2. Compute The Curl Of The Vector Field At The Given Point.a) F(x,y,z)=xyzi+ Xyzj+ Xyzk En El Punto (2,1,3)

Understanding the Concept of Curl in Vector Fields

In vector calculus, the curl of a vector field is a fundamental operation that measures the rotation or the swirling strength of the field at a given point in space. It provides insight into the local rotation tendencies of a vector field, which is essential in various fields such as fluid dynamics, electromagnetism, and engineering. In this article, we will explore how to compute the curl of a specific vector field at a particular point, focusing on the problem: **2. Compute The Curl Of The Vector Field At The Given Point.a) F(x,y,z)=xyzi+ Xyzj+ Xyzk En El Punto (2,1,3)**.

What Is the Curl of a Vector Field?

Definition of Curl

The curl of a vector field F = P i + Q j + R k is a vector that describes the infinitesimal rotation at a point. Mathematically, it is defined as:

$$\nabla \times F = (\partial R/\partial y - \partial Q/\partial z)\ i + (\partial P/\partial z - \partial R/\partial x)\ j + (\partial Q/\partial x - \partial P/\partial y)\ k$$

where $\nabla \times F$ signifies the curl operator applied to the vector field.

Significance of Curl in Physics and Engineering

- **Fluid dynamics:** The curl indicates the local rotation or vorticity of a fluid flow.
- **Electromagnetism:** The curl of magnetic and electric fields relates to their changing nature and the presence of currents.
- **Engineering:** Helps in analyzing rotational forces and fluid flow patterns.

Given Vector Field and Point

Our task is to compute the curl of the vector field:

```
F(x, y, z) = xyzi + xyzj + xyzk
```

at the point (2, 1, 3).

Expressing the Vector Field Components

Breaking down the vector field into its components:

- $P(x, y, z) = xy\ z$
- $Q(x, y, z) = xyz$
- $R(x, y, z) = xyz$

Note that in this case, Q and R are the same functions.

Step-by-Step Calculation of the Curl

Step 1: Compute Partial Derivatives

We need to find the following derivatives:

1. $\partial R/\partial y$ and $\partial Q/\partial z$ for the i-component
2. $\partial P/\partial z$ and $\partial R/\partial x$ for the j-component
3. $\partial Q/\partial x$ and $\partial P/\partial y$ for the k-component

Step 2: Calculate Each Derivative

- $\partial R/\partial y = \partial(xyz)/\partial y = xz$
- $\partial Q/\partial z = \partial(xyz)/\partial z = xy$
- $\partial P/\partial z = \partial(xyz)/\partial z = xy$
- $\partial R/\partial x = \partial(xyz)/\partial x = yz$
- $\partial Q/\partial x = \partial(xyz)/\partial x = yz$

- ∂P/∂y = ∂(xyz)/∂y = xz

Step 3: Assemble the Components of the Curl

Using the derivatives, apply the curl formula:

```
(∂R/∂y - ∂Q/∂z) i + (∂P/∂z - ∂R/∂x) j + (∂Q/∂x - ∂P/∂y) k
```

Step 4: Evaluate at the Point (2, 1, 3)

Substitute x=2, y=1, z=3 into the derivatives:

- ∂R/∂y = xz = 2 3 = 6
- ∂Q/∂z = xy = 2 1 = 2
- ∂P/∂z = xy = 2 1 = 2
- ∂R/∂x = yz = 1 3 = 3
- ∂Q/∂x = yz = 1 3 = 3
- ∂P/∂y = xz = 2 3 = 6

Final Calculation of the Curl at (2, 1, 3)

i-Component

```
∂R/∂y - ∂Q/∂z = 6 - 2 = 4
```

j-Component

```
∂P/∂z - ∂R/∂x = 2 - 3 = -1
```

k-Component

$$\partial Q/\partial x - \partial P/\partial y = 3 - 6 = -3$$

Resultant Curl Vector at the Point (2, 1, 3)

$$\nabla \times F\ (2, 1, 3) = 4\ i - 1\ j - 3\ k$$

Or, in component form:

Curl of F at (2, 1, 3): (4, -1, -3)

Implications of the Calculated Curl

The resulting curl vector indicates the local rotational behavior of the vector field at the specified point. A non-zero curl suggests that the field exhibits rotational or swirling properties at that location, which can be critical in understanding flow patterns or electromagnetic field behavior.

Conclusion

Calculating the curl of a vector field involves understanding the components of the field, computing the necessary partial derivatives, and evaluating these derivatives at the specific point of interest. The process is systematic and essential for analyzing physical phenomena modeled by vector fields. In our example, the curl at point (2, 1, 3) is (4, -1, -3), providing valuable insight into the local rotational characteristics of the given vector field.

Summary of Key Steps

1. Express the vector field components clearly.
2. Compute the necessary partial derivatives of each component.
3. Apply the curl formula component-wise.
4. Evaluate the derivatives at the given point.
5. Combine the results to find the curl vector.

Understanding and calculating the curl is vital for students and professionals working in physics, engineering, and applied mathematics, enabling them to analyze complex vector fields effectively.

Frequently Asked Questions

How do you compute the curl of the vector field F(x, y, z) = xyz i + xyz j + xyz k at the point (2, 1, 3)?

To compute the curl, use the formula $\nabla \times F = (\partial F_z/\partial y - \partial F_y/\partial z)i + (\partial F_x/\partial z - \partial F_z/\partial x)j + (\partial F_y/\partial x - \partial F_x/\partial y)k$. Calculate the partial derivatives at the point (2, 1, 3) to find the curl.

What are the components of the vector field F(x, y, z) = xyz i + xyz j + xyz k?

The components are $F_x = xyz$, $F_y = xyz$, and $F_z = xyz$, which are all identical functions of x, y, and z.

How do the partial derivatives for the curl calculation simplify for this specific field?

Since all components are the same (xyz), the partial derivatives simplify to derivatives of xyz with respect to different variables, e.g., $\partial(xyz)/\partial x = yz$, $\partial(xyz)/\partial y = xz$, and $\partial(xyz)/\partial z = xy$.

What is the value of the curl of F at the point (2, 1, 3)?

Calculating the derivatives: $\partial F_z/\partial y = \partial(xyz)/\partial y = xz = 2\ 3 = 6$, $\partial F_y/\partial z = \partial(xyz)/\partial z = xy = 2\ 1 = 2$, and similarly for others. Plugging in all derivatives, the curl at (2, 1, 3) is (6 - 2)i + (6 - 6)j + (2 - 2)k = (4, 0, 0).

What is the significance of the curl in the context of this vector field?

The curl measures the tendency of the field to rotate or circulate around a point. A non-zero curl indicates local rotation, while zero curl indicates irrotational flow.

Is the curl of this vector field at (2, 1, 3) zero or non-zero, and what does that imply?

The curl at (2, 1, 3) is (4, 0, 0), which is non-zero. This implies the vector field exhibits rotational behavior at that point.

How can understanding the curl help in physical applications like fluid flow or electromagnetism?

In fluid dynamics, the curl indicates vorticity or rotational motion of the fluid. In electromagnetism, it relates to the magnetic field's circulation. Knowing the curl helps analyze rotational properties in these fields.

Additional Resources

2. Compute The Curl Of The Vector Field At The Given Point. a) F(x,y,z) = xyzi + xyzj + xyzk en el punto (2,1,3)

Understanding the behavior of vector fields is a fundamental aspect of vector calculus, with applications spanning physics, engineering, fluid dynamics, and more. One of the key operations used to analyze such fields is the curl, which measures the tendency of the field to rotate around a point. In this article, we will delve into the process of computing the curl of a specific vector field at a given point, providing a comprehensive yet accessible explanation suitable for students, educators, and enthusiasts alike.

Introduction to the Concept of Curl in Vector Fields

Before jumping into the calculations, it's essential to grasp what the curl of a vector field represents. In simple terms, the curl quantifies the local rotation or swirling behavior of a vector field at a specific point. Imagine placing a tiny paddlewheel in a fluid flow; the curl indicates whether the wheel would spin and in which direction.

Mathematically, the curl of a vector field F = P i + Q j + R k is given by the vector operator:

$$
\nabla \times \mathbf{F} = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z} \right) \mathbf{i} + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x} \right) \mathbf{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) \mathbf{k}
$$

This operation involves taking partial derivatives of the component functions.

Defining the Given Vector Field

The vector field under consideration is:

$$
\mathbf{F}(x, y, z) = xy z \mathbf{i} + xyz \mathbf{j} + xyz \mathbf{k}
$$

Notice that the components are:

- $P(x, y, z) = xy z$
- $Q(x, y, z) = xyz$
- $R(x, y, z) = xyz$

Interestingly, the second and third components are identical functions, which simplifies some parts of the calculation.

The point at which the curl is to be computed is:

$$
(2, 1, 3)
$$

Step-by-Step Calculation of the Curl

Calculating the curl involves three main parts, each corresponding to a component of the resulting vector:

1. The i-component: $\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}$
2. The j-component: $\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}$
3. The k-component: $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}$

Let's analyze each component in detail.

Computing the i-Component: $\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}$

Step 1: Find $\frac{\partial R}{\partial y}$

Since $R(x, y, z) = xyz$,

$$
\frac{\partial R}{\partial y} = xz
$$

Step 2: Find $\frac{\partial Q}{\partial z}$

Given $Q(x, y, z) = xyz$,

$$
\frac{\partial Q}{\partial z} = xy
$$

Step 3: Evaluate at the point (2,1,3)

- $x = 2$
- $y = 1$
- $z = 3$

Thus,

$$
\frac{\partial R}{\partial y} = 2 \times 3 = 6
$$
$$
\frac{\partial Q}{\partial z} = 2 \times 1 = 2
$$

i-component:

\[
6 - 2 = 4
\]

Computing the j-Component: \(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x} \)

Step 1: Find \(\frac{\partial P}{\partial z} \)

Since \(P(x, y, z) = xy z \),

\[
\frac{\partial P}{\partial z} = xy
\]

Step 2: Find \(\frac{\partial R}{\partial x} \)

Recall \(R = xyz \),

\[
\frac{\partial R}{\partial x} = yz
\]

Step 3: Evaluate at the point (2,1,3)

\[
\frac{\partial P}{\partial z} = 1 \times 2 = 2
\]
\[
\frac{\partial R}{\partial x} = 1 \times 3 = 3
\]

j-component:

\[
2 - 3 = -1
\]

Computing the k-Component: \(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \)

Step 1: Find \(\frac{\partial Q}{\partial x} \)

\[
Q(x, y, z) = xyz
\]
\[

\frac{\partial Q}{\partial x} = yz
\]

Step 2: Find \(\frac{\partial P}{\partial y} \)

\[
P(x, y, z) = xy z
\]
\[
\frac{\partial P}{\partial y} = xz
\]

Step 3: Evaluate at the point (2,1,3)

\[
\frac{\partial Q}{\partial x} = 1 \times 3 = 3
\]
\[
\frac{\partial P}{\partial y} = 2 \times 3 = 6
\]

k-component:

\[
3 - 6 = -3
\]

Final Result: The Curl at (2,1,3)

Putting all components together, the curl vector at the specified point is:

\[
\boxed{
\nabla \times \mathbf{F}(2,1,3) = 4 \mathbf{i} - 1 \mathbf{j} - 3 \mathbf{k}
}
\]

In coordinate form, this is:

\[
\boxed{
\boxed{
\text{Curl} \mathbf{F} \big|_{(2,1,3)} = (4, -1, -3)
}
}
\]

Significance and Applications

Calculating the curl of a vector field at a point is more than an academic exercise; it has practical implications in various disciplines. For instance:

- Fluid Dynamics: The curl indicates local vorticity, helping to identify regions of rotational flow.
- Electromagnetism: The curl of magnetic and electric fields relates to induced currents and electromagnetic wave propagation.
- Mechanical Engineering: Analyzing rotational tendencies in force fields assists in designing systems with rotational dynamics.

Knowing how to compute the curl at a specific point enables engineers and scientists to understand the local behavior of complex fields, often guiding decision-making and innovation.

Final Remarks and Summary

The process of computing the curl involves:

1. Identifying component functions.
2. Calculating relevant partial derivatives.
3. Evaluating those derivatives at the specified point.
4. Combining the results into the curl vector.

In our example, the curl of the vector field $\mathbf{F}(x, y, z) = xy z \mathbf{i} + xyz \mathbf{j} + xyz \mathbf{k}$ at the point (2,1,3) is (4, -1, -3). This result offers a snapshot of the field's rotational behavior at that specific location, providing valuable insight into its local dynamics.

Understanding and calculating the curl is a vital skill in the toolkit of anyone working with vector fields, shedding light on the hidden rotational patterns that shape the physical world around us.

2 Compute The Curl Of The Vector Field At The Given Point A F X Y Z Xyzi Xyzj Xyzk En El Punto 2 1 3

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