

Find The Percent Increase In Volume When 1 Foot Is Added To Each Dimension Of The Prism. Round Your Answer

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Introduction

Understanding how changes in the dimensions of a three-dimensional object affect its volume is fundamental in geometry, engineering, architecture, and many practical applications. Specifically, calculating the percent increase in volume when each dimension of a prism is increased by a certain amount—here, by 1 foot—provides valuable insights into scalability and resource estimation. This article offers a comprehensive guide to solving this problem, including step-by-step calculations, important formulas, and practical examples. Whether you're a student preparing for an exam or a professional evaluating design modifications, mastering this concept is essential.

Understanding the Basics: Volume of a Prism

What Is a Prism?

A prism is a three-dimensional solid object with two parallel, congruent faces called bases, and the other faces are parallelograms. Common types include rectangular prisms (cuboids), triangular prisms, and more complex shapes.

Volume Formula for a Rectangular Prism

The most straightforward case is a rectangular prism, which has length, width, and height:

$$V = l \times w \times h$$

Where:

- (V) = volume
- (l) = length
- (w) = width
- (h) = height

Problem Statement and Approach

Given an initial rectangular prism with dimensions (l) , (w) , and (h) , we are asked to:

- Find the percentage increase in volume when each dimension is increased by 1 foot.
- Round the final answer to an appropriate decimal place.

Key steps:

1. Calculate the original volume.
2. Calculate the new volume after increasing each dimension by 1 foot.
3. Determine the difference between the new and original volumes.
4. Calculate the percent increase based on the original volume.
5. Round the answer to a specified decimal place.

Step-by-Step Solution

1. Define the original dimensions

Let:

- (l) = original length
- (w) = original width
- (h) = original height

2. Calculate the original volume

$$V_{\text{original}} = l \times w \times h$$

3. Calculate the new dimensions

Since each dimension is increased by 1 foot:

$$l_{\text{new}} = l + 1$$

$$w_{\text{new}} = w + 1$$

$$h_{\text{new}} = h + 1$$

4. Calculate the new volume

$$V_{\text{new}} = (l + 1) \times (w + 1) \times (h + 1)$$

5. Find the difference in volume

$$\Delta V = V_{\text{new}} - V_{\text{original}}$$

6. Calculate the percent increase

$$\text{Percent Increase} = \left(\frac{\Delta V}{V_{\text{original}}} \right) \times 100$$

Algebraic Expansion of the New Volume

To understand the change better, expand V_{new} :

$$V_{\text{new}} = (l + 1)(w + 1)(h + 1)$$

Using distributive property:

$$V_{\text{new}} =$$

$$V_{\text{new}} = (l w h) + (l w) + (l h) + (w h) + (l) + (w) + (h) + 1$$

\]

Similarly, the original volume is:

\[

$$V_{\text{original}} = l w h$$

\]

Therefore:

\[

$$\Delta V = V_{\text{new}} - V_{\text{original}} = (l w h) + (l w) + (l h) + (w h) + (l) + (w) + (h) + 1$$

\]

Final Formula for Percent Increase

Putting it all together:

\[

$$\text{Percent Increase} = \frac{(l w h + l w + l h + w h + l + w + h + 1)}{l w h} \times 100$$

\]

This formula allows calculation of the percent increase given specific dimensions.

Practical Example

Suppose the initial dimensions of the prism are:

- Length $(l = 10)$ ft
- Width $(w = 5)$ ft
- Height $(h = 4)$ ft

Step 1: Original volume

$$V_{\text{original}} = 10 \times 5 \times 4 = 200 \text{ ft}^3$$

Step 2: New dimensions

$$l_{\text{new}} = 11 \text{ ft}$$
$$w_{\text{new}} = 6 \text{ ft}$$
$$h_{\text{new}} = 5 \text{ ft}$$

Step 3: New volume

$$V_{\text{new}} = 11 \times 6 \times 5 = 330 \text{ ft}^3$$

Step 4: Volume difference

$$\Delta V = 330 - 200 = 130 \text{ ft}^3$$

Step 5: Percent increase

$$\text{Percent Increase} = \frac{130}{200} \times 100 = 65\%$$

Thus, increasing each dimension by 1 foot results in a 65% increase in the volume.

Generalized Formula Application

Using the example dimensions in the formula:

$$\text{Percent Increase} = \frac{(l w + l h + w h + l + w + h + 1)}{l w h} \times 100$$

Plugging in:

$$\frac{(10 \times 5 + 10 \times 4 + 5 \times 4 + 10 + 5 + 4 + 1)}{200} \times 100$$

Calculations:

$$\frac{(50 + 40 + 20 + 19)}{200} \times 100$$

$$\frac{(50 + 40 + 20 + 10 + 5 + 4 + 1)}{200} \times 100 = \frac{130}{200} \times 100 = 65\%$$

\\

Matching the previous calculation.

Additional Considerations

Rounding the Final Answer

- Always round to the appropriate decimal place as specified or as suitable for the context.
- In most cases, rounding to the nearest whole percent suffices; for more precision, round to one decimal place.

Extending to Other Shapes

While the example focuses on rectangular prisms, similar principles apply to other types of prisms and polyhedra, though the volume formulas may differ.

Real-World Applications

- Construction: Estimating material requirements when dimensions increase.
- Manufacturing: Scaling products to larger sizes and understanding volume changes.
- Design Optimization: Evaluating the impact of dimension modifications on volume and cost.
- Packaging: Adjusting container sizes and calculating volume capacity increases.

Summary

Calculating the percent increase in volume when each dimension of a prism is increased by 1 foot involves understanding the volume formula, expanding the new volume expression, and applying the percent increase formula. The key steps include determining the original volume, computing the new volume, finding the difference, and expressing this difference as a percentage of the original volume. Applying this method to specific examples demonstrates its usefulness and versatility across various fields.

Final Tips for Students and Professionals

- Always verify the original dimensions before calculations.
- Use algebraic expansion to simplify complex problems.
- Practice with different dimension values to strengthen understanding.
- Remember to round your final answer appropriately for clarity and presentation.

Conclusion

Understanding how incremental changes in dimensions affect the overall volume of a prism is a vital skill in geometry and practical problem-solving. By mastering the formula and approach detailed above, you can efficiently compute the percent increase in volume for any rectangular prism when each dimension is increased by a fixed amount. Approximate calculations, like the example provided, highlight the significance of even small dimensional changes and their substantial impact on volume, guiding better design, planning, and resource management.

Frequently Asked Questions

What is the first step to find the percent increase in volume when 1 foot is added to each dimension of a prism?

First, determine the original volume of the prism using its initial dimensions.

How do you calculate the new volume after increasing each dimension of the prism by 1 foot?

Multiply the new dimensions (original length + 1, width + 1, height + 1) to find the new volume.

What formula do you use to find the percent increase in volume?

Percent increase = $[(\text{New volume} - \text{Original volume}) / \text{Original volume}] \times 100\%$.

If the original dimensions are length l , width w , and height h , what is the formula for the original volume?

Original volume = $l \times w \times h$.

What is the formula for the new volume after adding 1 foot to each dimension?

New volume = $(l + 1) \times (w + 1) \times (h + 1)$.

How do you round your final answer when finding the percent increase in volume?

Round the percent increase to the nearest whole number or as specified in the problem instructions.

Why is it important to calculate the original and new volume accurately in this problem?

Accurate calculations ensure the percent increase is precise, reflecting the true change in volume.

Can you give an example with specific dimensions to illustrate the calculation?

Yes, for example, if the original dimensions are 3 ft, 4 ft, and 5 ft, then original volume = $3 \times 4 \times 5 = 60$.

New dimensions are 4 ft, 5 ft, and 6 ft, so new volume = $4 \times 5 \times 6 = 120$. Percent increase =

$[(120-60)/60] \times 100 = 100\%$.

Additional Resources

Find The Percent Increase In Volume When 1 Foot Is Added To Each Dimension Of The Prism is a classic problem that combines fundamental principles of geometry with algebraic manipulation. This problem not only tests one's understanding of volume calculations but also emphasizes the importance of percentage change, which is a common concept in various real-world applications such as manufacturing, construction, and data analysis. Addressing such a problem requires a clear grasp of how incremental changes to the dimensions of a shape affect its overall volume, and ultimately, its percentage increase.

Understanding the Problem

When given a rectangular prism (or any three-dimensional shape), its volume is calculated as the product of its length, width, and height. The problem asks: What is the percent increase in volume when each of these dimensions is increased by 1 foot? To solve this, it's essential to understand the

initial volume, the new volume after the increase, and how to express the change as a percentage of the original volume.

Key points to clarify:

- The original dimensions: length (L), width (W), height (H).
- The new dimensions after increase: (L + 1), (W + 1), (H + 1).
- The initial volume: $V = L \times W \times H$.
- The new volume: $V' = (L + 1) \times (W + 1) \times (H + 1)$.

The core goal is to find the percent increase:

$$\text{Percent Increase} = \left(\frac{V' - V}{V} \right) \times 100\%$$

Step-by-Step Solution Approach

1. Define the Original Dimensions and Volume

Suppose the original dimensions are given or known:

- Length = L
- Width = W
- Height = H

The original volume:

$$V = L \times W \times H$$

2. Calculate the New Volume After Increasing Each Dimension by 1 Foot

The new dimensions:

- Length = $L + 1$
- Width = $W + 1$
- Height = $H + 1$

The new volume:

$$V' = (L + 1)(W + 1)(H + 1)$$

3. Expand the New Volume Expression

Using algebra:

$$\begin{aligned} V' &= (L + 1)(W + 1)(H + 1) \\ &= (L + 1)(W + 1) \times (H + 1) \end{aligned}$$

First, expand $((L + 1)(W + 1))$:

$$LW + L + W + 1$$

Now, multiply this sum by $(H + 1)$:

$$\begin{aligned} & \end{aligned}$$

$$(LW + L + W + 1)(H + 1)$$

\]

\[

$$= (LW)(H + 1) + L(H + 1) + W(H + 1) + (H + 1)$$

\]

\[

$$= LWH + LW + LH + L + WH + W + H + 1$$

\]

Therefore, the expanded form of the new volume:

\[

$$V' = LWH + LW + LH + WH + L + W + H + 1$$

\]

4. Calculate the Change in Volume

Subtract the original volume from the new volume:

\[

$$V' - V = (LWH + LW + LH + WH + L + W + H + 1) - LWH$$

\]

\[

$$= LW + LH + WH + L + W + H + 1$$

\]

This expression represents the absolute increase in volume.

5. Compute the Percent Increase

Express the change as a percentage:

$$\text{Percent Increase} = \frac{V' - V}{V} \times 100\%$$

Substituting:

$$= \frac{LW + LH + WH + L + W + H + 1}{LWH} \times 100\%$$

This formula allows calculation of the percent increase given specific dimensions.

Special Case: When Dimensions Are Equal

Often, problems specify or imply equal dimensions for simplicity, such as a cube or a box with dimensions $L = W = H = \text{a number}$.

Example: Suppose each dimension is 10 feet

- Original volume:

$$V = 10 \times 10 \times 10 = 1000 \text{ cubic feet}$$

- New volume:

$$V' = (10 + 1)^3 = 11^3 = 1331 \text{ cubic feet}$$

- Change in volume:

$$\begin{aligned} & \backslash[\\ & 1331 - 1000 = 331 \text{ cubic feet} \\ & \backslash] \end{aligned}$$

- Percent increase:

$$\begin{aligned} & \backslash[\\ & \frac{331}{1000} \times 100\% = 33.1\% \\ & \backslash] \end{aligned}$$

This example illustrates how a uniform increase by 1 foot in all dimensions results in a significant percentage increase, especially for smaller initial dimensions.

Generalized Formula and Its Implications

The derived formula:

$$\begin{aligned} & \backslash[\\ & \text{Percent Increase} = \frac{L W + L H + W H + L + W + H + 1}{L W H} \times 100\% \\ & \backslash] \end{aligned}$$

provides a flexible way to evaluate the impact of adding 1 foot to each dimension across various scenarios.

Features:

- It considers the initial dimensions explicitly.
- It captures the compound effect of increasing each dimension.
- It simplifies to specific cases, such as cubes or rectangular prisms with known dimensions.

Pros and Cons of the Approach

Pros:

- Clear mathematical framework for volume change.
- Easily adaptable to different initial dimensions.
- Demonstrates the proportional impact of dimension increases.
- Useful for understanding geometric scaling in practical contexts.

Cons:

- Assumes dimensions are positive real numbers.
- Does not account for real-world constraints like material limits or structural integrity.
- The formula can become cumbersome with arbitrary dimensions, requiring calculator assistance.

Real-World Applications and Significance

Understanding the percent increase in volume when dimensions change is critical in various fields:

- Manufacturing & Production: Adjusting dimensions of containers or parts to optimize capacity.
- Construction: Estimating additional material costs when increasing size.
- Design & Engineering: Scaling models and understanding proportional changes.
- Data Analysis: Interpreting percentage changes in three-dimensional data sets.

Knowing how small changes in dimensions affect volume helps in making informed decisions, optimizing designs, and controlling costs.

Conclusion

The problem of finding the percent increase in volume when 1 foot is added to each dimension of a prism is a fundamental exercise in understanding dimensional relationships and percentage changes. By expanding the volume expressions and deriving a generalized formula, one gains insight into how incremental adjustments propagate through three-dimensional shapes. Whether dealing with cubes or rectangular prisms with varied dimensions, the approach remains consistent, emphasizing the importance of algebraic manipulation and proportional reasoning. This problem not only reinforces core mathematical skills but also highlights their practical relevance across numerous fields, making it a valuable exercise in both academic and real-world contexts.

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