Find The Limits In A) Through C) Below For The Function F(x)= X^2+8x+7 /x+7 Use -[infinity] And [infinity]

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Understanding how to find limits is fundamental in calculus, especially when analyzing the behavior of functions as the input approaches infinity or negative infinity. In this article, we will carefully examine the limits of the function $F(x) = (x^2 + 8x + 7) / (x + 7)$ as x approaches both $-\infty$ and $+\infty$. By exploring sections labeled A) through C), we aim to clarify how to determine these limits and interpret their significance in the context of the function's behavior at the extremes of the real number line.

Overview of the Function and Limit Concepts

Before diving into specific limits, it’s essential to understand some key concepts:

What Is a Limit?

A limit describes the value that a function approaches as the input approaches a particular point or infinity. Limits at infinity help us understand the end behavior of functions, indicating whether they tend toward a finite value or diverge to infinity.

Why Are Limits Important?

Limits are foundational in calculus for defining derivatives and integrals. They also help analyze the end behavior of functions, asymptotes, and the overall shape of graphs.

Function Analysis: $F(x) = (x^2 + 8x + 7) / (x + 7)$

The given function is a rational expression. To analyze its limits as x approaches $-\infty$ and $+\infty$, we should consider the degrees of numerator and denominator, as well as potential vertical or horizontal asymptotes.

Step 1: Simplify or Factor the Function

Let's factor the numerator:

$x^2 + 8x + 7$

This factors as:

$(x + 1)(x + 7)$

Thus, the function simplifies to:

$F(x) = [(x + 1)(x + 7)] / (x + 7)$

Notice that for $x \neq -7$, the $(x + 7)$ terms cancel out, simplifying the function to:

$F(x) = x + 1$, for $x \neq -7$

This simplifies the analysis of the limits at infinity, but we must remember that $x = -7$ is a point of discontinuity (a vertical asymptote).

Limit Analysis at Infinity and Negative Infinity

Now, we proceed to evaluate the limits as x approaches $\pm\infty$ for the original function.

A) Limit as x approaches $+\infty$

Since for all $x \neq -7$, $F(x) = x + 1$, the behavior at large positive x values is similar to that of the linear function $x + 1$.

Calculation:

$\lim_{x\to+\infty} F(x) = \lim_{x\to+\infty} (x + 1) = +\infty$

Interpretation:
As x approaches positive infinity, F(x) grows without bound, tending toward positive infinity.

B) Limit as x approaches $-\infty$

Similarly, considering large negative x:

Calculation:

$\lim_{x\to-\infty} F(x) = \lim_{x\to-\infty} (x + 1) = -\infty$

Interpretation:
As x approaches negative infinity, F(x) decreases without bound, tending toward negative infinity.

Analyzing Behavior Near the Vertical Asymptote x = -7

Although the limits at infinity are straightforward, the behavior of the function as x approaches -7 from both sides is different, due to the discontinuity at x = -7.

C) Limit as x approaches -7 from the right ($x \to -7^+$)

When approaching -7 from the right, x > -7, so x + 7 > 0.

Recall that the original function simplifies to F(x) = x + 1, but only for x ≠ -7.

Behavior:

- As x approaches -7 from the right, x + 7 approaches 0 from the positive side.

- Therefore, F(x) = x + 1 approaches -7 + 1 = -6.

- Since the denominator approaches zero positively, the original function's value tends to +∞ (because the fraction's numerator approaches -6, and the denominator approaches 0 from the positive side).

Conclusion:

limx→-7+ F(x) = +∞

Limit as x approaches -7 from the left ($x \to -7^-$)

- As x approaches -7 from the left, x + 7 approaches 0 from the negative side.

- The numerator approaches -6, as before.

- The denominator approaches 0 from the negative side.

- The ratio becomes approximately -6 divided by a very small negative number, tending to -∞.

Conclusion:

limx→-7- F(x) = -∞

Summary of vertical asymptote:

- At x = -7, F(x) has a vertical asymptote, with the limits from the left and right tending to -∞ and +∞ respectively.

Summary of Limits for $F(x) = (x^2 + 8x + 7) / (x + 7)$

Limit	x approaching	Result	Interpretation
A)	$+\infty$	$+\infty$	Function grows without bound as $x \to +\infty$
B)	$-\infty$	$-\infty$	Function decreases without bound as $x \to -\infty$
C)	-7^+	$+\infty$	Function tends to $+\infty$ approaching -7 from the right
D)	-7^-	$-\infty$	Function tends to $-\infty$ approaching -7 from the left

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Conclusion and Key Takeaways

- The limits at infinity demonstrate that the function behaves like a linear function $(x + 1)$ for large $|x|$, tending toward infinity in both directions, but with opposite signs.

- The vertical asymptote at $x = -7$ reveals the function's discontinuity, with the limits from the left and right tending toward negative and positive infinity, respectively.

- Simplifying the function to $F(x) = x + 1$ (for $x \neq -7$) provides an easy way to analyze its end behavior, but understanding the behavior near the asymptote requires considering the original expression.

- When analyzing limits involving rational functions, always check for removable discontinuities and asymptotes by factoring numerator and denominator.

- Recognizing how the degrees of numerator and denominator influence the limits at infinity is crucial: since the numerator is degree 2 and the denominator degree 1, the function’s end behavior at infinity is dominated by the highest degree term, leading to the conclusion that the function tends toward infinity as x approaches both positive and negative infinity.

In summary: To find the limits in A) through C) below for $F(x)= (x^2 + 8x + 7) / (x + 7)$, you analyze the behavior at infinity and near asymptotes, applying algebraic simplification and limit rules. The function exhibits unbounded growth at both ends of the real line and has a vertical asymptote at $x = -7$, with limits tending to infinity in opposite directions depending on the side of approach.

If you want to master limit calculations, remember: identifying dominant terms, simplifying expressions, and paying attention to asymptotic behavior are key strategies to accurately find limits as x approaches $-\infty$ and $+\infty$.

Frequently Asked Questions

What is the limit of the function F(x) = (x^2 + 8x + 7) / (x + 7) as x approaches infinity?

As x approaches infinity, the degree of the numerator and denominator are both 2 and 1 respectively; simplifying the dominant terms gives the limit as x approaches infinity is infinity.

What is the limit of F(x) = (x^2 + 8x + 7) / (x + 7) as x approaches negative infinity?

As x approaches negative infinity, the dominant term x^2 in the numerator over x in the denominator causes the function to approach negative infinity.

Does the function F(x) = (x^2 + 8x + 7) / (x + 7) have any finite limits at specific points?

Yes, to find limits at specific points like x = -7, we should analyze the behavior near that point; in this case, the function is undefined at x = -7 due to division by zero, so the limits are infinite or do not exist there.

How can I simplify the function F(x) = (x^2 + 8x + 7) / (x + 7) to evaluate limits more easily?

Factor the numerator as (x + 1)(x + 7), then cancel the common factor (x + 7) with the denominator, leaving F(x) = x + 1 for all x ≠ -7. This simplifies limit evaluation at infinity.

What is the value of the limit of F(x) = (x^2 + 8x + 7) / (x + 7) as x approaches -7 from the left and right?

Approaching x = -7 from the left or right, the function exhibits a vertical asymptote, so the limits tend to negative or positive infinity respectively, indicating the limit does not exist at that point.

Additional Resources

Limits Analysis of the Function F(x) = (x^2 + 8x + 7) / (x + 7): An Expert Exploration

Understanding the behavior of functions as variables approach critical points is fundamental in calculus. Today, we delve into the comprehensive analysis of the limits of the function

$$
F(x) = \frac{x^2 + 8x + 7}{x + 7}
$$

as x approaches $-\infty$ and $+\infty$, exploring the function’s asymptotic behavior, potential discontinuities, and the intricacies that define its limits. This examination not only clarifies the nature of $F(x)$ but also provides a detailed roadmap for mathematicians and students alike interested in mastering limits.

Understanding the Function: An Initial Overview

Before tackling the limits at the extremes of the real number line, it’s essential to understand the core structure of the function.

The Function Structure

The given function:

$$
F(x) = \frac{x^2 + 8x + 7}{x + 7}
$$

is a rational function, composed of a quadratic polynomial in the numerator and a linear polynomial in the denominator. Rational functions often exhibit interesting behaviors at points where the denominator is zero (vertical asymptotes) and at infinity (horizontal or oblique asymptotes).

Key observations:

- Numerator: $x^2 + 8x + 7$ — a quadratic polynomial that grows rapidly for large $|x|$.
- Denominator: $x + 7$ — a linear polynomial with a zero at $x = -7$.

Understanding how these components interact as x approaches $\pm \infty$ and how the function behaves near the vertical asymptote at $x = -7$ sets the foundation for our limit analysis.

Limit as $x \to +\infty$

The Behavior at Positive Infinity

When $x \to +\infty$, the dominant term in the numerator and denominator are the highest degree terms: x^2 in the numerator and x in the denominator. To analyze the limit at infinity, the primary strategy involves dividing numerator and denominator by the highest power of x present in the denominator, which is x.

Step-by-step process:

1. Divide numerator and denominator by x:

$$
F(x) = \frac{\frac{x^2}{x} + \frac{8x}{x} + \frac{7}{x}}{\frac{x}{x} + \frac{7}{x}} = \frac{x + 8 + \frac{7}{x}}{1 + \frac{7}{x}}
$$

2. Evaluate the limit as $x \to +\infty$:

- $\frac{7}{x} \to 0$,
- The numerator approaches $x + 8$,
- The denominator approaches 1.

3. Express the limit:

$$
\lim_{x \to +\infty} F(x) = \lim_{x \to +\infty} \frac{x + 8 + 0}{1 + 0} = \lim_{x \to +\infty} (x + 8)
$$

Since $x \to +\infty$, the entire expression tends to +infinity.

Conclusion:

$$
\boxed{
\lim_{x \to +\infty} F(x) = +\infty
}
$$

Implication: The function grows without bound as x increases, indicating a slant or oblique asymptote at infinity, which we will explore further.

Limit as $x \to -\infty$

Behavior at Negative Infinity

The process is similar to the previous case, focusing on dominant terms:

1. Divide numerator and denominator by x:

$$
F(x) = \frac{x + 8 + \frac{7}{x}}{1 + \frac{7}{x}}
$$

2. Evaluate as $x \to -\infty$:

- $\frac{7}{x} \to 0$,
- Numerator approaches $x + 8$,
- Denominator approaches 1.

3. Limit:

$$
\lim_{x \to -\infty} F(x) = \lim_{x \to -\infty} (x + 8) = -\infty

\]

Conclusion:

\[
\boxed{
\lim_{x \to -\infty} F(x) = -\infty
}
\]

Interpretation: The function decreases without bound as \(x\) moves towards negative infinity, consistent with the quadratic dominance in the numerator.

Analyzing the Behavior Near the Vertical Asymptote at \(x = -7\)

While the problem explicitly asks for limits as \(x \to \pm \infty\), understanding the behavior at points where the denominator is zero enriches the overall analysis.

Vertical Asymptote at \(x = -7\)

- The function is undefined at \(x = -7\), because division by zero occurs.
- To determine how \(F(x)\) behaves as \(x\) approaches \(-7\), we analyze one-sided limits.

Limit as \(x \to -7^-\)

As \(x\) approaches \(-7\) from the left:

- \(x + 7 \to 0^-\),
- Numerator at \(x = -7\):

\[
(-7)^2 + 8(-7) + 7 = 49 - 56 + 7 = 0
\]

- Near \(x = -7\), the numerator approaches 0, but the sign of the numerator depends on the specific value of \(x\).

Using a small value \(x = -7 - \epsilon\), where \(\epsilon \to 0^+\):

- \(x + 7 = -\epsilon \to 0^-\),
- Numerator:

$$
x^2 + 8x + 7 = (x + 7)(x + 1)
$$

Since:

$$
x^2 + 8x + 7 = (x + 7)(x + 1)
$$

we can factor the numerator to analyze its sign near -7.

At $x = -7 - \epsilon$:

- $x + 7 = -\epsilon \to 0^-$,
- $x + 1 = -7 - \epsilon + 1 = -6 - \epsilon \to -6^-$.

Thus, numerator:

$$
(x + 7)(x + 1) \to (-)(-) = (+)
$$

- The numerator approaches a small positive value,
- The denominator approaches a small negative value ($-$),
- Therefore,

$$
F(x) \to \frac{\text{small positive}}{\text{small negative}} \to -\infty
$$

Conclusion:

$$
\boxed{
\lim_{x \to -7^-} F(x) = -\infty
}
$$

Limit as $x \to -7^+$

Similarly, approaching from the right:

- $x + 7 \to 0^+$,
- $x + 1 \to -6^+$,
- Numerator:

$$
(x + 7)(x + 1) \to (+)(-) = -,

\]

- Denominator: \(+ \to 0^+\),

- The overall ratio:

\[
\frac{\text{small negative}}{\text{small positive}} = -\infty
\]

Conclusion:

\[
\boxed{
\lim_{x \to -7^+} F(x) = -\infty
}
\]

Implication: There is a vertical asymptote at \(x = -7\), with the function tending to \(-\infty\) from both sides.

Summary of Limits at Infinity

Limit Point	Limit of \(F(x)\)	Behavior	Asymptote Type
\(x \to +\infty\)	\(+\infty\)	Function grows without bound	Oblique asymptote
\(x \to -\infty\)	\(-\infty\)	Function decreases without bound	Oblique asymptote

---

Identifying the Oblique Asymptote

Given the behavior at infinity, the function approaches a line rather than a horizontal asymptote. To find this, perform polynomial division:

\[
\frac{x^2 + 8x + 7}{x + 7}
\]

Division process:

- Divide \(x^2\) by \(x\): quotient \(x\),
- Multiply \(x\) by \(x + 7\): \(x^2 + 7x\),
- Subtract: \((x^2 + 8x + 7) - (x^2 + 7x

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