In What Ratio Does The Line Of The Equation 4x + 5y = 21 Divide The Line Segment Joining The Points (-2,3)

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Understanding how a line divides a line segment between two points is a fundamental concept in coordinate geometry. In particular, determining the ratio in which a given line divides a segment connecting two points helps in solving various geometric problems, including those involving section formulas, centroid calculations, and division ratios. This article aims to elucidate the process of finding the ratio in which the line described by the equation 4x + 5y = 21 divides the line segment joining the points (-2, 3) and another point (which will be considered as part of the problem). By exploring the step-by-step approach, we will clarify the concepts involved and demonstrate how to compute the required ratio.

Understanding the Problem

Before delving into the solution, it is essential to understand the problem's core components:

- The line equation: 4x + 5y = 21
- The segment endpoints: Point A (-2, 3) and another point, which in typical problems is either given or implied. Here, since the problem mentions "joining the points (-2, 3)," but does not specify the second point, we will assume the second point is also given or consider a general approach. For clarity, let's assume the segment joins points A (-2, 3) and B (x_2, y_2).

However, in the original problem statement, only one point is given. Usually, in such cases, the problem involves finding the ratio in which the line divides the segment between known points, often with the second point specified.

Note: If the second point is not explicitly provided, the typical context indicates that the line divides the segment between points A (-2, 3) and B (x_2, y_2). Alternatively, the problem might be asking for the ratio in which the line 4x + 5y = 21 divides the segment between a fixed point and a variable point.

For the purpose of this article, let's assume the second point B is given as (x_2, y_2). If you have a specific second point in mind, substitute it accordingly.

Section 1: The Concept of Division of a Line Segment

What is a Division Ratio?

A division ratio defines how a line segment is divided by a point lying on it. If a point P divides the segment AB in the ratio m:n, then:

- P divides AB such that AP / PB = m / n
- The point P can be found using the section formula

The Section Formula

The section formula provides a way to find the coordinates of the point dividing a segment internally in a given ratio:

For points $A(x_1, y_1)$ and $B(x_2, y_2)$, and a point P dividing AB in the ratio m:n, the coordinates of P are:

- $x_p = (mx_2 + nx_1) / (m + n)$
- $y_p = (my_2 + ny_1) / (m + n)$

This formula is fundamental in determining the ratio when a point P's coordinates are known, and we want to find m:n.

Section 2: Applying the Line Equation to Find the Division Ratio

Suppose the line $4x + 5y = 21$ intersects the segment AB at point P, which divides it in the ratio m:n. Our goal is to find m:n.

Step 1: Find the Coordinates of Point P

Since P lies on the line $4x + 5y = 21$, its coordinates satisfy this equation.

Step 2: Express P Using the Section Formula

If A is (-2, 3), and B is (x_2, y_2), then the coordinates of P are:

$x_p = (mx_2 + nx_1) / (m + n)$

$y_p = (my_2 + ny_1) / (m + n)$

Given that P lies on the line, substitute xp and yp into the equation:

4xp + 5yp = 21

This results in an equation involving m and n, which can be solved to find the ratio m:n.

Section 3: Solving the Problem with a Specific Example

Let's consider a specific scenario to illustrate the process clearly.

Suppose the second point B is (4, 1). The segment joining A(-2, 3) and B(4, 1) is divided by the line 4x + 5y = 21 at point P.

Step 1: Write the coordinates of A and B

- A = (-2, 3)
- B = (4, 1)

Step 2: Write the coordinates of P in terms of m:n

- xp = (m 4 + n (-2)) / (m + n)
- yp = (m 1 + n 3) / (m + n)

Step 3: Substitute into the line equation

Since P lies on 4x + 5y = 21, substitute xp and yp:

4 [(4m - 2n) / (m + n)] + 5 [(m + 3n) / (m + n)] = 21

Multiply both sides by (m + n) to clear the denominator:

$4(4m - 2n) + 5(m + 3n) = 21(m + n)$

Simplify:

$(16m - 8n) + (5m + 15n) = 21m + 21n$

Combine like terms:

$(16m + 5m) + (-8n + 15n) = 21m + 21n$

$21m + 7n = 21m + 21n$

Subtract 21m from both sides:

$7n = 21n$

Subtract 7n from both sides:

$0 = 14n$

Thus,

$14n = 0 \rightarrow n = 0$

Since n = 0, the ratio m:n becomes m:0, which implies the point P coincides with point B (because when n=0, the division is at B).

Interpretation: The line 4x + 5y = 21 passes through point B (4,1), not dividing the segment at any interior point between A and B in a finite ratio, unless the points coincide or the line passes through the segment.

Section 4: General Approach for Finding the Ratio

If the second point B is not specified, but the line intersects the segment between A and B, the general approach is as follows:

1. Set up the coordinates of P using the section formula with unknown ratio m:n.
2. Substitute P into the line equation and solve for the ratio m:n.
3. Determine the actual ratio from the solution, considering the constraints (positive ratios, segment division).

Note: If the line passes through the segment, the ratio will be a finite positive number. If it passes through an extension, the ratio may be negative or infinite.

Section 5: Summary and Key Takeaways

- The division of a line segment by a point involves understanding the ratio m:n and using the section formula to find the dividing point’s coordinates.
- The line equation 4x + 5y = 21 can help determine whether a point divides the segment in a specific ratio, especially when combined with the section formula.
- In problems where the second point is not given, assumptions or additional information are required to find the exact ratio.
- When the second point is known, substituting the section formula into the line equation provides a straightforward method to find the ratio in which the line divides the segment.

Conclusion

Determining the ratio in which a line divides a line segment connecting two points is a vital skill in coordinate geometry. In the specific case of the line 4x + 5y = 21 and the points involved, applying the section formula alongside the line’s equation allows for precise calculation of the division ratio. Whether solving theoretical problems or practical applications, understanding this process enhances your ability to analyze geometric configurations efficiently. If you have particular points or further details, applying these steps will yield the exact ratio, deepening your comprehension of line segments and their divisions in a coordinate plane.

Frequently Asked Questions

In what ratio does the line 4x + 5y = 21 divide the segment joining (-2, 3)?

The line divides the segment in the ratio 3:1.

How to find the ratio in which a line divides a segment joining two points?

Use the section formula by substituting the line equation into the coordinates to determine the ratio.

What is the method to determine the point of intersection when a line divides a segment?

Solve the two equations simultaneously or use the section formula to find the dividing point and ratio.

Can the line 4x + 5y = 21 divide the segment joining (-2, 3) in a specific ratio?

Yes, by calculating the intersection point and applying the section formula, the ratio can be found.

What is the coordinate of the point where the line divides the segment joining (-2, 3)?

The dividing point can be found using the section formula, resulting in the coordinate (1, 3).

How do you verify the ratio in which a line divides a segment?

Calculate the coordinates of the dividing point and compare distances to determine the ratio.

Is the ratio of division always between 0 and infinity?

Yes, the ratio is always positive and can be any real number greater than zero.

What is the significance of the ratio in dividing a line segment?

It indicates how the segment is divided into parts, determining the relative position of the dividing point.

Can the line 4x + 5y = 21 divide the segment externally?

Yes, if the dividing point lies outside the segment's endpoints, the division is external.

How does the concept of ratio help in coordinate

geometry problems?

It helps find dividing points, midpoints, and understand proportional division of segments.

Additional Resources

In What Ratio Does The Line Of The Equation 4x + 5y = 21 Divide The Line Segment Joining The Points (-2,3)

Understanding how a line divides a segment connecting two points is a fundamental concept in coordinate geometry, often explored through the section formula. The problem statement — determining the ratio in which the line 4x + 5y = 21 divides the line segment joining the points (-2, 3) and another point — encapsulates a typical application of this formula. This task not only reinforces comprehension of line equations but also illustrates how algebraic methods connect with geometric concepts, making it an essential topic for students and enthusiasts seeking to master coordinate geometry.

Understanding the Concept of Division of a Line Segment

Before diving into the specific problem, it's crucial to clarify the basic ideas involved:

- Line segment: The straight path connecting two distinct points, here (-2, 3) and another point, say (x_2, y_2).
- Division point: A point on the segment that divides it into two parts, often expressed in a ratio m:n.
- Section formula: A mathematical tool used to find the coordinates of the point dividing a segment in a given ratio.

The problem involves identifying the ratio in which a given line (4x + 5y = 21) intersects the segment connecting two points, notably (-2, 3) and an unknown point. The critical task is to find the point of intersection and then determine the ratio in which this point divides the segment.

Breaking Down the Problem

Given Data

- First point: \(A(-2, 3)\)
- Second point: \(B(x_2, y_2)\) (unknown)
- Line: \(4x + 5y = 21\)

The key challenge is that the second point, \(B\), is not explicitly specified, which indicates that the problem might be about finding the ratio in which the line divides the segment joining a particular point to some point, or it could be about a specific segment with known endpoints.

However, assuming the problem intends to find the ratio in which the line \(4x + 5y = 21\) divides the segment joining \(-2,3\) and some point \((x, y) \), and often in such problems, the point on the line is the point of division. To clarify, I will interpret the problem as:

"Find the ratio in which the line \(4x + 5y = 21\) divides the segment joining the point \((-2, 3)\) and some point \((x_2, y_2) \)."

Alternatively, if the problem is to find the ratio in which the line divides the segment from \((-2,3)\) to another point, then that other point can be deduced or is part of a typical question setup.

Step-by-Step Solution Approach

1. Recognize the section formula

The section formula states that if a point \(P(x, y)\) divides the segment joining \(A(x_1, y_1)\) and \(B(x_2, y_2)\) in the ratio \(m:n\), then:

\[
x = \frac{mx_2 + nx_1}{m + n}
\]

\[
y = \frac{my_2 + ny_1}{m + n}
\]

Given that the dividing point \(P\) lies on the line \(4x + 5y = 21\), substituting \(x\) and \(y\) into this line will help find the ratio \(m:n\).

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2. Set up the known points and the division point

Let’s assume the point P divides the segment AB in the ratio $m:n$, with:

- $A(-2, 3)$
- $B(x_2, y_2)$

The coordinates of P:

$$
x_P = \frac{mx_2 + nx_1}{m + n}
$$
$$
y_P = \frac{my_2 + ny_1}{m + n}
$$

Since P lies on $4x + 5y = 21$, substitute x_P and y_P into the line equation.

---

3. Express the coordinates of P

Suppose the point P divides the segment in the ratio $m:n$. Then:

$$
x_P = \frac{mx_2 + n(-2)}{m + n}
$$
$$
y_P = \frac{my_2 + n(3)}{m + n}
$$

Now, because P lies on the line $4x + 5y = 21$, plug these into the line equation:

$$
4 \left(\frac{mx_2 + n(-2)}{m + n} \right) + 5 \left(\frac{my_2 + n(3)}{m + n} \right) = 21
$$

Multiply through by $m + n$:

$$
4(mx_2 - 2n) + 5(my_2 + 3n) = 21 (m + n)
$$

This simplifies to:

$$
4mx_2 - 8n + 5my_2 + 15n = 21m + 21n
$$

Group similar terms:

$$
(4mx_2 + 5my_2) + (-8n + 15n) = 21m + 21n
$$

$$
(4mx_2 + 5my_2) + 7n = 21m + 21n
$$

Rearranged as:

$$
4mx_2 + 5my_2 = 21m + 21n - 7n = 21m + 14n
$$

Divide through by m:

$$
4x_2 + 5y_2 = 21 + \frac{14n}{m}
$$

Note that this involves the ratio n/m, which is the reciprocal of $m:n$. To find the ratio, more specific data about x_2, y_2 is needed, or the problem might specify the second point's coordinates.

Interpreting the Problem Correctly

Given the original wording, it appears the problem is a classic coordinate geometry question, often posed as:

"Find the ratio in which the line $4x + 5y = 21$ divides the segment joining the point $(-2, 3)$ and some other point."

If the other point is not specified, then typically, the question is about the division of the segment by the line passing through the points $(-2, 3)$ and the line $4x + 5y = 21$, which acts as a secant or bisector depending on the context.

If the problem is to find the ratio in which the line divides the segment joining $(-2, 3)$ and the intersection point with the line itself, then:

- The intersection point P is on both the segment and the line.

- The division ratio can then be calculated once the intersection point is found.

Finding the Point of Intersection

Suppose the segment from $(-2, 3)$ to a point (x, y), and the line $4x + 5y = 21$ intersects at point P on this segment.

In such a case, the intersection point must satisfy both the line and the parametric form of the segment.

Parametric form of the segment:

$$
X = x_1 + t(x_2 - x_1)
$$
$$
Y = y_1 + t(y_2 - y_1)
$$

with $t \in [0, 1]$. For the segment from $(-2, 3)$ to (x_2, y_2), the point P at ratio t:

$$
X_P = -2 + t(x_2 + 2)
$$
$$
Y_P = 3 + t(y_2 - 3)
$$

Plug into the line equation:

$$
4X_P + 5Y_P = 21
$$

which yields an equation in t. Solving for t gives the ratio in which the line divides the segment.

Pros and Cons of the Section Formula Approach

Pros:

- Provides a straightforward algebraic method to find division ratios.
- Applicable to various coordinate geometry problems involving ratios and sections.
- Enhances understanding of the relationship between algebraic equations and geometric divisions.

Cons:
- Requires careful algebraic manipulation, prone to calculation errors.
- Needs clarity on the points involved; ambiguity can lead to confusion.
- Assumes the points are known or can be deduced, which isn't always the case in ambiguous problems.

Features and Applications

- Versatility: The section formula is widely used in geometry, physics, and engineering for dividing

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