Find The Solution Of Each Inequality In The Interval (0, 2). (Enter Your Answers Using Interval Notation.)

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Understanding how to solve inequalities within a specific interval is a fundamental skill in algebra and mathematical analysis. When working within the interval (0, 2), the goal is to find all the values of the variable that satisfy the given inequality while also falling strictly between 0 and 2. Properly solving these inequalities involves a combination of algebraic manipulation, understanding the properties of inequalities, and careful consideration of the interval constraints. This comprehensive guide aims to equip you with the techniques, strategies, and examples necessary to confidently find solutions to inequalities within the interval (0, 2), and to express your answers using precise interval notation.

Understanding the Interval (0, 2)

Before tackling inequalities, it’s critical to understand what the interval (0, 2) signifies.

Definition of the Interval (0, 2)

- The interval (0, 2) includes all real numbers greater than 0 and less than 2.
- It is an open interval, meaning:
- 0 and 2 are not included in the interval.
- The interval contains infinitely many points between 0 and 2.

Visual Representation of the Interval (0, 2)

- On the number line, (0, 2) is represented as:

---●====●---

where:
- The open circles (●) indicate that 0 and 2 are not part of the interval.
- The line between them indicates all points between 0 and 2.

Understanding this is crucial because solutions to inequalities must not only satisfy the inequality itself but also fall within this specific interval.

Types of Inequalities Commonly Encountered

Inequalities can be classified into various types, and each type may require different solving techniques.

Linear Inequalities

- Examples: $ax + b < c$, $ax + b \leq c$
- Generally straightforward to solve by isolating the variable.

Quadratic Inequalities

- Examples: $ax^2 + bx + c > 0$, $ax^2 + bx + c \leq 0$
- Usually involve finding roots and analyzing the sign of the quadratic expression.

Rational Inequalities

- Examples: $\frac{p(x)}{q(x)} > 0$, $\frac{p(x)}{q(x)} \leq 0$
- Require solving numerator and denominator separately and considering points where the denominator is zero.

Absolute Value Inequalities

- Examples: $|x - a| < b$, $|x - a| \geq b$
- Often involve splitting into two cases.

Each type has its peculiarities, but the core principles of solving and intersection with the interval remain consistent.

Step-by-Step Approach to Solving Inequalities in (0, 2)

To effectively find solutions within (0, 2), follow these systematic steps:

Step 1: Rewrite the Inequality in a Simplified Form

- Simplify expressions.
- Bring all terms to one side if necessary.
- Ensure the inequality is in a standard form suitable for analysis.

Step 2: Solve the Inequality as if No Interval Exists

- Use algebraic techniques:
- For linear inequalities: isolate the variable.
- For quadratic inequalities: find roots and analyze sign intervals.
- For rational inequalities: find critical points where numerator or denominator equals zero.
- For absolute value inequalities: split into cases based on the definition.

Step 3: Find the Solution Set on the Real Line

- Determine the solution intervals based on the inequality.
- Use test points or sign charts to verify solutions.

Step 4: Intersect the Solution Set with the Interval (0, 2)

- The solutions must be within the open interval (0, 2).
- Use intersection operations to restrict the solution set to this interval.

Step 5: Express the Final Answer in Interval Notation

- Present the solution as a union of intervals (if multiple disjoint intervals).
- Ensure that the endpoints respect the original interval (excluding 0 and 2).

Examples of Solving Inequalities in (0, 2)

To illustrate these steps, consider specific examples:

Example 1: Linear Inequality

Solve \(3x - 1 < 5 \) for \(x \in (0, 2) \).

Solution:
1. Simplify:
\[
3x - 1 < 5
\Rightarrow 3x < 6
\Rightarrow x < 2
\]
2. Since the inequality is \(x < 2 \), and \(x \in (0, 2) \), the solution within the interval is all \(x \) such that:
\[
0 < x < 2
\]
3. Final answer:
\[
(0, 2)
\]

Interpretation:
All points within (0, 2) satisfy the inequality.

Example 2: Quadratic Inequality

Solve \(x^2 - 3x + 2 \geq 0 \) for \(x \in (0, 2) \).

Solution:
1. Factor the quadratic:
\[
x^2 - 3x + 2 = (x - 1)(x - 2)
\]
2. Find the roots: \(x=1 \), \(x=2 \).
3. Sign analysis:
- For \(x < 1 \), both factors are negative and positive accordingly, but since \(x \in (0, 2) \), focus on this interval.
- The quadratic is \(\geq 0 \) outside the roots or at the roots.
4. Check intervals:
- \((0,1) \): pick \(x=0.5 \), then \((0.5 - 1)(0.5 - 2) = (-0.5)(-1.5) = 0.75 \geq 0 \). So, solution in \((0,1) \).
- At \(x=1 \): \(0 \), included because of \(\geq 0 \).
- \((1,2) \): pick \(x=1.5 \), \((1.5 - 1)(1.5 - 2) = (0.5)(-0.5) = -0.25 < 0 \), so not part of the solution.
- At \(x=2 \): \(0 \), included.
5. Final solution:
\[
[1, 2]

\]
6. Intersection with (0, 2):
- Since the interval (0, 2) excludes 0 but includes 2's endpoint? No, because 2 is not in (0, 2).
- The point \(x=2 \) is outside the interval (0, 2), so exclude it.
- For \(x \in (0, 2) \), the solution is \([1, 2) \), but since 2 is not included, we write:
\[
[1, 2)
\]
7. Final answer:
\[
[1, 2)
\]

Special Considerations When Solving in the Interval (0, 2)

While solving inequalities, always keep in mind the specific interval restrictions:

Endpoints Are Not Included

- Since the interval is open at 0 and 2, solutions equal to these points are not valid, even if the inequality is satisfied there.
- For example, if the solution set includes 0 or 2, exclude those endpoints.

Handling Strict Inequalities

- For \(< \) or \(> \), solutions do not include the boundary points.
- For \(\leq \) or \(\geq \), include the boundary points unless they fall outside the interval (0, 2).

Ensuring the Solution Lies Within the Interval

- When the inequality's solution set extends beyond the interval, intersect with (0, 2).
- For example, if the solution set is \((-\infty, 1.5] \), intersect with (0, 2):
\[
(0, 1.5]
\]
Since endpoints are open at 0 and the solution set includes 1.5, also within

(0, 2), the final answer is \((0, 1.5] \).

Advanced Techniques and Tips

For more complex inequalities, additional methods can be useful:

Using Sign Charts

- Plot critical points where the expression equals zero or undefined.
- Test the sign of the expression in each interval.
- Determine where the inequality holds based on the

Frequently Asked Questions

Solve the inequality 2x - 5 < 1 within the interval (0, 2). What is the solution set in interval notation?

(2/2, 3/2) or (0, 1.5)

Find the solution set for the inequality 3x + 4 > 10 in the interval (0, 2). Provide your answer in interval notation.

(0, 2/3)

Determine the solutions to the inequality 5 - 2x ≤ 3 within (0, 2). Express your answer in interval notation.

(0, 1)

Solve the inequality 4x - 7 > 1 for x in the interval (0, 2). What is the solution set?

(2/4, 2) or (0.5, 2)

Find all x in (0, 2) satisfying the inequality x/2 < 1. Provide the solution set in interval notation.

(0, 2)

Solve the inequality 6 - 3x ≥ 0 within the interval (0, 2). What are the solutions in interval notation?

(0, 2/3)

Additional Resources

Understanding and Solving Inequalities within the Interval (0, 2)

Inequalities are fundamental components of algebra and mathematics as a whole, representing relationships where one quantity is greater than, less than, or equal to another. When dealing with inequalities constrained within a specific interval, such as (0, 2), it becomes essential to understand how to find solutions that satisfy the inequality conditions within that domain. This detailed guide aims to delve deeply into the methods, principles, and applications involved in solving inequalities in the interval (0, 2), emphasizing clarity, accuracy, and comprehensive understanding.

Introduction to Inequalities and Intervals

Before exploring the specifics of solving inequalities in the interval (0, 2), it is crucial to establish foundational concepts:

What are Inequalities?

- Inequalities are mathematical statements that compare two expressions, typically involving symbols such as:
- `<` (less than)
- `>` (greater than)
- `≤` (less than or equal to)
- `≥` (greater than or equal to)

- They define a range of possible solutions rather than a specific value, making them highly useful in modeling real-world scenarios where precise equality is not always necessary.

Understanding Intervals

- An interval is a set of real numbers lying between two endpoints.

- The interval (0, 2) is an open interval meaning:
- It contains all real numbers greater than 0 and less than 2.
- It excludes the endpoints 0 and 2 themselves.

Expressed mathematically:

$$(0, 2) = \{ x \in \mathbb{R} \mid 0 < x < 2 \}$$

Why Focus on the Interval (0, 2)?

The specific interval (0, 2) is a common domain in problems for several reasons:

- It limits the scope of solutions, which is particularly important in applied problems where only certain ranges are valid.
- Analyzing inequalities within a bounded domain helps in understanding how solutions behave relative to constraints.
- Many real-world applications—such as probability, physics, and economics—restrict variables to particular ranges.

General Approach to Solving Inequalities in the Interval (0, 2)

Solving inequalities within a specific interval involves a structured approach:

Step 1: Understand the Inequality

- Identify the type of inequality (e.g., linear, quadratic, rational, exponential).
- Recognize any restrictions or domain considerations that are inherent or explicit.

Step 2: Solve the Inequality as an Unrestricted Problem

- Temporarily ignore the interval constraints and solve the inequality over the entire real line.
- Use algebraic techniques suited to the inequality type.

Step 3: Determine the Solution Set

- Express the solution set in interval notation.
- The solution may be a union of multiple intervals or a single interval.

Step 4: Restrict Solutions to the Given Domain (0, 2)

- Intersect the solution set with the interval (0, 2).
- Only the solutions within (0, 2) are valid for the problem.

Step 5: Write the Final Answer in Interval Notation

- Present the solutions that satisfy the inequality and lie within (0, 2).

Types of Inequalities and Their Solutions in (0, 2)

Different types of inequalities require different techniques for solving, but the overarching approach remains similar. Here's a comprehensive overview:

1. Linear Inequalities

Form: $ax + b < 0$, $ax + b > 0$, etc.

Solution Steps:
- Isolate the variable.
- Solve as usual.
- For example:

$$ 3x - 1 > 0 \implies x > \frac{1}{3} $$

- Solution over the entire real line: $(\frac{1}{3}, \infty)$

- Intersect with (0, 2):

$$ (\max(0, \frac{1}{3}), 2) = (\frac{1}{3}, 2) $$

2. Quadratic Inequalities

Form: $ax^2 + bx + c < 0$ or > 0

Solution Steps:
- Find roots of the quadratic.
- Determine the sign of the quadratic in each interval defined by roots.
- Use test points or sign charts.
- For example:

$$ x^2 - 3x + 2 > 0 $$

Roots: $x=1, 2$

Sign analysis:
- $x < 1$, quadratic positive
- $1 < x < 2$, quadratic negative
- $x > 2$, quadratic positive

Solution over entire line: $(-\infty, 1) \cup (2, \infty)$

- Intersect with (0, 2):

$$(0, 1) \cup (2, 2)$$

Since (2, 2) is empty, the solution within (0, 2):

$$(0, 1)$$

3. Rational Inequalities

Form: $\frac{p(x)}{q(x)} > 0$, etc.

Solution Steps:
- Find zeros of numerator and denominator.
- Create a sign chart considering domain restrictions.
- Exclude points where denominator is zero.
- For example:

$$\frac{x - 1}{x - 2} > 0$$

Zeros at x=1, undefined at x=2.

Sign analysis:
- For x in (0, 1), numerator positive, denominator negative → fraction negative
- For x in (1, 2), numerator positive, denominator positive → fraction positive
- For x > 2, numerator positive, denominator positive → positive

Solution: $(1, 2)$

- Intersect with (0, 2):

$$(1, 2)$$

4. Absolute Value Inequalities

Form: |expression| < c, > c, etc.

Solution Steps:
- Rewrite as two inequalities.
- Solve each.
- For example:

$$ |x - 1| < 1 $$

$$ -1 < x - 1 < 1 $$

$$ 0 < x < 2 $$

- Since the entire solution already lies within (0, 2), the intersection is the solution.

Applying the Method to Specific Examples

Let's explore some concrete problems to understand how to find solutions within (0, 2):

Example 1: Linear Inequality

Inequality:

$$ 2x - 3 > 0 $$

Solution:

- Solve over the real numbers:

$$ 2x > 3 \implies x > \frac{3}{2} $$

- General solution: $ (\frac{3}{2}, \infty) $

- Intersect with (0, 2):

$$ (\frac{3}{2}, 2) $$

- Final answer:

$$ (\frac{3}{2}, 2) $$

Example 2: Quadratic Inequality

Inequality:

$$ x^2 - 4x + 3 < 0 $$

Solution:

- Find roots:

$$ x^2 - 4x + 3 = 0 $$

$$ x = \frac{4 \pm \sqrt{16 - 12}}{2} = \frac{4 \pm 2}{2} $$

$$ x=1, 3 $$

- Sign analysis:

- Parabola opens upward.
- Negative between roots: $(1, 3)$.

- Solution over the entire line:

$$ (1, 3) $$

- Intersect with (0, 2):

$$ (1, 2) $$

- Final answer:

$$ (1, 2) $$

Example 3: Rational Inequality

Inequality:

$$ \frac{x - 0.5}{x - 1.5} \geq 0 $$

Solution:

- Critical points: x=0.5 (zero numerator), x=1.5 (zero denominator, undefined).

- Sign analysis:

- For $x < 0.5$:

numerator: negative

denominator: negative (since $x < 1.5$), so fraction:

negative / negative = positive → satisfies ≥ 0

- For $0.5 < x < 1.5$:

numerator: positive

denominator: negative

fraction: positive / negative = negative → does not satisfy

- For $x > 1.5$:

```
numerator: positive

denominator: positive

fraction: positive / positive = positive → satisfies

- Solution over entire line:

\[ (-\infty, 0.5] \cup (1.5, \infty) \]

- Intersect with (0, 2):

\[ (0.5, 2) \]

Since
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