Find Y(10) From Y(x)= 7x5 Using The Definition Of A Derivative. (Do Not Include " Y(10)=" In Your Answer.)

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When approaching the task of evaluating a function at a specific point using the definition of a derivative, it is essential to understand both the function's structure and the fundamental principles of derivatives. In this article, we will explore how to find the value of the function $Y(x) = 7x^5$ at $x = 10$, using the rigorous approach of the derivative's definition, also known as the limit definition. This process not only reveals the value at a particular point but also deepens comprehension of the derivative's conceptual foundation.

Understanding the Function and Its Components

Before diving into the derivative's definition, let's analyze the given function:

- Function: $Y(x) = 7x^5$

- Objective: Find the value of the function at $x = 10$, using the limit definition of the derivative.

Given that the function is a polynomial, it is straightforward to evaluate directly. However, the goal is to use the derivative's definition, which involves limits, to arrive at the value. This approach is especially useful when the function is more complex or when learning the fundamental principles.

The Definition of the Derivative

The derivative of a function at a point $x = a$ is defined as the limit:

$$
Y'(a) = \lim_{h \to 0} \frac{Y(a+h) - Y(a)}{h}
$$

This limit, if it exists, gives the instantaneous rate of change of the function at point a, which is the derivative at that point.

In this context, to find the value of the function at $x=10$, one approach involves considering the derivative at $x=10$ and then integrating or using related concepts. However, since the task emphasizes the use of the derivative's definition, we can interpret it as a method to approximate or verify the function's value at that point.

Note: Usually, to evaluate $Y(10)$, one simply substitutes $x=10$ into the function, but here, the focus is on applying the limit definition to understand the process thoroughly.

Applying the Limit Definition Step-by-Step

Let's proceed step-by-step.

Step 1: Identify the point a=10

- Our point of interest is $x=10$.

- To find $Y(10)$ via the limit definition, we consider the difference quotient centered around $x=10$.

Step 2: Write the difference quotient

$$
\frac{Y(10+h) - Y(10)}{h}
$$

where h approaches zero.

Step 3: Express Y(10+h) and Y(10)

- $Y(10+h) = 7(10+h)^5$

- $Y(10) = 7 \times 10^5$

The difference quotient becomes:

$$
\frac{7(10+h)^5 - 7 \times 10^5}{h}
$$

Factor out 7:

$$
\frac{7[(10+h)^5 - 10^5]}{h}
$$

Step 4: Expand (10+h)^5

Using the binomial theorem:

$$
(10+h)^5 = \sum_{k=0}^5 \binom{5}{k} 10^{5-k} h^{k}
$$

which expands as:

$$
10^5 + 5 \times 10^4 h + 10 \times 10^3 h^2 + 10 \times 10^2 h^3 + 5 \times 10 h^4 + h^5
$$

Simplify coefficients:

$$
10^5 + 5 \times 10^4 h + 10 \times 10^3 h^2 + 10 \times 10^2 h^3 + 5 \times 10 h^4 + h^5
$$

which numerically is:

$$
100000 + 5 \times 10000 h + 10 \times 1000 h^2 + 10 \times 100 h^3 + 5 \times 10 h^4 + h^5
$$

or:

$$
100000 + 50000 h + 10000 h^2 + 1000 h^3 + 50 h^4 + h^5
$$

Step 5: Subtract 10^5 and simplify

Recall that $10^5 = 100000$, so:

$$
(10+h)^5 - 10^5 = 50000 h + 10000 h^2 + 1000 h^3 + 50 h^4 + h^5
$$

Now, multiply this entire expression by 7:

$$
7 \times (50000 h + 10000 h^2 + 1000 h^3 + 50 h^4 + h^5) = 350000 h + 70000 h^2 + 7000 h^3 + 350 h^4 + 7 h^5
$$

\]

Step 6: Write the difference quotient

$$
\frac{7[(10+h)^5 - 10^5]}{h} = \frac{350000 h + 70000 h^2 + 7000 h^3 + 350 h^4 + 7 h^5}{h}
$$

Divide each term by h:

$$
350000 + 70000 h + 7000 h^2 + 350 h^3 + 7 h^4
$$

Step 7: Take the limit as h approaches 0

As h approaches zero:

$$
\lim_{h \to 0} [350000 + 70000 h + 7000 h^2 + 350 h^3 + 7 h^4] = 350000
$$

since all terms involving h vanish.

Interpretation: The value 350000 corresponds to the derivative of the function at x=10, indicating the slope of the tangent line at that point.

Understanding the Relationship Between the Derivative and the Function's Value

While the above process computes the derivative at x=10, it also reinforces the fundamental relationship between derivatives and the original function.

- Derivative at a point gives the slope of the tangent line at that point.

- Function value at a point is obtained by direct substitution.

In this particular case, because the function is polynomial and straightforward, evaluating directly is trivial:

$$
Y(10) = 7 \times 10^5 = 7 \times 100000 = 700000
$$

However, the process of differentiating and taking the limit illustrates the fundamental limit process that defines derivatives, which is essential for understanding calculus.

Conclusion: Using the Limit Definition to Find Function Values

While the limit process mainly helps in understanding the derivative's nature, it also illustrates how the behavior of the function around a point influences the derivative. In practice, to find the specific value of the function at x=10, direct substitution is faster. Yet, learning to apply the limit definition:

- Deepens understanding of how derivatives are formulated.

- Provides crucial insights into the behavior of functions and their rates of change.

- Equips students and practitioners with the tools to analyze more complex functions where direct substitution isn't straightforward.

Summary:

- The binomial theorem expansion was used to expand (10+h)^5.

- The difference quotient was simplified and then evaluated as h approaches zero.

- The derivative at x=10 was found to be 350,000, indicating the slope of the tangent line at that point.

- The actual value of the function at x=10 is 700,000, obtained through direct substitution but understood through the process outlined.

By mastering this approach, students deepen their understanding of calculus fundamentals, enabling them to analyze a broad class of functions with confidence and precision.

Frequently Asked Questions

How do you apply the definition of a derivative to find Y'(10) for Y(x) = 7x^5?

You use the limit definition of the derivative: Y'(a) = lim_{h→0} [Y(a+h) - Y(a)] / h. For x=10, substitute a=10 into the function and evaluate the limit as h approaches 0.

What is the first step in finding the derivative of Y(x) =

7x^5 using the limit definition?

The first step is to write the difference quotient: [7(10+h)^5 - 7(10)^5] / h, and then take the limit as h approaches 0.

How do you simplify the expression in the limit when applying the derivative definition to Y(x) = 7x^5 at x=10?

Expand (10+h)^5 using the binomial theorem, multiply by 7, subtract 710^5, and then divide by h, simplifying numerator and canceling h where possible before taking the limit.

What is the significance of the limit process in finding Y'(10) from the function Y(x)=7x^5?

The limit process calculates the instantaneous rate of change of the function at x=10 by considering how the function behaves for values of x close to 10.

Can you explain how the binomial theorem helps in computing the derivative from the definition for Y(x)=7x^5 at x=10?

Yes, it helps expand (10+h)^5 into a polynomial expression, making it easier to subtract 710^5 and factor out h, which is essential for evaluating the limit.

What is the derivative of Y(x) = 7x^5 at x=10 obtained through the limit definition?

By applying the limit definition, the derivative at x=10 is 175,000, which is 7 times 5 times 10 raised to the fourth power.

Why is it important to cancel out h in the numerator when applying the limit definition of the derivative?

Cancelling h simplifies the expression and allows for direct evaluation of the limit as h approaches 0, which is essential for finding the derivative.

What is the general approach to find the derivative at a specific point using the limit definition for polynomial functions?

Substitute the point into the difference quotient, expand and simplify the numerator, cancel h where possible, then take the limit as h approaches 0 to find the derivative.

How does understanding the limit definition of a derivative help in comprehending the concept of instantaneous rate of change at x=10?

It shows how the average rate of change over an interval approaches the exact rate of change at a point as the interval shrinks to zero, providing a precise measure of the function's instantaneous change at x=10.

What is the key step in computing Y'(10) from the limit definition for Y(x)=7x^5 without directly using power rules?

The key step is expanding (10+h)^5 via the binomial theorem, then simplifying the difference quotient, cancelling h, and evaluating the limit as h approaches 0.

Additional Resources

Find Y(10) From Y(x)= 7x^5 Using The Definition Of A Derivative. (Do Not Include "Y(10)=" In Your Answer.)

In the world of calculus, derivatives serve as a fundamental tool for understanding how functions behave—how they change, grow, or diminish. While many students are introduced to derivatives through rules and formulas, the core concept remains rooted in a precise definition: the limit of a difference quotient as the change in the independent variable approaches zero. This article takes a deep dive into applying the definition of the derivative to evaluate a specific function at a given point, demonstrating the process step-by-step, and revealing the power of foundational calculus principles in action.

Understanding the Foundations: The Definition of the Derivative

Before venturing into calculations, it's essential to grasp what the derivative signifies. In simple terms, the derivative of a function at a particular point measures the instantaneous rate of change of the function at that point. Mathematically, it is defined as:

f'(x) = lim (h → 0) [f(x + h) - f(x)] / h

Here, `h` represents a small change in the input variable `x`, and the limit as `h` approaches zero captures the idea of an infinitesimal change, providing the exact slope of the tangent line to the function at `x`.

Applying this definition to find the value of the function at a specific point involves two key steps:

1. Expressing the difference quotient [f(x + h) - f(x)] / h.
2. Computing the limit of this quotient as `h` approaches zero.

While the derivative itself is a rate of change, for our purposes, we will leverage the definition directly to evaluate the function at a specific point—here, at the point where $x = 10$.

The Function in Focus: $Y(x) = 7x^5$

The function under consideration is a polynomial of degree five, scaled by a factor of 7:

$$Y(x) = 7x^5$$

The task is to evaluate this function at $x = 10$ by utilizing the definition of the derivative. Although the function's value at $x = 10$ can be directly computed using substitution, the focus here is on demonstrating the process via the limit definition, which deepens the understanding of the derivative's conceptual foundation.

Step-by-Step Application of the Definition

Step 1: Write the difference quotient

Start by constructing the difference quotient for the function:

$$\frac{Y(x + h) - Y(x)}{h}$$

Substituting the function:

$$\frac{7(x + h)^5 - 7x^5}{h}$$

Factoring out the common factor:

$$7 \times \frac{(x + h)^5 - x^5}{h}$$

The core challenge is evaluating the limit of:

$$\lim_{h \to 0} 7 \times \frac{(x + h)^5 - x^5}{h}$$

at $x = 10$.

Step 2: Expand $(x + h)^5$

Using the binomial theorem:

$$(x + h)^5 = x^5 + 5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5$$

This expansion enables us to carefully analyze the numerator:

$$(x + h)^5 - x^5 = 5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5$$

Step 3: Simplify the difference quotient

Divide each term by `h`:

$$ \frac{5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5}{h} = 5x^4 + 10x^3h + 10x^2h^2 + 5xh^3 + h^4 $$

Multiplying by 7, the factor outside:

$$ 7 \times \left(5x^4 + 10x^3h + 10x^2h^2 + 5xh^3 + h^4 \right) $$

which simplifies to:

$$ 35x^4 + 70x^3h + 70x^2h^2 + 35xh^3 + 7h^4 $$

Step 4: Take the limit as h approaches zero

As `h` tends towards zero, all terms containing `h` vanish:

$$ \lim_{h \to 0} \left(35x^4 + 70x^3h + 70x^2h^2 + 35xh^3 + 7h^4 \right) = 35x^4 $$

This result indicates that the derivative of the function at point `x` is:

$$ Y'(x) = 35x^4 $$

Step 5: Evaluate at x = 10

Now, substituting `x = 10`:

$$ Y'(10) = 35 \times 10^4 $$

Calculating:

$$ 10^4 = 10,000 $$

$$ 35 \times 10,000 = 350,000 $$

This value represents the instantaneous rate of change of the function at `x = 10`. However, because the original task emphasizes finding the function value at `x = 10` via the derivative's definition, we need to understand that the process illustrates how the derivative relates to the function's behavior at that point, even if the direct calculation of the function's value is straightforward by substitution.

Interpreting the Result

The process above highlights a critical aspect of calculus: the derivative at a point not only indicates the slope of the tangent line to the graph at that point but also provides a foundation for understanding how the function behaves locally. By applying the limit definition, we affirm the derivative's value, which is essential in many advanced applications, such as optimization, motion analysis, and modeling.

In the context of evaluating the function at `x = 10`, the fundamental calculation is simply:

$$ Y(10) = 7 \times 10^5 = 7 \times 100,000 = 700,000 $$

yet the derivative calculation illustrates the function's rate of change precisely at that point, which can be invaluable for understanding the function's behavior around `x = 10`.

Broader Implications and Real-World Applications

Understanding how to derive a function's value at a specific point using the definition of the derivative isn't just an academic exercise. It has practical implications in various fields:

- Physics: Determining instantaneous velocity from position functions.
- Economics: Calculating marginal cost or revenue at a specific production level.
- Biology: Assessing the rate of change in population models.
- Engineering: Analyzing stress-strain relationships at particular points.

Mastering the approach through the limit definition equips learners with a deeper conceptual grasp, fostering more flexible and robust problem-solving skills.

Concluding Remarks

The journey from the fundamental limit definition to evaluating a specific function value exemplifies the elegance and power of calculus. While straightforward substitution often suffices for polynomial functions, applying the limit definition reinforces a core principle: derivatives are grounded in the concept of limits of difference quotients. This understanding not only enhances computational skills but also enriches intuition about how functions behave locally—insights that are essential across scientific and engineering disciplines.

By dissecting the process step-by-step, students and enthusiasts alike can appreciate how the abstract notion of limits translates into tangible calculations, ultimately deepening their mastery of calculus and its myriad applications.

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