points where $\cos(x) = 0$ (at $x = \frac{\pi}{2} + k\pi$).

Domain Considerations

- $\tan(x)$ is defined for all real x except at $x = \frac{\pi}{2} + k\pi$, $k \in \mathbb{Z}$.
- Near $x=0$, $\tan(x)$ is continuous and differentiable.
- Since the point of interest is $x=0$, which is within the domain, the analysis around this point is valid.

Calculating the Derivative $F'(x)$

To determine where $F(x)$ is increasing, we need to compute its derivative and analyze its sign.

Applying the Product Rule

Given $F(x) = 16x \tan(x)$, the derivative is:

$$
F'(x) = 16 \left[\tan(x) + x \cdot \sec^2(x) \right]
$$

This follows from the product rule:

$$
\frac{d}{dx}[u(x) \cdot v(x)] = u'(x) v(x) + u(x) v'(x)
$$

where:

- $u(x) = 16x$, so $u'(x) = 16$,
- $v(x) = \tan(x)$, so $v'(x) = \sec^2(x)$.

Therefore,

$$
F'(x) = 16 \left(\tan(x) + x \sec^2(x) \right)
$$

Determining When $F(x)$ Is Increasing

Since the constant 16 is positive, the sign of $F'(x)$ depends solely on:

$$
G(x) = \tan(x) + x \sec^2(x)
$$

- $F(x)$ is increasing where $F'(x) > 0$,
- which occurs where $G(x) > 0$.

Our goal reduces to analyzing the inequality:

$$
G(x) = \tan(x) + x \sec^2(x) > 0
$$

around $x=0$.

Behavior of $G(x)$ Near $x=0$

- At $x=0$:

$$
G(0) = \tan(0) + 0 \cdot \sec^2(0) = 0 + 0 = 0
$$

- To determine whether $G(x)$ is positive or negative near 0, consider a small neighborhood around 0.

Series Expansion of $G(x)$ Near 0

Using Maclaurin series:

- $\tan(x) \approx x + \frac{x^3}{3} + \mathcal{O}(x^5)$,
- $\sec^2(x) = 1 + 2 \tan^2(x) \approx 1 + 2x^2 + \mathcal{O}(x^4)$.

Substituting these into $G(x)$:

$$
G(x) \approx \left(x + \frac{x^3}{3} \right) + x \left(1 + 2x^2 \right) = x + \frac{x^3}{3} + x + 2x^3
$$

Simplify:

$$
G(x) \approx 2x + \left(\frac{x^3}{3} + 2x^3 \right) = 2x + \frac{7}{3}x^3
$$

Near $x=0$, this approximation suggests:

$$
G(x) \approx 2x
$$

Thus,

- For $x>0$ close to 0, $G(x) \approx 2x > 0$,
- For $x<0$ close to 0, $G(x) \approx 2x < 0$.

This indicates:

- $F(x)$ is increasing for x slightly greater than 0,
- $F(x)$ is decreasing for x slightly less than 0.

Conclusion: The Interval Where $F(x)$ Is Increasing Around Zero

From the above analysis:

- At $x=0$, $G(0)=0$,
- For $x>0$ near 0, $G(x) > 0$,
- For $x<0$ near 0, $G(x) < 0$.

Therefore, $F(x)$ is increasing immediately to the right of 0 and decreasing immediately to the left of 0. The function transitions from decreasing to increasing at $x=0$.

Answer: The function $F(x) = 16x \tan(x)$ is increasing on the open interval $\boxed{(0, \frac{\pi}{2})}$.

Note: The upper limit $\frac{\pi}{2}$ is due to the vertical asymptote of $\tan(x)$ at $x=\frac{\pi}{2}$. Within this interval, $F(x)$ remains increasing until approaching the asymptote.

Summary of the Key Findings

- The derivative $F'(x) = 16 (\tan(x) + x \sec^2(x))$ determines the increasing behavior of $F(x)$.
- Near $x=0$, $F'(x)$ changes from negative to positive at $x=0$, indicating a local minimum at the origin.
- The function is increasing for all x in the interval $(0, \frac{\pi}{2})$, where $\tan(x)$ is defined and positive.
- Since the problem asks for the open interval containing $x=0$, the answer is:

$$
\boxed{(0, \frac{\pi}{2})}
$$

which is the maximal interval immediately to the right of zero where $F(x)$ is increasing.

Additional Considerations and Domain Restrictions

- The interval $(0, \frac{\pi}{2})$ is open, excluding the asymptote at $\frac{\pi}{2}$.
- On the negative side, $F(x)$ is decreasing near zero, so it does not satisfy the increasing condition on the negative side.
- If the question also considers the interval to the left of zero, the analysis shows $F(x)$ is decreasing there, so it is not part of the interval where $F(x)$ is increasing.

Final Remarks

The method applied here—finding the derivative, analyzing its sign near the point of interest, and considering the domain—are standard procedures in calculus for investigating the monotonicity of functions. In particular, the key step was understanding the behavior of the derivative $F'(x)$ around $x=0$, which revealed the transition point and the interval of increase.

In summary, the open interval containing $x=0$ where $F(x)=16x \tan(x)$ is increasing is:

$$
\boxed{(0, \frac{\pi}{2})}
$$

This result aligns with the behavior of the derivative and the properties of the tangent function within its domain.

Frequently Asked Questions

How do we determine where the function F(x) = 16x tan(x) is increasing around x=0?

To find where $F(x)$ is increasing, we analyze its derivative $F'(x)$ and identify intervals where $F'(x) > 0$, especially near $x=0$.

What is the derivative of F(x) = 16x tan(x)?

Using the product rule, $F'(x) = 16 [\tan(x) + x \sec^2(x)]$.

How do we find the critical points of F(x) to determine increasing intervals?

Set $F'(x) = 0$ to find critical points, solving $16 [\tan(x) + x \sec^2(x)] = 0$, or equivalently $\tan(x) + x \sec^2(x) = 0$.

What is the behavior of F'(x) near x=0?

As x approaches 0, $\tan(x) \approx x$ and $\sec^2(x) \approx 1$, so $F'(x) \approx 16 (x + x) = 32x$, which is positive for $x > 0$ and negative for $x < 0$.

Is F(x) increasing on the interval containing x=0?

Yes, near $x=0$, since $F'(x) \approx 32x$, F(x) is increasing for $x > 0$ and decreasing for $x < 0$.

What open interval containing x=0 corresponds to where F(x) is increasing?

The interval is $(0, \pi/2)$, where $F'(x) > 0$, indicating F(x) is increasing there.

Are there any other intervals near x=0 where F(x) is increasing besides (0, π/2)?

No, because for $x<0$, $F'(x) < 0$, so F(x) is decreasing there; F'(x) changes sign at $x=0$.

What is the final answer for the interval where F(x) is increasing around x=0?

$(0, \pi/2)$

Additional Resources

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Introduction

Understanding where a function is increasing or decreasing is a fundamental aspect of calculus, vital for analyzing the behavior of functions and their graphs. In this article, we focus on the function $F(x) = 16x \tan(x)$, specifically aiming to identify the open interval containing $x=0$ where $F(x)$ is increasing. This exploration involves calculating derivatives, analyzing their signs, and interpreting the results within the context of open intervals. Such an analysis not only deepens our understanding of the function's behavior but also exemplifies essential calculus techniques applicable across numerous mathematical and real-world scenarios.

Understanding the Function $F(x) = 16x \tan(x)$

Before delving into the increasing intervals, it’s crucial to understand the structure of the function:

- Components:
- The linear term $16x$
- The trigonometric component $\tan(x)$

- Domain considerations:
- Since $\tan(x)$ has vertical asymptotes at $x = \pm \frac{\pi}{2} + k\pi$ for all integers k, the domain of $F(x)$ excludes these points.
- Near $x=0$, $\tan(x)$ is well-behaved, and the function is continuous and differentiable.

The behavior of $F(x)$ around $x=0$ is of particular interest because the problem asks for an interval containing zero where the function increases.

Deriving the First Derivative $F'(x)$

The key to finding where $F(x)$ is increasing lies in analyzing its first derivative:

$$
F'(x) = \frac{d}{dx} \left(16x \tan(x) \right)
$$

Applying the product rule:

$$
F'(x) = 16 \left(\frac{d}{dx}(x \tan(x)) \right) = 16 \left(\tan(x) \cdot \frac{d}{dx}(x) + x \cdot \frac{d}{dx}(\tan(x)) \right)
$$

Recall:

$$
\frac{d}{dx}(x) = 1
$$

$$
\frac{d}{dx}(\tan(x)) = \sec^2(x)
$$

Thus:

$$
F'(x) = 16 \left(\tan(x) + x \sec^2(x) \right)
$$

Simplified derivative:

$$
F'(x) = 16 \left(\tan(x) + x \sec^2(x) \right)
$$

The sign of $F'(x)$ determines whether $F(x)$ is increasing ($F'(x) > 0$) or decreasing ($F'(x) < 0$).

Analyzing the Sign of $F'(x)$

To find the intervals where the function is increasing, we need to solve:

$$
F'(x) > 0
$$

which simplifies to:

$$
\tan(x) + x \sec^2(x) > 0
$$

Let’s analyze this inequality step-by-step, especially around $x=0$.

Behavior Near $x=0$

At $x=0$:

$$
F'(0) = 16 \left(\tan(0) + 0 \cdot \sec^2(0) \right) = 16(0 + 0) = 0
$$

The derivative is zero at $x=0$. To understand whether $F(x)$ is increasing immediately to the right or left of zero, analyze the sign of $F'(x)$ near zero.

Using Taylor series expansions:

- $\tan(x) \approx x + \frac{x^3}{3} + \dots$
- $\sec^2(x) = 1 + 2x^2 + \dots$

Substituting into $F'(x)$:

$$
F'(x) \approx 16 \left(x + \frac{x^3}{3} + x (1 + 2x^2) \right)
$$

Simplify:

$$
F'(x) \approx 16 \left(x + \frac{x^3}{3} + x + 2x^3 \right) = 16 \left(2x + \frac{x^3}{3} + 2x^3 \right)
$$

Combine like terms:

$$
F'(x) \approx 16 \left(2x + \frac{x^3}{3} + 2x^3 \right) = 16 \left(2x + \frac{x^3}{3} + \frac{6x^3}{3} \right) = 16 \left(2x + \frac{7x^3}{3} \right)
$$

Further simplified:

$$
F'(x) \approx 32x + \frac{16 \times 7 x^3}{3} = 32x + \frac{112}{3} x^3
$$

Near zero:

- For small positive x, $F'(x) \approx 32x > 0$
- For small negative x, $F'(x) \approx 32x < 0$

Thus, immediately to the right of zero, $F'(x)$ is positive, indicating $F(x)$ is increasing for small positive x. To the left of zero, $F'(x)$ is negative, so $F(x)$ is decreasing.

Solving the Inequality $F'(x) > 0$

To find the precise interval where $F(x)$ is increasing around zero, analyze the inequality:

$$
\tan(x) + x \sec^2(x) > 0
$$

Rearranged as:

$$
\tan(x) > -x \sec^2(x)
$$

Since:

$$
\sec^2(x) = 1 + \tan^2(x)
$$

the inequality becomes:

$$
\tan(x) > -x (1 + \tan^2(x))
$$

This is a transcendental inequality, but for small x, the approximations above reveal the behavior:

- For small positive x, the inequality holds.
- For small negative x, the inequality does not hold; $F'(x)$ is negative.

The critical point at $x=0$ is a point of inflection where the derivative crosses zero.

Determining the Maximal Interval Containing $x=0$

Given the derivative's behavior:

- $F'(x) > 0$ immediately to the right of zero
- $F'(x) < 0$ immediately to the left of zero

and considering the asymptotes at $x = \pm \frac{\pi}{2}$, the function is increasing in an interval around $x=0$, extending until the derivative becomes zero or the function approaches an asymptote.

The critical point is at $x=0$, with the derivative positive just to the right and negative just to the left. To find where $F(x)$ transitions from decreasing to increasing or vice versa, examine the sign of the derivative for x within the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$.

Key observations:

- For $x \in (0, \frac{\pi}{2})$, $F'(x) > 0$, so $F(x)$ is increasing.
- For $x \in (-\frac{\pi}{2}, 0)$, $F'(x) < 0$, so $F(x)$ is decreasing.
- At $x=0$, $F'(x)=0$, indicating a potential local extremum.

Therefore, the interval containing $x=0$ where $F(x)$ is increasing is:

$$
\boxed{(0, \frac{\pi}{2})}
$$

This is an open interval, as per the problem's requirement, and captures the region immediately to the right of zero where the function is increasing.

Final Answer

The open interval containing $x=0$ where $F(x) = 16x \tan(x)$ is increasing is:

$$
\boxed{(0, \frac{\pi}{2})}
$$

This conclusion aligns with the derivative analysis showing the derivative is positive immediately to the right of zero, and considering the domain restrictions imposed by the asymptote at $x=\frac{\pi}{2}$.

Concluding Remarks

The analysis of $F(x)$ demonstrates the importance of derivative sign analysis in understanding the increasing or decreasing behavior of functions. The key steps involved deriving the first derivative, examining its behavior around critical points, leveraging Taylor series approximations, and considering domain restrictions due to asymptotes. Such methods are fundamental tools in calculus, providing insights not only into the behavior of specific functions but also into the broader principles governing mathematical analysis.

Whether for academic purposes or real-world applications, identifying intervals where functions increase or decrease enables better modeling, optimization, and understanding of various phenomena

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