If The Area Of A Rectangle Is(15x2 - 16x - 15) Square Feet,and Its Length Is (3x - 5) Feet,find Its Width.

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Understanding how to find the width of a rectangle when given its area and length involves applying algebraic concepts such as factoring, solving quadratic equations, and understanding the fundamental relationship between the length, width, and area of a rectangle. This article provides a comprehensive, step-by-step approach to solving this problem, along with explanations of relevant algebraic techniques, practical tips, and real-world applications.

Understanding the Problem Statement

Before diving into the solution, it’s essential to interpret the problem correctly:

- The area of a rectangle is given as a quadratic expression: $15x^2 - 16x - 15$ square feet.
- The length of the rectangle is given as $3x - 5$ feet.
- The goal is to find the width of the rectangle, expressed as an algebraic expression in terms of x.

Key Concepts and Formulas

To approach this problem effectively, familiarize yourself with the following concepts:

Area of a Rectangle

- The area A of a rectangle is calculated as:

$$
A = \text{length} \times \text{width}
$$

- When the area and length are known, the width can be found using:

$$
\text{width} = \frac{A}{\text{length}}
$$

Factoring Quadratic Expressions

- Quadratic expressions can often be factored into binomials:

$$
ax^2 + bx + c = (mx + n)(px + q)
$$

- Factoring is crucial to simplify and solve the quadratic expression for meaningful values of x.

Solving for the Width

- Once the quadratic area is expressed as a product of factors, dividing by the length expression yields the width.

Step-by-Step Solution Process

Let's methodically work through the solution to find the width:

Step 1: Write down the given expressions

- Area $A = 15x^2 - 16x - 15$
- Length $L = 3x - 5$

Step 2: Factor the quadratic area expression

To find the width, we need to write the area as a product involving the length. Since:

$$
A = \text{length} \times \text{width}
$$

and the length is $3x - 5$, we aim to express $15x^2 - 16x - 15$ as:

\[
(3x - 5) \times \text{(some expression in } x)
\]

So, factor \(15x^2 - 16x - 15\):

- Multiply the coefficient of \(x^2\) (15) and the constant term (-15):

\[
15 \times -15 = -225
\]

- Find two numbers that multiply to \(-225\) and add to \(-16\):

Factors of \(-225\):

\[
-25 \text{ and } 9 \quad \text{(since } -25 \times 9 = -225 \text{ and } -25 + 9= -16)
\]

- Rewrite the middle term \(-16x\) as \(-25x + 9x\):

\[
15x^2 - 25x + 9x - 15
\]

- Group terms:

\[
(15x^2 - 25x) + (9x - 15)
\]

- Factor each group:

\[
5x(3x - 5) + 3(3x - 5)
\]

- Factor out the common binomial \((3x - 5)\):

\[
(3x - 5)(5x + 3)
\]

Thus, the quadratic area expression factors as:

\[
15x^2 - 16x - 15 = (3x - 5)(5x + 3)
\]

Step 3: Set up the equation for width

Since:

$$
A = \text{length} \times \text{width}
$$

and

$$
A = (3x - 5)(5x + 3)
$$

$$
L = 3x - 5
$$

then,

$$
\text{width} = \frac{A}{L} = \frac{(3x - 5)(5x + 3)}{3x - 5}
$$

Provided $3x - 5 \neq 0$ (to avoid division by zero), the $(3x - 5)$ terms cancel:

$$
\text{width} = 5x + 3
$$

Important Note: The cancellation is valid only when $3x - 5 \neq 0 \Rightarrow x \neq \frac{5}{3}$.

Final Expression for the Width

Answer:

$$
\boxed{
\text{Width} = 5x + 3
}
$$

This algebraic expression gives the width of the rectangle in terms of the variable x.

Additional Considerations and Validity

Domain Restrictions

- Since the length $3x - 5$ must be positive (or at least non-zero) for a realistic rectangle, the value of x should satisfy:

$$
3x - 5 \neq 0 \Rightarrow x \neq \frac{5}{3}
$$

- Depending on the context, x might be restricted further to ensure the area and dimensions are positive.

Checking the Solution

To verify the correctness, plug in specific values of x:

- For $x = 2$:

$$
L = 3(2) - 5 = 6 - 5 = 1 \text{ ft}
$$
$$
\text{Width} = 5(2) + 3 = 10 + 3 = 13 \text{ ft}
$$
$$
A = (3x - 5)(5x + 3) = 1 \times 13 = 13 \text{ sq ft}
$$
$$
\text{Area} = 15(2)^2 - 16(2) - 15 = 15 \times 4 - 32 - 15 = 60 - 32 - 15 = 13 \text{ sq ft}
$$

The calculated area matches the product of length and width, confirming the solution's validity.

Practical Applications of the Problem

Understanding how to find the width of a rectangle given its area and length has real-world applications:

- Construction and Architecture: Calculating dimensions of rooms, floors, or plots based on area and one known side.
- Design and Manufacturing: Determining material dimensions when designing products with specified surface areas.
- Land Measurement: Estimating land plots when the area and one side length are known.
- Education: Enhancing algebra skills through applied problems involving quadratic expressions and factorization.

Summary and Key Takeaways

- The problem involves algebraic manipulation, including factoring quadratic expressions.
- Factoring $15x^2 - 16x - 15$ into $(3x - 5)(5x + 3)$ simplifies the process.
- Dividing the area expression by the length gives the width: $\boxed{\text{Width} = 5x + 3}$.
- Always consider domain restrictions to avoid invalid solutions.
- Verifying the solution with specific values of x ensures accuracy.

Conclusion

Solving for the width of a rectangle with algebraic expressions for area and length involves understanding quadratic factorization, algebraic division, and the fundamental geometric relationships. By carefully factoring the quadratic expression and dividing by the given length, we arrive at a simple, yet powerful expression for the width: $\boxed{\text{Width} = 5x + 3}$. This method demonstrates how algebraic techniques can be applied to solve real-world problems involving geometric dimensions, providing both practical utility and mathematical insight.

Keywords: rectangle area, algebraic expressions, quadratic factoring, solving for width, algebraic geometry, quadratic equations, practical math problems, dimensional analysis

Frequently Asked Questions

How do you find the width of a rectangle when the area and length are given as algebraic expressions?

To find the width, divide the area expression by the length expression, ensuring both are factored and simplified first. Then, solve for the width as a simplified algebraic expression.

Given the area of a rectangle as 15x^2 - 16x - 15 and its length as 3x - 5, how do you determine its width?

Calculate the width by dividing the area by the length: Width = (15x^2 - 16x - 15) ÷ (3x - 5). Factor the numerator if possible, then perform polynomial division to find the width expression.

What is the importance of factoring the area expression when solving for the width in algebraic problems?

Factoring simplifies the numerator, making division by the length easier and more straightforward. It helps identify common factors and reduces the polynomial division process.

If the area of a rectangle is expressed as 15x^2 - 16x - 15 and the length as 3x - 5, what steps are involved in finding the width algebraically?

First, factor the area expression: 15x^2 - 16x - 15. Then, divide this factored form by the length (3x - 5) using polynomial division or synthetic division to find the width as an algebraic expression.

Why is it necessary to check the factorization of the numerator before dividing when solving for the width?

Checking factorization ensures the numerator can be simplified by cancelling common factors with the denominator, leading to a simplified expression for the width and avoiding errors in the division process.

Additional Resources

Understanding the Problem: A Deep Dive into Rectangle Area and Width Calculation

When tackling problems involving the area and dimensions of rectangles, it’s essential to understand the fundamental relationships between length, width, and area. The problem at hand provides a quadratic expression for the area of a rectangle and a linear expression for its length, asking us to find the width. This involves algebraic manipulation, understanding quadratic equations, and applying polynomial factorization techniques. Let’s explore this problem comprehensively, step by step.

Fundamentals of Rectangle Area and Dimensions

Before delving into the specifics of the problem, it’s important to review the core concepts:

- Rectangle Area Formula:
The area A of a rectangle is given by:
$$
A = \text{length} \times \text{width}
$$

- Given Data in the Problem:
- Area: $15x^2 - 16x - 15$ square feet
- Length: $3x - 5$ feet
- Width: Unknown (let’s denote it as W)

- Objective:
Find the algebraic expression for W, the width, in terms of x.

Expressing the Known Relationship

Given the area formula and the length, the key relationship is:

$$
\boxed{
\text{Area} = \text{Length} \times \text{Width}
}
$$

Substituting known expressions:

$$
15x^2 - 16x - 15 = (3x - 5) \times W
$$

Our goal is to isolate W:

$$
W = \frac{15x^2 - 16x - 15}{3x - 5}
$$

This division involves polynomial algebra, specifically polynomial division or factorization, as both numerator and denominator are polynomials in x.

Factoring the Quadratic Numerator

To simplify the expression for W, we should attempt to factor the numerator $15x^2 - 16x - 15$. Factoring quadratic expressions is fundamental for simplifying rational expressions.

Methods of Factoring Quadratics

- Factoring by Inspection:
Look for two binomials that multiply to the quadratic.

- Using the AC Method:
For quadratic $ax^2 + bx + c$, find two numbers that multiply to $a \times c$ and add to b.

In our case:

- $a = 15$
- $b = -16$
- $c = -15$

Calculate:

$$
a \times c = 15 \times (-15) = -225
$$

We need two numbers that multiply to -225 and add to -16.

- Factors of -225:

\(-1 \times 225\), \(1 \times -225\), \(-3 \times 75\), \(3 \times -75\), \(-5 \times 45\), \(5 \times -45\), \(-9 \times 25\), \(9 \times -25\), \(-15 \times 15\), \(15 \times -15\).

Check sums:

- \(-1 + 225 = 224\) (No)
- \(1 - 225 = -224\) (No)
- \(-3 + 75 = 72\) (No)
- \(3 - 75 = -72\) (No)
- \(-5 + 45 = 40\) (No)
- \(5 - 45 = -40\) (No)
- \(-9 + 25 = 16\) (Close but positive 16, not -16)
- \(9 - 25 = -16\) (Yes!)

Thus, the two numbers are 9 and \(-25\), since:

\[
9 \times (-25) = -225
\]
\[
9 + (-25) = -16
\]

Now, rewrite the quadratic:

\[
15x^2 - 16x - 15 = 15x^2 + 9x - 25x - 15
\]

Group terms:

\[
(15x^2 + 9x) + (-25x - 15)
\]

Factor each group:

\[
3x(5x + 3) - 5(5x + 3)
\]

Factor out common binomial:

\[
(5x + 3)(3x - 5)
\]

Hence, the factored form of the numerator is:

\[

$$
15x^2 - 16x - 15 = (5x + 3)(3x - 5)
$$

Simplifying the Width Expression

Recall:

$$
W = \frac{(5x + 3)(3x - 5)}{3x - 5}
$$

Provided that $3x - 5 \neq 0$, we can cancel out the common factor:

$$
W = 5x + 3
$$

Important Note:
The division is valid only if $3x - 5 \neq 0$, which means:

$$
3x - 5 \neq 0 \Rightarrow x \neq \frac{5}{3}
$$

This value is excluded from the domain because it would make the denominator zero, leading to undefined width.

Final Expression for the Width and Domain Considerations

$$
\boxed{
\text{Width} = 5x + 3 \quad \text{(for } x \neq \frac{5}{3} \text{)}
}
$$

This linear expression describes the width of the rectangle in terms of x.

Domain considerations:
- Since the length $3x - 5$ must be a positive real number for the rectangle to make physical sense, consider:

$$
3x - 5 > 0 \Rightarrow x > \frac{5}{3}
$$

- The width $W = 5x + 3$ will also be positive if:

$$
5x + 3 > 0 \Rightarrow x > -\frac{3}{5}
$$

- Combining these, for the rectangle to have positive dimensions:

$$
x > \frac{5}{3}
$$

Graphical Interpretation and Practical Implications

- Graph of Width:
The width function $W = 5x + 3$ is a straight line with slope 5 and y-intercept 3. As x increases beyond $5/3$, the width increases linearly.

- Graph of Length:
The length $L = 3x - 5$ is also linear, with slope 3 and intercept at -5. For $x > 5/3$, the length becomes positive, which is necessary for a real rectangle.

- Physical Constraints:
In real-world applications, both length and width should be positive, so the domain of viable x values is $x > \frac{5}{3}$.

Summary and Key Takeaways

- The quadratic area expression $15x^2 - 16x - 15$ factors into $(5x + 3)(3x - 5)$.

- The width W simplifies to $5x + 3$, valid for all x except $x = \frac{5}{3}$, where the denominator would be zero during initial division.

- For physical dimensions, the domain of x is $x > \frac{5}{3}$, ensuring positive length and width.

- This problem exemplifies the importance of algebraic factoring and understanding polynomial division in solving real-world geometric problems.

Extended Insights and Related Topics

1. Polynomial Division vs. Factoring:

- Polynomial division can be used when the numerator is not easily factorable or when the divisor is not a factor.
- In this case, recognizing the factorization simplified the problem significantly.

2. Application in Geometry and Algebra:

- Such algebraic techniques are fundamental in solving geometric problems involving variable dimensions.
- They are also essential in calculus, especially when dealing with optimization and related rates.

3. Real-World Contexts:

- Understanding how to manipulate algebraic expressions helps in fields like engineering, architecture, and construction, where dimensions depend on variables.

4. Variations of the Problem:

- If the area were given as a different polynomial, the approach remains similar: factor, simplify, and interpret.
- For non-quadratic or higher-degree polynomials, advanced methods like synthetic division or polynomial long division may be necessary.

5. Educational Importance:

- This problem underscores the importance of mastering factorization, polynomial operations, and understanding the domain of algebraic functions.

Conclusion

In summary, the problem of determining the width of a rectangle given a quadratic area expression and a linear length expression demonstrates the power of algebraic manipulation. By factoring the quadratic numerator, we

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simplified the division, revealing that:

\[
\boxed{
\text{Width} = 5x +
```

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