(8 Points) Consider The Vector Field F (x, Y, Z) = (z + 4y) I + (5z + 4x)j + (5y + X) K. A) Find A Function

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Understanding vector fields and their associated scalar functions is fundamental in vector calculus, physics, and engineering. In this article, we will explore how to analyze the vector field F(x, y, z) = (z + 4y) i + (5z + 4x) j + (5y + x) k, focusing specifically on finding a scalar function A(x, y, z) such that F can be expressed as the gradient of A. This process involves understanding the concepts of gradient fields, potential functions, and the methods to determine these functions for a given vector field.

Understanding the Vector Field F

Before diving into the process of finding the potential function A, it's essential to understand the structure of the vector field F.

Components of F

The vector field F(x, y, z) is given as:
- F(x, y, z) = (z + 4y) i + (5z + 4x) j + (5y + x) k

Breaking it down:
- F_x = z + 4y
- F_y = 5z + 4x
- F_z = 5y + x

This component-wise representation helps analyze whether F is a conservative vector field, which is a prerequisite for the existence of a scalar potential A.

What is a Gradient (Potential) Function?

A scalar function A(x, y, z) is called a potential function (or scalar potential) for F if:
- ∇A = F, which means:
- ∂A/∂x = F_x
- ∂A/∂y = F_y
- ∂A/∂z = F_z

Finding A involves integrating the components of F with respect to their variables, ensuring consistency across all partial derivatives.

Step-by-Step Guide to Find the Function A(x, y, z)

To find the potential function A(x, y, z), follow these steps:

Step 1: Integrate F_x with respect to x

Begin with the first component:
- $F_x = z + 4y$

Integrate F_x with respect to x:
- $A(x, y, z) = \int (z + 4y)\, dx = x(z + 4y) + C(y, z)$

Here, $C(y, z)$ is an arbitrary function of y and z, since differentiation with respect to x eliminates any x-independent terms.

Step 2: Differentiate A with respect to y and compare with F_y

Differentiate $A(x, y, z) = x(z + 4y) + C(y, z)$ with respect to y:
- $\partial A/\partial y = 4x + \partial C/\partial y$

Recall $F_y = 5z + 4x$

Set the derivatives equal:
- $4x + \partial C/\partial y = 5z + 4x$

Subtract 4x from both sides:
- $\partial C/\partial y = 5z$

Integrate $\partial C/\partial y$ with respect to y:
- $C(y, z) = 5z\,y + D(z)$

$D(z)$ is an arbitrary function of z.

Step 3: Differentiate A with respect to z and compare with F_z

Now, differentiate $A(x, y, z) = x(z + 4y) + 5z\,y + D(z)$ with respect to z:
- $\partial A/\partial z = x + 5\,y + D'(z)$

Set this equal to $F_z = 5\,y + x$:
- $x + 5\,y + D'(z) = 5\,y + x$

Subtract $x + 5\,y$ from both sides:
- $D'(z) = 0$

Integrate:
- $D(z) = \text{constant}$

Thus, $D(z) = \text{constant}$, which can be set to zero without loss of generality.

Final Form of the Potential Function A(x, y, z)

Putting all the parts together:
- $A(x, y, z) = x(z + 4\,y) + 5\,z\,y + \text{constant}$

For simplicity, set the constant to zero:
- $A(x, y, z) = x\,z + 4\,x\,y + 5\,z\,y$

This scalar function A satisfies the condition $\nabla A = F$, making F a conservative vector field with potential A.

Conclusion and Significance of Finding A

Determining the potential function $A(x, y, z)$ for a vector field is vital in various applications:
- In physics, it helps identify conservative forces like gravity and electrostatics.
- In engineering, it simplifies complex vector fields into scalar functions for analysis.
- It provides insights into the field's behavior, such as whether work done around closed loops is zero.

In our case, the potential function $A(x, y, z) = x z + 4 x y + 5 z y$ encapsulates the essence of the vector field F and confirms its conservative nature.

Additional Tips for Finding Potential Functions

- Always verify that the mixed partial derivatives are consistent (Clairaut's theorem).
- Ensure the vector field satisfies the curl-free condition $\nabla \times F = 0$ before assuming it has a potential function.
- When integrating, keep track of functions that depend on variables other than the one you're integrating with respect to.

Summary

- The vector field $F(x, y, z)$ was analyzed component-wise.
- Integration and differentiation steps led to the potential function $A(x, y, z) = x z + 4 x y + 5 z y$.
- The process underscores the importance of understanding gradient fields and potential functions in vector calculus.

By mastering these techniques, students and professionals can efficiently analyze complex vector fields and apply these concepts across physics, engineering, and mathematics.

Meta note: This detailed guide provides a comprehensive approach to finding a potential function for a given vector field, tailored for educational and SEO purposes, and ensures clarity for readers at various levels of understanding.

Frequently Asked Questions

What is the given vector field F(x, y, z) in the problem?

The vector field is $F(x, y, z) = (z + 4y)\, i + (5z + 4x)\, j + (5y + x)\, k$.

What is the main goal when asked to 'Find A Function' related to the vector field?

The main goal is to determine a scalar potential function $\phi(x, y, z)$ such that $F = \nabla \phi$, meaning F is a gradient field.

How do you verify if the vector field F is conservative (has a potential

function)?

You verify if the curl of F is zero, i.e., $\nabla \times F = 0$, indicating F is conservative and has a potential function.

What are the steps to find the potential function φ(x, y, z) for the given vector field?

Integrate the first component with respect to x, then find the remaining functions by matching derivatives with other components, ensuring consistency across all variables.

How can you check the correctness of the potential function φ(x, y, z)?

By computing its gradient $\nabla \phi$ and verifying that it equals the original vector field F.

What is the significance of finding the potential function for the vector field in physics or engineering?

It helps in understanding work done by the field, potential energy, and simplifies calculations of line integrals and flux.

Additional Resources

In-Depth Analysis of the Vector Field $\mathbf{F}(x, y, z) = (z + 4y) \mathbf{i} + (5z + 4x) \mathbf{j} + (5y + x) \mathbf{k}$: Finding a Potential Function

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Introduction

The process of understanding vector fields is foundational in various branches of physics and mathematics, including fluid dynamics, electromagnetism, and vector calculus. One crucial aspect of analyzing vector fields is determining whether they are conservative (i.e., whether they can be expressed as the gradient of some scalar potential function). If such a potential function exists, it simplifies the analysis of the field considerably, enabling easier computation of line integrals, understanding flow patterns, and analyzing energy functions.

In this detailed review, we focus on the vector field:

$$
\mathbf{F}(x, y, z) = (z + 4y) \mathbf{i} + (5z + 4x) \mathbf{j} + (5y + x) \mathbf{k}
$$

Our goal is to find a scalar potential function $\phi(x, y, z)$ such that:

$$
\nabla \phi = \mathbf{F}
$$

In other words, we want to find ϕ satisfying:

$$
\frac{\partial \phi}{\partial x} = P(x, y, z) = z + 4y
$$

$$
\frac{\partial \phi}{\partial y} = Q(x, y, z) = 5z + 4x
$$

$$
\frac{\partial \phi}{\partial z} = R(x, y, z) = 5y + x
$$

Understanding the Concept of Potential Functions and Conservative Fields

What is a potential function?
A scalar function $\phi(x, y, z)$ is called a potential function of a vector field $\mathbf{F}$ if:

$$
\nabla \phi = \mathbf{F}
$$

which implies:

$$
\frac{\partial \phi}{\partial x} = P, \quad \frac{\partial \phi}{\partial y} = Q, \quad \frac{\partial \phi}{\partial z} = R
$$

When does a vector field have a potential function?
A vector field is conservative if it can be expressed as the gradient of a scalar potential. For a vector field $\mathbf{F}$, this condition is met when:

- The field is defined over a simply connected domain
- The curl of $\mathbf{F}$ is zero:

$$
\nabla \times \mathbf{F} = \mathbf{0}
$$

Why check the curl?
The curl condition ensures the field is irrotational; thus, a potential function exists locally. If the domain is simply connected, the potential function exists globally.

Step 1: Verify if $\mathbf{F}$ is Conservative

Before attempting to find ϕ, confirm whether $\mathbf{F}$ is conservative.

Calculate:

$$
\nabla \times \mathbf{F} =
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
P & Q & R
\end{vmatrix}
$$

Where:

- $P = z + 4y$
- $Q = 5z + 4x$
- $R = 5y + x$

Compute each component:

$(\nabla \times \mathbf{F})_x$:

$$
\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z} = \frac{\partial (5y + x)}{\partial y} - \frac{\partial (5z + 4x)}{\partial z} = 5 - 5 = 0
$$

$(\nabla \times \mathbf{F})_y$:

$$
\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x} = \frac{\partial (z + 4y)}{\partial z} - \frac{\partial (5y + x)}{\partial x} = 1 - 1 = 0
$$

$(\nabla \times \mathbf{F})_z$:

$$
\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = \frac{\partial (5z + 4x)}{\partial x} - \frac{\partial (z + 4y)}{\partial y} = 4 - 4 = 0
$$

Result:
Since the curl is zero everywhere, $\nabla \times \mathbf{F} = \mathbf{0}$, and the field is irrotational. Assuming the domain is simply connected, $\mathbf{F}$ is conservative, and a potential function ϕ exists.

Step 2: Find the Potential Function ϕ

Given that:

$$
\nabla \phi = (P, Q, R) = (z + 4y, 5z + 4x, 5y + x)
$$

we proceed by integrating component-wise, starting with the partial derivative with respect to x:

$$
\frac{\partial \phi}{\partial x} = z + 4y
$$

Step 3: Integrate P with respect to x

$$
\phi(x, y, z) = \int (z + 4y) \, dx + h(y, z)
$$

Since $z + 4y$ is independent of x, the integral yields:

$$
\phi(x, y, z) = x(z + 4y) + h(y, z)
$$

Here, $h(y, z)$ is an arbitrary function of y and z, since integrating with respect to x treats

y and z as constants.

Step 4: Use $\frac{\partial \phi}{\partial y} = Q$

Differentiate ϕ:

$$
\frac{\partial \phi}{\partial y} = \frac{\partial}{\partial y} [x(z + 4y) + h(y, z)] = x(4) + \frac{\partial h(y, z)}{\partial y}
$$

Set equal to $Q = 5z + 4x$:

$$
4x + \frac{\partial h(y, z)}{\partial y} = 5z + 4x
$$

Subtract $4x$ from both sides:

$$
\frac{\partial h(y, z)}{\partial y} = 5z
$$

Integrate with respect to y:

$$
h(y, z) = 5z y + g(z)
$$

where $g(z)$ is an arbitrary function of z.

Step 5: Use $\frac{\partial \phi}{\partial z} = R$

Differentiate ϕ:

$$
\frac{\partial \phi}{\partial z} = \frac{\partial}{\partial z} [x(z + 4y) + 5z y + g(z)] = x(1) + 0 + 5 y + g'(z)
$$

Set equal to $R = 5 y + x$:

$$
x + 5 y + g'(z) = 5 y + x
$$

Subtract $x + 5 y$ from both sides:

$$
g'(z) = 0
$$

This implies:

$$
g(z) = C

\]

where \(C \) is a constant.

Final Potential Function

Putting all parts together, the potential function is:

\[
\boxed{
\phi(x, y, z) = x(z + 4 y) + 5 z y + C
}
\]

or, equivalently,

\[
\phi(x, y, z) = x z + 4 x y + 5 y z + \text{constant}
\]

Summary and Insights

- The vector field \(\mathbf{F} \) was confirmed to be conservative by verifying the curl condition.
- The potential function \(\phi \) was systematically derived by integrating component-wise, ensuring consistency across all partial derivatives.
- The final potential function simplifies to a polynomial expression, which is typical for fields derived from linear combinations of coordinate functions.

Implications of the Potential Function

Having identified the potential function, several important applications become accessible:

- Line integrals: For any path \(C \), the line integral of \(\mathbf{F} \) becomes:

\[
\int_C \mathbf{F} \cdot d\mathbf{r} = \phi(\text{endpoint}) - \phi(\text{start point})
\]

- Energy interpretations: In physics, \(\phi \) can represent potential

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