

One Number Is Four More Than Four Times Another. If Their Sum Is Decreased By Six, The Result Is Thirteen.

One Number Is Four More Than Four Times Another. If Their Sum Is Decreased By Six, The Result Is Thirteen. This intriguing statement introduces a classic algebraic problem that involves setting up equations based on relationships between numbers and then solving for those numbers. Such problems are common in math education to develop students' understanding of variables, equations, and problem-solving strategies. In this article, we will explore how to translate this word problem into mathematical equations, solve for the unknown numbers, and understand the concepts involved. Whether you're a student preparing for exams or a math enthusiast, understanding this problem enhances your algebra skills and logical reasoning.

Understanding the Problem

Before jumping into equations and calculations, it's essential to carefully interpret the problem statement. Let's examine it in parts:

- Part 1: "One number is four more than four times another."
- Part 2: "If their sum is decreased by six, the result is thirteen."

This statement involves two unknown numbers, which we can represent with variables. The goal is to find the specific values of these numbers based on the relationships described.

Defining Variables

To translate the word problem into algebraic expressions, assign variables to the unknowns:

- Let x = the first number.
- Let y = the second number.

With these variables, we can now express the relationships described in the problem.

Formulating Equations from the Word Problem

Based on the problem statement, we can derive two equations:

Equation 1: Relationship Between the Two Numbers

- The first number x is four more than four times the second number y .
- Mathematically:

$$x = 4y + 4$$

Equation 2: Sum Decreased by Six Equals Thirteen

- The sum of the two numbers is $x + y$.
- When this sum is decreased by six, the result is thirteen.
- Mathematically:

$$(x + y) - 6 = 13$$

Now, we have a system of two equations:

1. $x = 4y + 4$

2. $x + y - 6 = 13$

Let's proceed to solve this system.

Solving the System of Equations

The goal is to find the values of x and y that satisfy both equations.

Step 1: Substitute x from Equation 1 into Equation 2

Since $x = 4y + 4$, replace x in the second equation:

$$(4y + 4) + y - 6 = 13$$

Step 2: Simplify and Solve for y

Combine like terms:

$$4y + y + 4 - 6 = 13$$

$$5y - 2 = 13$$

Add 2 to both sides:

$$5y = 15$$

Divide both sides by 5:

$$y = 3$$

Step 3: Find x Using the Value of y

Substitute $y = 3$ into Equation 1:

$$x = 4(3) + 4 = 12 + 4 = 16$$

Summary of Solutions

- First number (x): 16
- Second number (y): 3

These are the numbers satisfying the conditions of the problem.

Verifying the Solution

It's good practice to verify the solutions by plugging them back into the original statements.

- Check the first statement:

Is x four more than four times y?

4 times y: $4 \times 3 = 12$

Four more: $12 + 4 = 16$

Since $x = 16$, the condition holds.

- Check the second statement:

Sum: $16 + 3 = 19$

Decreased by six: $19 - 6 = 13$

Result: 13, which matches the requirement.

Thus, the solutions are correct.

Practical Applications of Such Problems

Problems like this are not just academic exercises—they have practical applications in various fields.

Real-World Scenarios

- Financial calculations: Determining investments or loans based on relationships between amounts.
- Engineering: Calculating component sizes or forces where relationships are defined algebraically.
- Business planning: Budgeting and resource allocation based on proportional relationships.

Educational Benefits

- Enhances logical thinking and problem-solving skills.
- Builds a foundation for understanding more complex algebraic concepts.
- Develops the ability to translate word problems into mathematical models.

Steps to Approach Similar Word Problems

When encountering similar problems, follow these systematic steps:

1. Read Carefully: Understand what is being asked.
2. Identify Variables: Assign symbols to unknown quantities.
3. Translate Words into Equations: Express relationships mathematically.
4. Solve the System: Use substitution or elimination methods.
5. Verify Solutions: Check that the solutions satisfy the original conditions.
6. Interpret Results: Relate the solutions back to the context.

Additional Tips for Solving Algebraic Word Problems

- Write down what each part of the problem states.
- Keep equations organized.
- Use graphing as a visual aid when applicable.
- Practice with varied problems to build confidence.

Conclusion

The statement "One number is four more than four times another. If their sum is decreased by six, the result is thirteen" exemplifies how algebra can be applied to solve real-world problems expressed in words. By defining variables, translating the problem into equations, and systematically solving, we can find the specific numbers that satisfy the given conditions. This process not only enhances mathematical skills but also fosters critical thinking and problem-solving abilities applicable in many areas of life. By mastering such problems, students and enthusiasts develop a strong foundation in algebra that opens doors to more complex mathematical concepts and practical applications.

Frequently Asked Questions

How can we represent the two numbers in the given problem using variables?

Let the smaller number be x . Then, the larger number is four more than four times x , which is $4x + 4$.

What equation can be formed from the statement 'Their sum is decreased by six, the result is thirteen'?

The sum of the two numbers is $(x + 4x + 4) = 5x + 4$. Decreasing this sum by six gives $(5x + 4) - 6 = 13$, which simplifies to $5x - 2 = 13$.

How do you solve for the value of x in this problem?

Set up the equation $5x - 2 = 13$, then add 2 to both sides to get $5x = 15$, and divide both sides by 5 to find $x = 3$.

What are the actual values of the two numbers based on the solution?

The smaller number x is 3, and the larger number is $4x + 4 = 4(3) + 4 = 12 + 4 = 16$.

What is the significance of the given problem in understanding algebraic expressions?

It demonstrates how to translate word problems into algebraic equations, solve for variables, and interpret the solutions in context.

Additional Resources

One number is four more than four times another. If their sum is decreased by six, the result is thirteen.

This intriguing statement presents a classic algebraic problem that combines simple algebraic expressions with logical reasoning. It encapsulates the essence of word problems that not only test mathematical skills but also challenge one's ability to translate verbal descriptions into solvable equations. As we delve into this problem, we'll explore its components, interpret the conditions carefully, and analyze the solutions step-by-step. This article aims to provide a comprehensive understanding of the problem, its underlying principles, and its broader educational significance.

Understanding the Problem: Breaking Down the Statement

Before jumping into equations and calculations, it's crucial to interpret the problem carefully. The statement contains two primary conditions:

1. One number is four more than four times another.
2. If their sum is decreased by six, the result is thirteen.

Each of these conditions involves relationships between two unknown numbers, which we can denote as variables for clarity.

The Variables

Let's assign:

- x = the first number

- y = the second number

The goal is to find the values of x and y that satisfy both conditions simultaneously.

The Conditions in Detail

Condition 1: "One number is four more than four times another."

- This suggests a relationship between x and y . It could be interpreted as:
- x is four more than four times y , or
- y is four more than four times x .

Since the statement is symmetrical, and often in algebra problems, the phrasing "one number is ..." generally refers to a particular variable in relation to the other. For clarity, we'll analyze both possibilities:

- Case 1: $x = 4 + 4y$
- Case 2: $y = 4 + 4x$

We will examine both cases to see which yields a consistent solution.

Condition 2: "If their sum is decreased by six, the result is thirteen."

- Mathematically, this can be expressed as:

$$(x + y) - 6 = 13$$

Simplifying:

$$x + y = 19$$

This second condition is straightforward and will be used as a key equation in solving the problem.

Formulating Equations Based on the Conditions

Having interpreted the problem, we now translate it into algebraic equations.

Case 1: $x = 4 + 4y$

From this, and knowing $x + y = 19$, we can substitute x :

$$x + y = 19$$

$$\Rightarrow (4 + 4y) + y = 19$$

$$\Rightarrow 4 + 4y + y = 19$$

$$\Rightarrow 4 + 5y = 19$$

Subtract 4 from both sides:

$$5y = 15$$

Divide both sides by 5:

$$y = 3$$

Now, substitute y back into x :

$$x = 4 + 4(3) = 4 + 12 = 16$$

Solution for Case 1:

$$- x = 16$$

$$- y = 3$$

$$\text{Case 2: } y = 4 + 4x$$

Using the same sum equation:

$$x + y = 19$$

Substitute y:

$$x + (4 + 4x) = 19$$

$$\Rightarrow x + 4 + 4x = 19$$

$$\Rightarrow 5x + 4 = 19$$

Subtract 4:

$$5x = 15$$

Divide both sides by 5:

$$x = 3$$

Now, find y:

$$y = 4 + 4(3) = 4 + 12 = 16$$

Solution for Case 2:

$$- x = 3$$

$$- y = 16$$

Analyzing the Solutions and Their Validity

Both solutions— $(x=16, y=3)$ and $(x=3, y=16)$ —satisfy the algebraic conditions derived from the problem.

Let's examine each in detail:

Solution 1: $(x=16, y=3)$

- Is the first condition true?

$$x = 4 + 4y$$

$$16 = 4 + 4(3) = 4 + 12 = 16 \quad \square \square$$

- Does the second condition hold?

$$x + y = 19$$

$$16 + 3 = 19 \quad \square \square$$

Solution 2: $(x=3, y=16)$

- Check the first condition:

$$y = 4 + 4x?$$

$$16 = 4 + 4(3) = 4 + 12 = 16 \quad \square\square$$

- Check the sum:

$$3 + 16 = 19 \quad \square\square$$

Both solutions satisfy the algebraic equations derived from the problem, indicating that either number could be the "first" or "second" depending on the context.

Interpreting the Meaning and Context of the Solutions

The symmetrical nature of the solutions reflects the problem's inherent symmetry: the two numbers are interchangeable in the context of the conditions. This is common in algebraic word problems, where the phrasing might suggest multiple interpretations, each leading to valid solutions.

Which solution is more appropriate?

In practical terms, the question is often designed to have a unique solution. The ambiguity arises from the phrase "One number is four more than four times another," which can be read in either direction. Both solutions are mathematically valid, but in real-world contexts, additional information might clarify which number is which.

The significance of the solutions:

- (16, 3): Interpreted as the first number being 16, the second number 3.
- (3, 16): The first number being 3, the second number 16.

Both satisfy the original conditions and the secondary condition involving their sum.

Broader Educational and Mathematical Significance

This problem exemplifies several important concepts in algebra and problem-solving:

1. Variable Assignment and Equations

Assigning variables and translating verbal descriptions into algebraic equations is a fundamental skill. It demonstrates how language can be systematically converted into mathematical expressions.

2. Symmetry and Multiple Solutions

Many real-world problems are symmetrical, leading to multiple valid solutions. Recognizing this symmetry helps students understand the importance of context and interpretation.

3. Substitution and Solving Systems of Equations

The process of substitution illustrates a core algebraic technique—solving systems of equations—that extends to more complex scenarios involving multiple variables.

4. Interpreting Word Problems

Understanding the precise meaning of words and phrases is crucial. Phrases like "more than" and "the sum decreased by" require careful parsing to avoid misinterpretation.

5. Critical Thinking and Logical Reasoning

Analyzing the solutions involves checking their validity, understanding their implications, and considering alternate interpretations, fostering critical thinking skills.

Applications and Extensions of the Problem

While this problem is a straightforward algebraic exercise, its structure lends itself to various applications and extensions:

Real-World Contexts

- Financial calculations: Estimating income and expenses based on relationships.
- Physics problems: Relating velocities or forces with specific ratios.
- Statistics: Comparing data points with known relationships.

Variations and Complexity

- Introducing additional conditions, such as constraints or inequalities.
- Extending to systems with more variables.
- Considering non-integer solutions or real-world constraints.

Educational Use

- As a teaching tool to develop problem-solving strategies.
- To illustrate the importance of precise language interpretation.
- To demonstrate the versatility of algebra in modeling real-world scenarios.

Conclusion: Synthesis and Reflection

The problem "One number is four more than four times another. If their sum is decreased by six, the result is thirteen." encapsulates fundamental algebraic principles, demonstrating how verbal descriptions translate into equations, how multiple solutions can emerge, and how careful analysis leads to meaningful interpretations. Both solutions—(16, 3) and (3, 16)—highlight the importance of context and interpretation in math problems.

This exploration underscores that algebra is not merely about computations but about logical reasoning, language precision, and analytical thinking. Problems like this serve as stepping stones toward mastering more complex mathematical concepts and developing critical problem-solving skills applicable across disciplines.

By dissecting such problems thoroughly, learners not only grasp the mechanics of algebra but also appreciate the elegance and utility of mathematical reasoning in understanding the world around us.

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