Given The Following Transfer Function: H(z): 1.7/1 + 3.6 Z^-1 - 0.5/1-0.9z^-1 A. Calculate Its Right-sided

Understanding the Transfer Function H(z): Analyzing Its Right-Sidedness

Given The Following Transfer Function: H(z): 1.7/1 + 3.6 Z^-1 - 0.5/1-0.9z^-1 A. Calculate Its Right-sided — this statement introduces a fundamental problem in discrete-time signal processing and system analysis. The task involves examining the transfer function's properties to determine whether it is right-sided, left-sided, or two-sided. Understanding these classifications is essential for analyzing system stability, causality, and realizability.

In this article, we will explore the transfer function provided, interpret its structure, and systematically calculate whether it is right-sided. We will also delve into the importance of right-sidedness in digital filter design, the mathematical tools used to analyze transfer functions, and practical steps for such calculations.

What is a Transfer Function in Discrete-Time Systems?

Before analyzing the specific transfer function, it’s vital to understand what a transfer function represents in discrete-time systems.

Definition and Significance

- The transfer function, denoted as $H(z)$, describes the relationship between the input and output of a discrete-time linear time-invariant (LTI) system in the z-domain.
- It encapsulates the system's behavior, including stability, causality, and frequency response.
- Expressed as a ratio of polynomials in z or z^{-1}, it facilitates easy analysis and filter implementation.

Standard Form of Transfer Functions

- Typically written as:

$$H(z) = N(z) / D(z)$$

where $N(z)$ and $D(z)$ are polynomials in z^{-1} (or z).
- The order of these polynomials indicates the system's order, influencing its dynamic properties.

Analyzing the Given Transfer Function

The provided transfer function appears to be a combination of two rational functions:

$$H(z) = 1.7 / (1 + 3.6 z^{-1}) - 0.5 / (1 - 0.9 z^{-1})$$

However, the original statement's notation could be misinterpreted. To clarify, let's express the transfer function properly:

$$H(z) = \frac{1.7}{1 + 3.6 z^{-1}} - \frac{0.5}{1 - 0.9 z^{-1}}$$

This form suggests a difference of two rational functions, each with specific pole structures.

Expressing the Transfer Function in a Common Denominator

To analyze the system's properties thoroughly, it is helpful to combine these fractions into a single rational function:

1. Find the common denominator:

Denominator = $(1 + 3.6 z^{-1})(1 - 0.9 z^{-1})$

2. Rewrite each term with this common denominator:

$H(z) = [1.7 (1 - 0.9 z^{-1}) - 0.5 (1 + 3.6 z^{-1})] / [(1 + 3.6 z^{-1})(1 - 0.9 z^{-1})]$

3. Expand numerator:

Numerator = $1.7(1 - 0.9 z^{-1}) - 0.5(1 + 3.6 z^{-1})$

$= 1.7 - 1.53 z^{-1} - 0.5 - 1.8 z^{-1}$

$= (1.7 - 0.5) + (-1.53 z^{-1} - 1.8 z^{-1})$

$= 1.2 - 3.33 z^{-1}$

4. Expand denominator:

$(1 + 3.6 z^{-1})(1 - 0.9 z^{-1}) = 1(1 - 0.9 z^{-1}) + 3.6 z^{-1}(1 - 0.9 z^{-1})$

$= 1 - 0.9 z^{-1} + 3.6 z^{-1} - 3.24 z^{-2}$

$= 1 + (3.6 - 0.9) z^{-1} - 3.24 z^{-2}$

$= 1 + 2.7 z^{-1} - 3.24 z^{-2}$

So, the combined transfer function becomes:

$$H(z) = \frac{1.2 - 3.33 z^{-1}}{1 + 2.7 z^{-1} - 3.24 z^{-2}}$$

Determining Right-Sidedness of the Transfer Function

The classification of a transfer function as right-sided, left-sided, or two-sided depends on the properties of its impulse response and the pole-zero configuration.

Concepts of Right-Sided and Left-Sided Sequences

- Right-sided sequences: Impulse responses that are causal and non-zero for t □ 0.
- Left-sided sequences: Responses that are non-zero for t □ 0.
- Two-sided sequences: Responses that are non-zero for both t □ 0 and t □ 0.

In the z-domain, these correspond to different regions of convergence (ROC):

- Right-sided: ROC extends outward beyond the outermost pole (|z| > |poles|).
- Left-sided: ROC extends inward, |z| < |poles|.
- Two-sided: ROC lies between poles.

Poles and Zeros Analysis for Right-Sidedness

To determine if H(z) is right-sided:

1. Find the poles by solving the denominator:

$1 + 2.7 z^{-1} - 3.24 z^{-2} = 0$

2. Rewrite as a standard polynomial:

Multiply through by z^2:

$z^2 + 2.7 z - 3.24 = 0$

3. Solve quadratic:

$z = \frac{-2.7 \pm \sqrt{(2.7)^2 - 4 \times 1 \times (-3.24)}}{2}$

Calculate discriminant:

$\Delta = 7.29 + 13.0 = 20.29$

$\sqrt{\Delta} \approx 4.505$

4. Find roots:

- $z = \frac{-2.7 + 4.505}{2} \approx \frac{1.805}{2} \approx 0.9025$

- $z = \frac{-2.7 - 4.505}{2} \approx \frac{-7.205}{2} \approx -3.6025$

The poles are approximately at:

- $z_1 \approx 0.9025$
- $z_2 \approx -3.6025$

Implications:

- For right-sided (causal) systems, the ROC must satisfy $|z| > \max |poles|$. Since the poles are at approximately 0.9025 and 3.6025, the ROC for a right-sided system is $|z| > 3.6025$.

- The system's impulse response is causal if the ROC includes $|z| > 3.6025$.

Additional considerations:

- The zeros' locations can further influence the system's properties, but poles primarily determine stability and causality.

Stability and Causality of the System

A key aspect of transfer function analysis is understanding system stability.

Conditions for Stability

- The system is BIBO stable if all poles lie inside the unit circle, i.e., $|z| < 1$.

- In our case, poles are at approximately 0.9025 (inside the unit circle) and -3.6025 (outside the unit circle).

- Since one pole lies outside $|z| = 1$, the system is unstable if considering the unit circle as the stability boundary.

Implication for Right-Sidedness

- Despite the pole at 0.9025 being inside the unit circle, the pole at -3.6025 is outside, indicating potential instability.

- For right-sided sequences, the ROC must be outside the outermost pole; here, that would be $|z| > 3.6025$.

- The system's impulse response will be causal but not stable because it has an outside pole.

Practical Steps to Confirm Right-Sidedness

To conclusively determine whether H(z) is right-sided, follow these steps:

1. Identify poles and zeros from the factored form.
2. Determine the ROC:

- For right-sided, ROC: $|z| > \text{largest pole magnitude}$.
- For left-sided, ROC: $|z| < \text{smallest pole magnitude}$.

3. Check whether the ROC includes the unit circle:

- If yes, the system can be stable.
- If no, the system is unstable.

4. Verify if the transfer function is causal:

- Causality aligns with right-sidedness, provided the ROC extends outward beyond all poles.

Conclusion: Given the pole locations, the transfer function H

Frequently Asked Questions

What does the transfer function H(z) = (1.7) / (1 + 3.6 z^{-1}) - 0.5 / (1 - 0.9 z^{-1}) represent in discrete-time systems?

It represents the system's behavior in the z-domain, characterizing how input signals are transformed to outputs, including poles and zeros that influence stability and response.

How do you determine the right-sided nature of the transfer function H(z)?

By analyzing the region of convergence (ROC), which for a right-sided sequence is outside the outermost pole, typically ensuring system causality and stability if the ROC includes the unit circle.

What is the significance of calculating the right-sided transfer function for H(z)?

Calculating the right-sided transfer function helps identify the causal and stable part of the system, ensuring the system's response is physically realizable and stable.

How can you compute the right-sided transfer function from the given H(z)?

By expressing H(z) as a sum of terms corresponding to right-sided sequences, then identifying the regions of convergence and ensuring the Z-transform corresponds to causal signals.

What role do poles and zeros play in analyzing the right-sided nature of H(z)?

Poles determine the stability and ROC, while zeros affect the zero locations in the Z-plane; for right-sided sequences, the ROC is outside the outermost pole.

Can you provide a step-by-step method to find the right-sided transfer function for the given H(z)?

Yes, first factor the transfer function into partial fractions, then identify terms corresponding to causal sequences with ROC outside the outermost pole, and verify stability conditions.

What are common challenges faced when calculating the right-sided transfer function from a complex H(z)?

Challenges include correctly performing partial fraction expansion, determining the appropriate ROC, and ensuring the resulting sequence is causal and stable.

How does the value of the poles influence whether H(z) is right-sided or not?

If the poles are inside the ROC that includes the unit circle (outside the outermost pole), then H(z) is right-sided, indicating a causal and stable system.

Additional Resources

Given the following transfer function: H(z): 1.7 / (1 + 3.6 z^{-1}) - 0.5 / (1 - 0.9 z^{-1}) — A comprehensive analysis of its right-sidedness and implications

---

Introduction

In digital signal processing and control systems, understanding the nature of a transfer function is fundamental to analyzing system stability, causality, and the type of signals it can process effectively. The transfer function provided, H(z), combines two rational functions with negative powers of z, indicating a discrete-time system, likely a digital filter or control system component.

This article aims to dissect the properties of this transfer function, focusing on its "right-sided" nature, a technical term referring to whether the system's impulse response is causal, anti-causal, or two-sided. We will explore the mathematical structure, analyze the poles and zeros, interpret the implications of right-sidedness, and discuss how these properties influence the system's behavior.

Understanding the Transfer Function: Formulation and Components

Expression of the Transfer Function

The given transfer function is:

$$ H(z) = \frac{1.7}{1 + 3.6 z^{-1}} - \frac{0.5}{1 - 0.9 z^{-1}} $$

This expression is a sum of two rational functions, each with a single pole (denominator) and a numerator constant. To work with it more conveniently, it is customary to rewrite it in terms of positive powers of z:

$$
H(z) = \frac{1.7 z}{z + 3.6} - \frac{0.5 z}{z - 0.9}
$$

by multiplying numerator and denominator of each term by z.

Note: These forms are typical in digital filter design, where transfer functions are expressed in terms of z^{-1} for convenience related to recursive implementations.

Key Components: Poles and Zeros

- Poles: Values of z that make the denominator zero. They determine the system's stability and causality.
- Zeros: Values of z that make the numerator zero. They influence the frequency response and filter characteristics.

For each term:

1. First term:
- Zero at $z = 0$ (since numerator is proportional to z)
- Pole at $z = -3.6$

2. Second term:
- Zero at $z = 0$
- Pole at $z = 0.9$

The overall transfer function's poles are at $z = -3.6$ and $z = 0.9$, and zeros at $z = 0$ (double zero considering the sum).

System Classification: Causality and Right-Sidedness

What Does "Right-Sided" Mean?

In the context of discrete-time systems, "right-sided" refers to the nature of the system's impulse response:

- Right-sided (causal): The impulse response $h[n]$ is zero for $n < 0$. This is typical for physically realizable systems, which cannot respond before an input is applied.
- Left-sided (anti-causal): $h[n] = 0$ for $n > 0$. Such systems are theoretical and often used in analysis or non-causal filters.
- Two-sided: Non-zero for both positive and negative n.

The key to determining whether the system is right-sided lies in the system's impulse response and the region of convergence (ROC) of its z-transform.

Implications of System Poles and Zeros on Causality

The causality of a system is primarily governed by its pole locations relative to the unit circle and the ROC:

- For causal (right-sided) systems, the ROC must be outside the outermost pole, including the unit circle if the system is stable.

- For anti-causal (left-sided) systems, the ROC is inside the innermost pole.
- For two-sided systems, the ROC is a ring between poles.

Given the poles at $z = -3.6$ and $z = 0.9$, the system's causality hinges on the ROC:

- The pole at $z = 0.9$ is inside the unit circle ($|0.9| < 1$), which favors stability and often causality.
- The pole at $z = -3.6$ is outside the unit circle ($|-3.6| > 1$), which suggests the system might be non-causal or unstable depending on the ROC.

Analyzing the Impulse Response and Right-Sidedness

Region of Convergence (ROC) Considerations

The ROC determines the system's causality:

- For a causal system, the ROC extends outward from the outermost pole, i.e., $|z| > 3.6$.
- For an anti-causal system, the ROC is inside the innermost pole, i.e., $|z| < 0.9$.
- For a two-sided system, the ROC is between the poles: $0.9 < |z| < 3.6$.

Stability Criterion: For a system to be BIBO (bounded-input bounded-output) stable, the ROC must include the unit circle ($|z|=1$). This implies that the ROC must satisfy $|z| > 3.6$ or $|z| < 0.9$, but only the latter is within the unit circle, favoring anti-causality.

Conclusion on Right-Sidedness:

- Since the pole at 3.6 lies outside the unit circle, a causal, stable system would require the ROC to be $|z| > 3.6$, which does not include the unit circle. Therefore, the system cannot be both causal and stable.

- Conversely, if the system is anti-causal with ROC inside the pole at 0.9, the system is stable but non-causal.

Thus, based on pole locations, the transfer function is not right-sided (causal and stable) in the classical sense. It appears to be either non-causal or unstable if the ROC doesn't include the unit circle.

Implications for Practical Systems

In practical digital filter design, the goal is usually to create causal and stable filters. The pole at $z = -3.6$ suggests that if this system were causal, it would be unstable because the ROC would be outside the outermost pole, which does not include the unit circle.

Hence, the transfer function as given likely represents a non-causal or theoretical construct, or perhaps a part of an analysis with a specific ROC chosen for mathematical completeness rather than physical realizability.

Frequency Response and Phase Characteristics

Understanding the frequency response of $H(z)$ involves evaluating $H(z)$ on the unit circle, $z = e^{j\omega}$:

$$
H(e^{j\omega}) = \frac{1.7}{1 + 3.6 e^{-j\omega}} - \frac{0.5}{1 - 0.9 e^{-j\omega}}
$$

This can be analyzed over the range $\omega \in [0, 2\pi]$:

- The first term's magnitude varies depending on ω, with potential peaks where denominator approaches zero near the pole at $z = -3.6$.
- The second term's response is more stable due to the pole at $z = 0.9$.

This frequency response analysis reveals filter characteristics such as passbands, stopbands, and phase shift, which are critical in applications like filtering noise, signal extraction, or system stabilization.

Conclusion: The Right-Sidedness of H(z) and Its Significance

The detailed examination of the transfer function H(z) draws a clear picture of its mathematical and practical properties:

- Pole locations at $z = -3.6$ and $z = 0.9$ indicate a complex system with potential stability and causality issues.
- Region of convergence considerations suggest that the system is not right-sided—meaning it is not causal and stable in the classical sense—due to the pole outside the unit circle.
- Implications for system design involve recognizing that such a transfer function, if realized physically, would require non-causal or unstable configurations, or perhaps the model is used for analysis rather than implementation.

In sum, understanding the "right-sidedness" of a transfer function like H(z) is pivotal for engineers and researchers designing digital systems. It informs whether a system can be implemented in real-time, its stability, and how it responds to various inputs. The analysis underscores the importance of pole placement, ROC selection, and the fundamental principles governing digital filter design.

Future investigations might involve designing stable, causal approximations of this transfer function or exploring how modifications to pole locations can achieve desired stability and causality properties, aligning theoretical models with practical constraints.

References

- Oppenheim, A. V., & Schafer, R. W. (2010). Discrete-Time Signal Processing. Pearson.
- Proakis, J. G., & Manolakis, D. G. (2006). Digital Signal Processing: Principles, Algorithms, and Applications. Pearson.
- Rabiner, L. R., & Gold, B. (1975). Theory and Application

Given The Following Transfer Function H Z 1 7 1 3 6 Z 1 0 5 1 0 9z 1 A Calculate Its Right Sided

Given The Following Transfer Function H Z 1 7 1 3 6 Z 1 0 5 1 0 9z 1 A Calculate Its Right Sided

Related Articles

- Consider The Reaction Between Ozone And A Metal Cation, M2+, To Form The Metal Oxide, MO2, And Dioxygen:O3
- Many Theorists Believe That Walking Is Simply A Matter Of Biological Programming. What Would Esther Thelen
- Perform The Indicated Multiplication If Possible. (Enter NONE In Any Unused Answer Blanks. If The Matrices

Back to Home