Given A State-space Model:x= [0 1] X +=[0] [-5 -21/4] [1] Uy = [5 4]xa) Find The Controllability Matrix.

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Understanding State-space Models in Control Systems

State-space models are fundamental in modern control theory, providing a mathematical framework to model and analyze dynamic systems. These models describe the behavior of a system using a set of first-order differential (or difference) equations. They are especially powerful for multi-input, multi-output (MIMO) systems, offering a comprehensive way to design controllers and analyze system properties such as controllability and observability.

In this article, we will delve into the specific state-space model provided, interpret its components, and guide you through the process of computing the controllability matrix, a critical step in control system analysis.

Components of the Given State-space Model

Let's break down the given model:

- State Equation:

$$
\dot{x} = A x + B u
$$

- Output Equation:

$$
y = C x + D u
$$

From the problem statement, the model is expressed as:

$$
x = \begin{bmatrix} 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ -5 \end{bmatrix} x + \begin{bmatrix} -21/4 \\ 1 \end{bmatrix} u
$$

and the output:

$$
y = \begin{bmatrix} 5 & 4 \end{bmatrix} x
$$

However, the notation appears somewhat inconsistent. Clarifying the standard form:

- The state matrix A:

$$
A = \begin{bmatrix}
0 & 1 \\
-5 & -\frac{21}{4}
\end{bmatrix}
$$

- The input matrix B:

$$
B = \begin{bmatrix}
0 \\
1
\end{bmatrix}
$$

- The output matrix C:

$$
C = \begin{bmatrix}
5 & 4
\end{bmatrix}
$$

- The feedforward matrix D is typically zero in many systems unless specified otherwise.

Given this interpretation, the model describes a second-order system with the matrices as above.

What Is the Controllability Matrix?

Controllability is a key concept in control theory that determines whether it is possible to steer the system's state from any initial state to any desired final state within finite time, using appropriate inputs.

The controllability matrix, denoted as $\mathcal{C}$, is constructed from the matrices A and B and is used to assess this property. Specifically:

$$
\mathcal{C} = \left[B \quad A B \quad A^2 B \quad \dots \quad A^{n-1} B \right]
$$

where:

- \(n\) is the order of the system (dimension of the state vector, here \(n=2\))
- \(A^k B\) represents the matrix \(B\) multiplied by the matrix \(A\) raised to the power \(k\)

The system is controllable if and only if the controllability matrix \(\mathcal{C} \) has full rank, i.e., rank equal to \(n\).

In our case, since the system is second order (\(n=2\)), the controllability matrix will be:

\[
\mathcal{C} = [B \quad A B]
\]

Why is controllability important? Because it informs whether a state feedback controller can be designed to move the system from any initial state to any final desired state, which is crucial for effective control system design.

Step-by-step Calculation of the Controllability Matrix

Let's proceed to compute the controllability matrix for the provided system.

Step 1: Define Matrices \(A\) and \(B\)

Based on the earlier interpretation:

\[
A = \begin{bmatrix}
0 & 1 \\
-5 & -\frac{21}{4}
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
0 \\
1
\end{bmatrix}
\]

Step 2: Calculate \(A B\)

Multiply matrix \(A\) by \(B\):

\[

A B = \begin{bmatrix}
0 & 1 \\
-5 & -\frac{21}{4}
\end{bmatrix}
\begin{bmatrix}
0 \\
1
\end{bmatrix}
= \begin{bmatrix}
(0)(0) + (1)(1) \\
(-5)(0) + \left(-\frac{21}{4}\right)(1)
\end{bmatrix}
= \begin{bmatrix}
1 \\
-\frac{21}{4}
\end{bmatrix}
\]

Note that \(-\frac{21}{4} = -5.25\).

Step 3: Form the Controllability Matrix \(\mathcal{C} \)

Now, assemble the controllability matrix:

\[
\mathcal{C} = [B \quad A B] = \left[\begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 1 \\ -\frac{21}{4} \end{bmatrix} \right]
\]

Expressed as a \(2 \times 2\) matrix:

\[
\mathcal{C} = \begin{bmatrix}
0 & 1 \\
1 & -\frac{21}{4}
\end{bmatrix}
\]

Assessing Controllability

To determine whether the system is controllable, compute the rank of \(\mathcal{C} \). For a \(2 \times 2\) matrix, the rank can be found by calculating the determinant:

\[
\det(\mathcal{C}) = (0)(-\frac{21}{4}) - (1)(1) = 0 - 1 = -1
\]

Since the determinant is non-zero ($-1 \neq 0$), the matrix is full rank, i.e., rank 2.

Conclusion: The system is controllable because the controllability matrix has full rank.

Implications of Controllability in System Design

Knowing that the system is controllable opens up several possibilities:

- State Feedback Control: You can design a state feedback controller $u = -Kx + r$ to place the eigenvalues of the closed-loop system at desired locations, ensuring stability and performance.

- System Stabilization: With controllability confirmed, the system can be stabilized through appropriate feedback control strategies.

- Observer Design: While controllability pertains to inputs, observability (which can be analyzed similarly) relates to the ability to estimate the system's states from outputs.

Summary of Key Steps in Computing the Controllability Matrix

To recap, here's a structured approach for calculating the controllability matrix for a given linear system:

1. Identify the matrices A and B from the state-space equations.

2. Calculate $A B$, $A^2 B$, ..., up to $A^{n-1} B$, where n is the system order.

3. Form the controllability matrix $\mathcal{C} = [B \quad A B \quad \dots \quad A^{n-1} B]$.

4. Determine the rank of $\mathcal{C}$. If full rank, the system is controllable.

This process is fundamental in control system analysis and controller design.

Additional Considerations and Practical Tips

- Numerical Stability: When dealing with large matrices or systems with parameters close to singularity, use numerical methods with appropriate precision to evaluate rank and determinants.

- Controllability vs. Stabilizability: While controllability ensures the ability to move between states, in some systems, even uncontrollable modes are stable. Stabilizability is a weaker condition relevant for certain control strategies.

- Software Tools: Utilize control system toolboxes in MATLAB, Python (e.g., Control library), or other software to automate the computation of controllability matrices and rank analysis.

Conclusion

Analyzing the controllability of a system is a cornerstone in control theory. For the given state-space model, we successfully determined the controllability matrix and confirmed the system's controllability. This foundational step paves the way for designing effective controllers, ensuring the system can be driven to desired states, and optimizing system performance.

Understanding how to formulate and compute the controllability matrix empowers control engineers and students alike to assess and influence dynamic systems effectively. Mastery of these concepts is essential for advancing in control system design, stability analysis, and real-world applications such as robotics, aerospace, automotive systems, and industrial

Frequently Asked Questions

What is the state-space model given in the problem?

The state-space model is given as x' = A x + B u, where A = [[0, 1], [-5, -21/4]] and B = [[5], [4]].

How do you define the controllability matrix for a given state-space system?

The controllability matrix is defined as C = [B, A B], which involves concatenating B and A multiplied by B to assess controllability.

What are the steps to compute the controllability matrix for the given system?

First, calculate A B by multiplying matrix A with B. Then, form the controllability matrix C by placing B as the first block column and A B as the second block column.

Can you compute the controllability matrix for the provided matrices A and B?

Yes. First, compute A B = [[0, 1], [-5, -21/4]] [[5], [4]] = [[14], [-55 + (-21/4)4]] = [[4], [-25 - 21]] = [[4], -46]. The controllability matrix is C = [[5, 4], [4, -46]].

Why is the controllability matrix important in the context of the given state-space model?

The controllability matrix determines whether the system states can be fully controlled by the input

u. If the matrix has full rank (equal to the number of states), the system is controllable.

Additional Resources

Given A State-space Model: x = [0 1] x + [0; -5 - 21/4] u = [5 4], find the controllability matrix.

This problem presents an essential aspect of control systems analysis: determining the controllability of a linear time-invariant (LTI) system. Controllability is a fundamental property that indicates whether it is possible to steer the system's state from any initial condition to any desired final state within a finite time using an appropriate input. Understanding how to compute the controllability matrix from the state-space representation is crucial for control system design, especially when designing controllers or observers. In this article, we will analyze the given model, derive the controllability matrix, and discuss its implications thoroughly.

Understanding the State-space Model

Before diving into the controllability matrix, it's vital to interpret the provided state-space model correctly.

System Representation

The general form of a continuous-time LTI system is:

$$
\dot{x}(t) = A x(t) + B u(t)
$$

where:
- $x(t) \in \mathbb{R}^n$ is the state vector.
- $u(t) \in \mathbb{R}^m$ is the input vector.
- $A \in \mathbb{R}^{n \times n}$ is the system matrix.
- $B \in \mathbb{R}^{n \times m}$ is the input matrix.

From the problem statement:
- $x = [x_1, x_2]^T$
- $A = \begin{bmatrix} 0 & 1 \\ -5 & -\frac{21}{4} \end{bmatrix}$
- $B = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

This model describes a second-order system, which is quite common in mechanical and electrical systems. The specific values of A and B determine the system's dynamics and controllability.

Key features:
- The system matrix A has a companion form, typical for second-order systems.

- The input matrix B indicates how the control input influences the states.

Controllability in State-space Systems

Controllability assesses whether the system's states can be fully manipulated through the input $u(t)$.

Definition of Controllability

A system is controllable if, for any initial state x_0, there exists an input $u(t)$ that transfers the system to any final state x_f within finite time T.

Mathematically, the controllability is examined via the controllability matrix:

$$
\mathcal{C} = [B \quad A B \quad A^2 B \quad \dots \quad A^{n-1} B]
$$

where n is the system order (here $n=2$).

If $\mathcal{C}$ has full rank n, the system is controllable.

Features:
- The controllability matrix encapsulates the influence of input over multiple steps.
- Full rank indicates complete controllability.

Constructing the Controllability Matrix

Given the second-order system, the controllability matrix simplifies to:

$$
\mathcal{C} = [B \quad A B]
$$

since $n=2$.

Calculating $A B$

Given:

$$
A = \begin{bmatrix} 0 & 1 \\ -5 & -\frac{21}{4} \end{bmatrix}
$$
and
$$
B = \begin{bmatrix} 5 \\ 4 \end{bmatrix}
$$

Compute $A B$:

$$
A B = \begin{bmatrix} 0 & 1 \\ -5 & -\frac{21}{4} \end{bmatrix} \begin{bmatrix} 5 \\ 4 \end{bmatrix}
= \begin{bmatrix} (0)(5) + (1)(4) \\ (-5)(5) + \left(-\frac{21}{4}\right)(4) \end{bmatrix}
$$

Calculate each component:

- First row: $0 + 4 = 4$
- Second row: $-25 + (-21) = -46$

Note: The second component simplifies as:

$$
-\frac{21}{4} \times 4 = -21
$$

So,

$$
A B = \begin{bmatrix} 4 \\ -46 \end{bmatrix}
$$

Forming the Controllability Matrix

Now, assemble:

$$
\mathcal{C} = [B \quad A B] = \begin{bmatrix} 5 & 4 \\ 4 & -46 \end{bmatrix}
$$

Assessing the Controllability

To determine if the system is controllable, evaluate the rank of $\mathcal{C}$.

Calculating the Determinant

$$
\det(\mathcal{C}) = (5)(-46) - (4)(4) = -230 - 16 = -246
$$

Since $\det(\mathcal{C}) \neq 0$, the controllability matrix is full rank.

Implication:

- The rank of $\mathcal{C}$ is 2, which equals the system order $n=2$.
- Therefore, the system is controllable.

Implications of Controllability

Knowing that the system is controllable allows for various control design strategies, such as:

- State feedback control: Designing controllers that assign desired eigenvalues.
- Trajectory planning: Guaranteeing the ability to reach any state.
- Observer design: Ensuring the system's states can be estimated and controlled effectively.

Pros of a controllable system:
- Flexibility in control design.
- Ability to stabilize and achieve desired performance.
- Facilitates pole placement and optimal control strategies.

Cons or limitations:
- Controllability does not guarantee robustness or optimality.
- Practical implementation may encounter actuator constraints or disturbances.
- System may be controllable theoretically but challenging to control in real-world conditions.

Summary and Final Remarks

In this analysis, we examined the given state-space model, derived the controllability matrix, and confirmed the system's controllability through the full rank of the matrix. The procedure involved:

- Interpreting the system matrices.
- Computing $A B$ explicitly.
- Forming the controllability matrix $\mathcal{C}$.
- Checking its determinant for full rank.

The key takeaway is that the given second-order system is controllable, which opens the door for various advanced control techniques. Control engineers can now proceed confidently to design controllers, observers, or perform further analysis knowing that the system's states can be fully manipulated through the control input.

Final thoughts:

- The method demonstrated is standard for analyzing controllability in linear systems.
- Understanding the structure of A and B simplifies the process.
- Ensuring controllability is a critical step before moving onto controller design.

This comprehensive approach underscores the importance of the controllability matrix in the control systems toolbox and highlights its role in ensuring the desired system performance can be achieved.

Given A State Space Model X 0 1 X 0 5 21 4 1 Uy 5 4 Xa Find The Controllability Matrix

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