10 [Maximum Mark: 8] For The Points A(0, 4), B(2, 8) And C(1, 3): Find The Line Perpendicular To AB Through

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Introduction

In the realm of coordinate geometry, understanding how to find equations of lines, especially perpendicular lines, is fundamental for solving numerous mathematical problems. Whether you're preparing for exams, practicing for competitive tests, or simply enhancing your understanding of geometry, mastering these concepts is essential.

Today, we focus on a specific problem involving three points: A(0, 4), B(2, 8), and C(1, 3). The task is to determine the equation of the line that is perpendicular to the segment AB and passes through a certain point. Typically, such problems are designed to test your ability to calculate slopes, understand the concept of perpendicularity, and formulate the equations of lines in the coordinate plane.

This article provides a comprehensive, step-by-step explanation of how to approach and solve this type of problem. We will cover the necessary background concepts, detailed calculations, and tips for avoiding common mistakes, all optimized for clarity and SEO relevance to help students, educators, and enthusiasts alike.

Understanding the Problem

Before diving into calculations, let's clarify what the problem entails:

- Points Given:
- A(0, 4)
- B(2, 8)
- C(1, 3) (though in this problem, C may not be directly involved unless specified)

- Objective:
- Find the line that is perpendicular to the segment AB
- The line must pass through a specified point (often point C or another point on the plane)

In many similar problems, the goal is to find the equation of the line perpendicular to AB that passes through point C. If the problem explicitly states the point through which the line passes, focus on that; otherwise, the general approach applies.

Step 1: Calculating the Slope of Line AB

The first vital step is to find the slope of line AB because the perpendicular line's slope depends on this.

Formula for Slope:

$$
m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}
$$

Applying to Points A(0, 4) and B(2, 8):

$$
m_{AB} = \frac{8 - 4}{2 - 0} = \frac{4}{2} = 2
$$

Result: The slope of AB is 2.

Step 2: Determining the Slope of the Perpendicular Line

Lines perpendicular to each other have slopes that are negative reciprocals. The negative reciprocal of slope m is:

$$
m_{\perp} = -\frac{1}{m}
$$

Calculation:

$$
m_{\perp} = -\frac{1}{2}
$$

Result: The slope of the perpendicular line to AB is $-\frac{1}{2}$.

Step 3: Identifying the Point Through Which the Perpendicular Line Passes

In many problems, you need to find the perpendicular line passing through a specific point. Based on the initial points provided, common options include:

- The point C(1, 3)
- Or, perhaps, the midpoint of AB (if the problem specifies)

Assuming the task is to find the line perpendicular to AB passing through point C(1, 3), then:

Point for the perpendicular line:

$$
(x_0, y_0) = (1, 3)
$$

Step 4: Writing the Equation of the Perpendicular Line

Using the point-slope form:

$$
y - y_0 = m (x - x_0)
$$

Substituting the known values:

$$
y - 3 = -\frac{1}{2} (x - 1)
$$

Simplify the Equation:

$$
y - 3 = -\frac{1}{2}x + \frac{1}{2}
$$

$$
y = -\frac{1}{2}x + \frac{1}{2} + 3
$$

$$
y = -\frac{1}{2}x + \frac{1}{2} + \frac{6}{2}
$$

$$
y = -\frac{1}{2}x + \frac{7}{2}
$$

Final Equation in Slope-Intercept Form:

$$
\boxed{y = -\frac{1}{2}x + \frac{7}{2}}
$$

This is the equation of the line perpendicular to AB passing through point C.

Step 5: Verifying the Solution

To ensure accuracy, verify that:

- The line is perpendicular to AB
- The line passes through point C(1, 3)

Check perpendicularity:

- Slope of AB: 2
- Slope of the perpendicular line: $-\frac{1}{2}$

Since the product of the slopes:

$$
2 \times -\frac{1}{2} = -1
$$

which confirms the lines are perpendicular.

Check if point C lies on the line:

$$
y = -\frac{1}{2}x + \frac{7}{2}
$$

Plug in $x=1$:

$$
y = -\frac{1}{2} \times 1 + \frac{7}{2} = -\frac{1}{2} + \frac{7}{2} = \frac{6}{2} = 3
$$

which matches the y-coordinate of C. Therefore, point C indeed lies on the line.

Additional Considerations

If the problem involves a different point

Suppose the line must pass through a different point, such as the midpoint of AB or another coordinate. Adjust the point accordingly in the point-slope formula.

If the problem asks for the perpendicular line passing through a different point:

- Use the same slope $-\frac{1}{2}$
- Replace (x_0, y_0) with the new point's coordinates
- Derive the equation following the same steps

If the problem asks for the perpendicular line to AB at point A or B:

- Use the respective point in the point-slope form
- Confirm the line's slope remains $-\frac{1}{2}$

Practical Tips for Solving Perpendicular Line Problems

1. Always find the slope of the original line first: This forms the basis for calculating the perpendicular slope.
2. Remember the negative reciprocal rule: To find the slope of the perpendicular line.
3. Use the point-slope form for the equation, especially when a specific point is given.
4. Verify your answers: Check for perpendicularity (product of slopes = -1) and confirm the point lies on the line.
5. Convert to slope-intercept or standard form as needed for clarity or specific problem requirements.
6. Be cautious with fractions: Simplify calculations to avoid errors.

Common Mistakes to Avoid

- Using incorrect points for calculations

- Forgetting the negative reciprocal when finding the perpendicular slope
- Mixing up points when applying the point-slope formula
- Miscalculating slopes when points have the same x or y coordinates
- Ignoring the verification step, which can help catch errors early

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Conclusion

Understanding how to find a line perpendicular to a given segment and passing through a specific point is an essential skill in coordinate geometry. The key steps involve calculating the slope of the original line, finding its negative reciprocal for the perpendicular line, and then applying the point-slope form to derive the equation.

In our example with points A(0, 4), B(2, 8), and C(1, 3), we demonstrated how to:

- Calculate the slope of AB
- Determine the perpendicular slope
- Find the equation of the line passing through C
- Verify the solution's correctness

Mastering these steps will empower you to solve similar problems confidently, whether in exams, competitive tests, or practical applications involving geometry.

Additional Resources

- Coordinate Geometry Formulas: A comprehensive list of formulas for slopes, equations of lines, and distances.
- Practice Problems: Try solving similar problems with different points to enhance understanding.
- Visual Aids: Use graphing tools or software like GeoGebra to visualize lines and verify solutions.

By following the detailed approach outlined above, you can confidently tackle problems involving perpendicular lines in coordinate geometry, boosting both your mathematical skills and exam performance.

Frequently Asked Questions

What is the first step to find the line perpendicular to AB passing through a point?

First, find the slope of the line AB, then determine the negative reciprocal of that slope for the perpendicular line.

How do you calculate the slope of line AB given

points A(0, 4) and B(2, 8)?

The slope m = (8 - 4) / (2 - 0) = 4 / 2 = 2.

What is the slope of the line perpendicular to AB?

Since the slope of AB is 2, the perpendicular line's slope is -1/2.

Which point do we use to find the equation of the perpendicular line?

We use the point through which the perpendicular line passes; in this case, the problem asks for the line through point C(1, 3).

How do you write the equation of the line perpendicular to AB passing through point C(1, 3)?

Using point-slope form: y - 3 = (-1/2)(x - 1).

What is the simplified form of the perpendicular line's equation passing through C(1, 3)?

Simplify to y - 3 = (-1/2)x + 1/2, then y = (-1/2)x + 3.5.

Can you verify that the line is perpendicular to AB?

Yes, by confirming the slopes are negative reciprocals: AB has slope 2, and the perpendicular line has slope -1/2.

What is the significance of the point C in this problem?

Point C(1, 3) is the point through which the perpendicular line to AB is drawn.

What is the final equation of the line perpendicular to AB passing through C?

The equation is y = (-1/2)x + 3.5.

Additional Resources

10 [Maximum Mark: 8] For The Points A(0, 4), B(2, 8) And C(1, 3): Find The Line Perpendicular To AB Through

Understanding how to find the line perpendicular to a given line segment through a specific point is a fundamental skill in coordinate geometry. In this guide, we will explore the process step-by-step for the points A(0, 4), B(2, 8), and C(1, 3), with a focus on finding the line perpendicular to segment AB through point C. This comprehensive analysis aims to clarify the concepts, calculations, and reasoning involved, providing a clear pathway for

students and enthusiasts alike.

Introduction to the Problem

In coordinate geometry, the ability to analyze line segments, determine slopes, and find perpendicular lines is crucial. The problem at hand involves three points:

- A(0, 4)
- B(2, 8)
- C(1, 3)

The key task is to find the equation of the line that is perpendicular to the line segment AB and passes through point C.

This problem combines several core concepts:

- Calculating the slope of a line segment
- Understanding perpendicular slopes
- Using point-slope form to derive the equation of the perpendicular line

Let's explore each step in detail.

Step 1: Find the Slope of Line AB

The first step involves calculating the slope of the line passing through points A and B.

How to Calculate the Slope

The slope m_{AB} of the line through points (x_1, y_1) and (x_2, y_2) is given by:

$$
m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}
$$

Applying to Points A and B:

$$
x_1 = 0,\quad y_1 = 4
$$
$$
x_2 = 2,\quad y_2 = 8
$$

So,

$$
m_{AB} = \frac{8 - 4}{2 - 0} = \frac{4}{2} = 2
$$

Result: The slope of AB is 2.

Step 2: Determine the Slope of the Perpendicular Line

Lines perpendicular to each other have slopes that are negative reciprocals.

Negative Reciprocal

Given a slope m, its perpendicular slope m_{perp} is:

$$
m_{perp} = -\frac{1}{m}
$$

Calculation:

$$
m_{perp} = -\frac{1}{2}
$$

Result: The slope of the line perpendicular to AB is -$\frac{1}{2}$.

Step 3: Write the Equation of the Perpendicular Line Through Point C

Now, we need the equation of the line passing through point C(1, 3) with slope $-\frac{1}{2}$.

Using Point-Slope Form

The point-slope form of a line:

$$
y - y_1 = m(x - x_1)
$$

Substituting:

$$
x_1 = 1,\quad y_1 = 3,\quad m = -\frac{1}{2}
$$

$$
y - 3 = -\frac{1}{2}(x - 1)
$$

Simplify:

$$
y - 3 = -\frac{1}{2}x + \frac{1}{2}
$$

Adding 3 to both sides:

$$
y = -\frac{1}{2}x + \frac{1}{2} + 3
$$

$$
y = -\frac{1}{2}x + \frac{1}{2} + \frac{6}{2}
$$

\]

\[
y = -\frac{1}{2}x + \frac{7}{2}
\]

Final Equation:

\[
\boxed{y = -\frac{1}{2}x + \frac{7}{2}}
\]

This is the equation of the line perpendicular to segment AB passing through point C.

Additional Insights and Applications

1. Graphical Representation

Plotting the points A(0, 4), B(2, 8), and C(1, 3):

- Point A at (0, 4)
- Point B at (2, 8)
- Point C at (1, 3)

Drawing line AB and then the perpendicular line through C will give a visual understanding of the relationship.

2. Verifying the Perpendicularity

- The slope of AB is 2.
- The slope of the perpendicular line is \(-\frac{1}{2}\).

Since \(2 \times -\frac{1}{2} = -1 \), the slopes are negative reciprocals, confirming perpendicularity.

3. Applications in Geometry and Real Life

- Construction of perpendicular bisectors
- Designing roads or pathways intersecting at right angles
- Analyzing geometric shapes and angles

Summary and Key Takeaways

Here's a quick summary of the process:

1. Calculate the slope of the given line segment:

\[
m_{AB} = \frac{8 - 4}{2 - 0} = 2
\]

2. Find the slope of the perpendicular line:

\[

$$
m_{perp} = -\frac{1}{m_{AB}} = -\frac{1}{2}
$$

3. Use point C and the perpendicular slope to find the line's equation:

$$
y - 3 = -\frac{1}{2}(x - 1)
$$

Simplified to:

$$
y = -\frac{1}{2}x + \frac{7}{2}
$$

This comprehensive approach ensures clarity, accuracy, and confidence in solving similar coordinate geometry problems.

Final Remarks

Mastering the skills of calculating slopes, understanding perpendicularity, and formulating line equations are foundational for higher-level mathematics and practical applications. Whether you're preparing for exams or tackling real-world problems, these steps serve as essential tools in your mathematical toolkit.

Remember, practice with various points and different configurations will strengthen your understanding and proficiency in coordinate geometry.

Happy solving! If you have any further questions or need additional examples, feel free to reach out.

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