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Introduction

The Laplace transform is a powerful mathematical tool widely used in engineering, physics, and applied mathematics to analyze linear differential equations, control systems, signal processing, and more. In many applications, after transforming a function into the Laplace domain, the critical step is to find its inverse Laplace transform to retrieve the original time-domain function.

This article provides a comprehensive, step-by-step guide on how to find the inverse Laplace transform of the function:

$$
F(s) = \frac{6s + 18}{\text{(denominator)}}
$$

with detailed explanations and methods, ensuring clarity for learners and practitioners. We focus on the specific function $F(s) = \frac{6s + 18}{1}$, which simplifies to $6s + 18$, but the principles and techniques apply broadly to similar rational functions.

Understanding the Inverse Laplace Transform

What Is the Inverse Laplace Transform?

The inverse Laplace transform, denoted as $\mathcal{L}^{-1} \{F(s)\}$, converts a function from the complex frequency domain back into the time domain. It is defined as a complex integral, but in practice, it is often computed using tables, partial fraction decompositions, or properties of Laplace transforms.

Why Is It Important?

- Solving differential equations: Many differential equations are easier to solve in the Laplace domain, but solutions are meaningful only in the time domain.
- Control systems analysis: Understanding system responses requires inverse transforms.
- Signal processing: Reconstructing signals from their transforms.

Key Techniques for Finding the Inverse Laplace Transform

- Direct lookup in Laplace tables
- Partial fraction decomposition
- Completing the square for quadratic denominators
- Using properties and theorems (linearity, shifting, etc.)
- Residue theorem for complex functions

Step-by-Step Solution for $F(s) = 6s + 18$

Step 1: Recognize the Form of $F(s)$

The given function:

$$
F(s) = 6s + 18
$$

is a simple polynomial in s. Note that it can be written as:

$$
F(s) = 6(s + 3)
$$

which is a linear function in s.

Step 2: Recall Corresponding Inverse Transforms

- $\mathcal{L}^{-1}\{s\} = \delta'(t)$ (distributional derivative), but in standard functions, more common is the inverse of functions involving $\frac{1}{s - a}$.
- For functions like $\frac{1}{s - a}$, the inverse is $e^{a t}$.
- For polynomial functions like s or constants, the inverse transforms often involve derivatives of delta functions or derivatives of exponential functions.

However, since our $F(s)$ is a polynomial, it corresponds to derivatives of delta functions. But in practice, when dealing with rational functions of s, the function is usually expressed as a ratio where the numerator and denominator are polynomials, and the inverse transform is obtained via partial fractions.

In our case, because the numerator is $6s + 18$, which is a polynomial, the inverse Laplace transform corresponds to derivatives of delta functions, an advanced concept typically not used in standard engineering problems.

Step 3: Clarify the Context

Important: The initial function $F(s) = 6s + 18$ may be derived from a ratio $\frac{6s + 18}{1}$. If the original problem intended a ratio with a denominator—say, $F(s) = \frac{6s + 18}{s^2 + 4s + 5}$—the approach would involve partial fractions.

Assumption: For this tutorial, we interpret the problem as finding the inverse Laplace transform of $F(s) = \frac{6s + 18}{\text{(denominator)}}$, where the denominator is missing in your statement. To make the problem meaningful, let's consider a common scenario where the function is:

$$
F(s) = \frac{6s + 18}{s^2 + 4s + 5}
$$

which is a typical rational function encountered in differential equations.

Inverse Laplace Transform of Rational Functions: A General Approach

Step 1: Express the Function Clearly

Suppose:

$$
F(s) = \frac{6s + 18}{s^2 + 4s + 5}
$$

Our goal is to find:

$$
f(t) = \mathcal{L}^{-1} \{F(s)\}
$$

Step 2: Factor or Complete the Square in the Denominator

Complete the square:

$$
s^2 + 4s + 5 = (s + 2)^2 + 1
$$

This form suggests the inverse transform involves exponential and sinusoidal functions.

Step 3: Partial Fraction Decomposition

Express numerator as a combination of terms that match derivatives of the denominator:

$$
F(s) = \frac{6s + 18}{(s + 2)^2 + 1}
$$

Rewrite numerator:

$$
6s + 18 = 6(s + 3)
$$

But note that:

$$
s + 3 = (s + 2) + 1
$$

Thus:

$$
F(s) = \frac{6[(s + 2) + 1]}{(s + 2)^2 + 1} = 6 \frac{s + 2}{(s + 2)^2 + 1} + 6 \frac{1}{(s + 2)^2 + 1}
$$

Now, the inverse transforms are standard:

- $\mathcal{L}^{-1}\left\{ \frac{s + a}{(s + a)^2 + \omega^2} \right\} = e^{-a t} \cos \omega t$
- $\mathcal{L}^{-1}\left\{ \frac{\omega}{(s + a)^2 + \omega^2} \right\} = e^{-a t} \sin \omega t$

Since our numerator is $s + 2$ (matching the denominator's shift), and the numerator's coefficient aligns with the cosine term, we proceed accordingly.

Step 4: Find the Inverse Transform

Using standard Laplace pairs:

$$
\mathcal{L}^{-1} \left\{ \frac{s + a}{(s + a)^2 + \omega^2} \right\} = e^{-a t} \cos \omega t
$$

and

$$
\mathcal{L}^{-1} \left\{ \frac{\omega}{(s + a)^2 + \omega^2} \right\} = e^{-a t} \sin \omega t
$$

Our terms are:

- $6 \frac{s + 2}{(s + 2)^2 + 1}$ corresponds to $6 e^{-2 t} \cos t$
- $6 \frac{1}{(s + 2)^2 + 1}$ can be written as $6 \times \frac{1}{(s + 2)^2 + 1}$, which is $6 \times \left(\frac{\omega}{(s + a)^2 + \omega^2} \right)$ with $\omega = 1$

But since the numerator is 1, and the standard inverse is for $\frac{\omega}{(s + a)^2 + \omega^2}$, we need to adjust accordingly.

Note:

$$
\mathcal{L}^{-1}\left\{ \frac{1}{(s + a)^2 + \omega^2} \right\} = e^{-a t} \frac{\sin \omega t}{\omega}
$$

Therefore, the inverse transform of $6 \times \frac{1}{(s + 2)^2 + 1}$ is:

$$
6 \times e^{-2 t} \frac{\sin t}{1} = 6 e^{-2 t} \sin t
$$

Final Expression:

$$
f(t) = 6 e^{-2 t} \cos t + 6 e^{-2 t} \sin t
$$

which simplifies to:

$$
f(t) = 6 e^{-2 t} (\cos t + \sin t)
$$

Summary of the Process

To recap, the process to find the inverse Laplace transform of such rational functions involves:

1. Expressing the function clearly, preferably in a factored or completed square form.
2. Decomposing the function into simpler fractions or terms matching standard inverse Laplace transforms.
3. Identifying standard transforms for exponential, sine, and cosine functions.
4. Applying inverse transforms individually and summing the results.

Additional Tips for Finding Inverse Laplace Transforms

Frequently Asked Questions

How do you find the inverse Laplace transform of F(s) = (6s + 18)/s?

First, decompose F(s) into simpler parts: F(s) = (6s + 18)/s = 6s/s + 18/s = 6 + 18/s. The inverse Laplace transform of 6 is δ(t) scaled appropriately, but more straightforwardly, since 6 is a constant, its inverse is 6 times δ(t). For 18/s, the inverse Laplace transform is 18 1 = 18. However, because these are functions multiplied by s or constants, the proper approach is to recognize F(s) as 6 + 18/s. The inverse Laplace transform of 6 is 6δ(t), and of 18/s is 18u(t), where u(t) is the unit step function. Therefore, the inverse Laplace transform is f(t) = 6δ(t) + 18u(t). When considering causal functions, the delta function at t=0 is often ignored in practical signals, so the main part is f(t) = 18.

What is the step-by-step method to find the inverse Laplace transform of F(s) = (6s + 18)/s?

Step 1: Simplify F(s): (6s + 18)/s = 6s/s + 18/s = 6 + 18/s. Step 2: Recall that the inverse Laplace transform of 6 is 6δ(t), and of 18/s is 18u(t). Step 3: Combine the results: f(t) = 6δ(t) + 18u(t). In many cases, especially for t>0, the delta function can be ignored, leaving f(t) = 18.

Can you explain the inverse Laplace transform of a function like F(s) = (6s + 18)/s in terms of standard transform pairs?

Yes. The function F(s) can be split into 6 + 18/s. The inverse Laplace of a constant c is c times δ(t), and the inverse of 1/s is the unit step function u(t). Therefore, the inverse Laplace transform of F(s) is f(t) = 6δ(t) + 18u(t). For causal systems, the delta function at t=0 is often considered as an initial impulse, so the main response for t > 0 is 18.

What is the significance of decomposing F(s) = (6s + 18)/s when finding its inverse Laplace transform?

Decomposing F(s) into simpler terms like 6 + 18/s allows us to apply standard inverse Laplace transform pairs directly. Recognizing these parts makes it straightforward to determine the inverse transform without complex partial fractions, especially since 6 corresponds to an impulse at t=0, and 18/s corresponds to a step function starting at t=0.

Is it necessary to perform partial fraction decomposition for F(s) = (6s + 18)/s?

No, in this case, partial fraction decomposition isn't necessary because the expression can be directly separated into simpler terms: 6 + 18/s. This simplifies the process, allowing us to directly apply known inverse Laplace transforms.

How does the presence of the delta function δ(t) affect the interpretation of the inverse Laplace transform in this problem?

The delta function δ(t) represents an instantaneous impulse at t=0. In the context of the inverse Laplace transform, the term 6δ(t) indicates an initial impulse response. For many physical applications where t > 0, the delta function's effect is localized at t=0, so the primary response is 18u(t).

What is the final expression for the inverse Laplace transform of F(s) = (6s + 18)/s?

The inverse Laplace transform is f(t) = 6δ(t) + 18u(t). For practical purposes, especially for t > 0, it simplifies to f(t) = 18.

Additional Resources

Inverse Laplace Transform: Unlocking the Secrets of Function Reversal

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Introduction: The Power and Precision of the Inverse Laplace Transform

In the vast landscape of engineering, mathematics, and physics, the Laplace transform stands out as a fundamental tool for analyzing complex systems—be it electrical circuits, mechanical dynamics, or control systems. It simplifies differential equations into algebraic forms, making them more manageable. However, the true strength of this method is realized when we need to revert back to the original function in the time domain, which is where the Inverse Laplace Transform comes into play.

Understanding how to find the inverse of a Laplace transform is essential for interpreting the physical or real-world significance of a solution. It bridges the gap between the transformed (frequency or complex domain) and the original (time domain) functions. Today, we'll scrutinize a specific function:

$$
F(s) = \frac{6s + 18}{s}
$$

and explore how to determine its inverse Laplace transform with meticulous detail and clarity, akin to a comprehensive product review or expert analysis. This exercise not only demonstrates a straightforward example but also elucidates the systematic approach one can apply to similar functions.

Understanding the Given Function: $F(s) = \frac{6s + 18}{s}$

Before diving into the inverse process, it's crucial to analyze the structure of the function:

- Numerator: $6s + 18$

- Denominator: s

This rational function is straightforward but requires some algebraic manipulation to identify its components in terms of standard Laplace transform pairs. Recognizing common forms simplifies the process of finding the inverse.

Step-by-Step Approach to Find the Inverse Laplace Transform

The process involves several systematic steps:

1. Simplify the Given Function

Start with rewriting $F(s)$ into a form that's easier to interpret:

$$
F(s) = \frac{6s + 18}{s}
$$

Divide numerator and denominator where possible, or split into simpler terms:

$$
F(s) = \frac{6s}{s} + \frac{18}{s}
$$

which simplifies to:

$$
F(s) = 6 + \frac{18}{s}
$$

This step is crucial because it expresses $F(s)$ as a sum of terms involving basic Laplace transforms.

2. Recognize Standard Laplace Transform Pairs

To find the inverse, recall the fundamental Laplace transform pairs:

Function $f(t)$	Laplace Transform $F(s)$	Comments
1	$\frac{1}{s}$	Unit step, constant in time
$t^n / n!$	$\frac{1}{s^{n+1}}$	Polynomial functions

Applying these, the inverse of $F(s) = 6 + \frac{18}{s}$ can be obtained by recognizing each term as a Laplace transform of a specific function.

3. Decompose and Identify the Inverse Components

- The term 6 in the s-domain corresponds to the Laplace transform of $6\delta(t)$ only in the context of impulse functions. But since $\delta(t)$ is a Dirac delta, and our terms are simple constants, it is better to think in terms of the inverse of the entire function.

- The term $\frac{18}{s}$ is directly associated with the constant function 18×1 in the time domain, because:

$$
\mathcal{L}\{1\} = \frac{1}{s}
$$

Thus,

$$
\mathcal{L}^{-1}\left\{\frac{18}{s}\right\} = 18 \times 1 = 18
$$

- For the constant 6, note that the Laplace transform of $6 \times t^0$ (a constant function in time) is $\frac{6}{s}$. But in our case, the term is just 6, which suggests it corresponds to a scaled delta function at $t=0$. Since our goal is to find a standard inverse Laplace transform, it's more straightforward to interpret the entire $F(s)$ after splitting as the sum of functions corresponding to known transforms.

Final Expression and Its Inverse

Having simplified and identified the components, the inverse Laplace transform of $F(s)$ is:

$$
f(t) = \mathcal{L}^{-1}\left\{6 + \frac{18}{s}\right\} = \mathcal{L}^{-1}\{6\} + \mathcal{L}^{-1}\left\{\frac{18}{s}\right\}
$$

- Inverse of 6: Since 6 is a constant with respect to s, and the Laplace of a Dirac delta at zero is 1, this doesn't directly translate to a standard function unless considered in a generalized function sense. For practical purposes, the inverse of a constant in Laplace space often corresponds to a scaled delta function:

$$
\mathcal{L}^{-1}\{c\} = c \delta(t)
$$

but delta functions are distributions, and unless specified, for most physical problems, the inverse of a constant in $F(s)$ would be zero for $t > 0$.

- Inverse of $\frac{18}{s}$: As noted, this corresponds to the constant function 18:

$$
\mathcal{L}^{-1}\left\{\frac{18}{s}\right\} = 18 \times 1 = 18
$$

Final Result: The Inverse Laplace Transform

Considering the above, and assuming the function represents a causal system (which typically involves functions for $t \geq 0$), the inverse Laplace transform of $F(s) = 6 + \frac{18}{s}$ is:

$$
\boxed{
f(t) = 6 \delta(t) + 18
}
$$

This indicates that the original function in the time domain is a combination of an impulse at $t=0$ scaled by 6, plus a constant function of 18 for $t > 0$.

In a more typical physical context, especially when dealing with causal systems, the delta function represents an instantaneous impulse, and the constant term signifies a steady-state or equilibrium value.

Alternative Approach: Using Partial Fraction Decomposition

While the above approach simplifies the process, another robust method involves partial fraction decomposition, especially useful for more complex functions.

1. Express the function as a sum of simpler fractions

Starting from:

$$
F(s) = \frac{6s + 18}{s}
$$

which we previously simplified to:

$$
F(s) = 6 + \frac{18}{s}
$$

2. Recognize standard transforms

As previously discussed, leads directly to the inverse:

$$
f(t) = 6 \delta(t) + 18
$$

3. Summary of Partial Fraction Method

- Decompose complex rational functions into sums of simpler fractions.
- Match each term with known Laplace inverse pairs.
- Sum the inverse transforms to get the final solution.

Implications and Applications of the Result

Understanding this inverse Laplace transform is not merely an academic exercise—it has practical significance:

- Electrical Engineering: The impulse response (represented by $\delta(t)$) combined with a steady-state response (constant 18) can describe how circuits respond to sudden inputs and reach equilibrium.

- Mechanical Systems: The delta function signifies an instantaneous force applied at $t=0$, while the constant term indicates a sustained force or displacement.

- Control Systems: Recognizing the inverse transform helps in designing controllers and understanding system stability.

Conclusion: Mastering the Inverse Laplace Transform for Clarity and Control

The journey from the function $F(s) = \frac{6s + 18}{s}$ to its inverse in the time domain highlights the elegance and utility of the Laplace transform methodology. By methodically simplifying, recognizing standard pairs, and understanding the physical significance of each term, one can confidently decode the original system behavior.

This detailed walkthrough underscores the importance of familiarity with Laplace transform pairs, algebraic manipulation skills, and interpretative insight—a trifecta essential for engineers, mathematicians, and physicists alike. Whether dealing with impulses, steady states, or dynamic responses, mastering the inverse

Laplace transform unlocks a deeper understanding of the systems that shape our world.

In essence, this example exemplifies how a simple rational function can reveal complex insights about system behavior—an invaluable skill for anyone striving to analyze, design, or troubleshoot dynamic systems with precision and confidence.

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