A Rectangle Has Perimeter 52 Feet. [a] Express Its Length L In Terms Of Its Width W. [b] Uso [a] To Express

Understanding the Perimeter of a Rectangle

A Rectangle Has Perimeter 52 Feet. [a] Express Its Length L In Terms Of Its Width W. [b] Uso [a] To Express is a common problem in geometry that helps students understand the relationship between a rectangle's dimensions and its perimeter. In this article, we will explore how to derive a formula for the length of a rectangle when the perimeter is known, and how to use that information to solve related problems.

A rectangle is a four-sided polygon with opposite sides equal in length. The perimeter of a rectangle is the total distance around it, calculated by adding the lengths of all four sides. Since opposite sides are equal, the perimeter can be expressed as:

$$P = 2L + 2W$$

where:
- P is the perimeter,
- L is the length,
- W is the width.

In our problem, the perimeter P is given as 52 feet, and our goal is to express the length L in terms of the width W.

Expressing Length L in Terms of Width W

Step 1: Write the Perimeter Formula

Given:

$$P = 52 \text{ feet}$$

And the perimeter formula:

$$P = 2L + 2W$$

Substitute the known perimeter:

$$52 = 2L + 2W$$

Step 2: Simplify the Equation

Divide both sides of the equation by 2 to make calculations easier:

$$\frac{52}{2} = L + W$$

$$26 = L + W$$

Step 3: Solve for L

Isolate L to express it in terms of W:

$$L = 26 - W$$

This is the key formula that relates the length of the rectangle to its width when the perimeter is fixed at 52 feet.

Using the Expression to Solve Practical Problems

Scenario 1: Find the Length When the Width Is Known

Suppose the width W is known, for example, 10 feet. Then, the length L can be calculated as:

$$L = 26 - 10 = 16 \text{ feet}$$

Similarly:

- If $W = 15$ feet:

$$L = 26 - 15 = 11 \text{ feet}$$

- If $W = 5$ feet:

$$L = 26 - 5 = 21 \text{ feet}$$

This demonstrates how the length varies inversely with the width when the perimeter remains constant.

Scenario 2: Find the Width When the Length Is Known

If, for example, the length L is 20 feet:

$$W = 26 - L = 26 - 20 = 6 \text{ feet}$$

The flexibility of the formula allows you to determine the missing dimension based on the known one, facilitating design and layout decisions in real-world applications.

Graphical Interpretation of the Relationship

Understanding the relationship between L and W can be enhanced by visualizing the equation:

$$ L = 26 - W $$

This is a linear equation with:

- Slope: -1,
- Y-intercept: 26.

Plotting this on a coordinate plane where W is on the x-axis and L on the y-axis, the line will slope downward, illustrating that as the width increases, the length decreases, maintaining a constant perimeter.

Constraints and Practical Considerations

While the formula $L = 26 - W$ provides a mathematical relationship, practical constraints must be considered:

- Both L and W should be positive values.
- The width W must be less than or equal to 26 feet to ensure a positive length L.

Thus:

$$ 0 < W \leq 26 $$

Similarly, the corresponding length:

$$ 0 < L \leq 26 $$

Any choice of W outside these bounds would result in negative or zero dimensions, which are not physically meaningful for a rectangle’s sides.

Extending the Problem: Using the Relationship to Find Area

Once the length L is expressed in terms of the width W, it becomes possible to derive the area A of the rectangle:

$$A = L \times W$$

Substitute $L = 26 - W$:

$$A = (26 - W) \times W$$

$$A = 26W - W^2$$

This quadratic function describes the area in terms of the width, and its maximum can be found using calculus or vertex formula, which is useful in optimization problems.

Finding the Width That Maximizes the Area

Since $A = -W^2 + 26W$, the parabola opens downward, and the maximum occurs at the vertex:

$$W_{max} = -\frac{b}{2a}$$

Where:

- $a = -1$,
- $b = 26$.

Calculate:

$$W_{max} = -\frac{26}{2 \times -1} = -\frac{26}{-2} = 13$$

Corresponding length:

$$L = 26 - 13 = 13$$

Maximum area:

$$A_{max} = 13 \times 13 = 169 \text{ square feet}$$

This demonstrates that for a rectangle with a perimeter of 52 feet, the maximum area is achieved when the rectangle is a square with sides of 13 feet each.

Summary of Key Points

- The perimeter of a rectangle relates its length and width through the formula $P = 2L + 2W$.
- When the perimeter is fixed at 52 feet, the length L can be expressed as $L = 26 - W$.
- The dimensions are constrained by the condition $0 < W \leq 26$ and $0 < L \leq 26$.
- The relationship between length and width is linear, with increasing width resulting in decreasing length.
- The area of the rectangle can be expressed as $A = 26W - W^2$, allowing for optimization to find maximum area.

Practical Applications and Real-World Relevance

Understanding how to manipulate and interpret formulas relating to the perimeter and dimensions of rectangles has numerous practical applications:
- Designing fencing for a rectangular garden with a fixed amount of fencing material.
- Planning floor layouts where the total boundary length is constrained.
- Optimization in manufacturing to maximize usable space within size constraints.
- Educational purposes, illustrating the relationships between geometric properties.

Conclusion

Mastering the relationship between the perimeter, length, and width of a rectangle is fundamental in geometry. By expressing the length \(L \) in terms of the width \(W \), students and professionals can solve a variety of problems involving dimensions, areas, and optimization. The key formula \(L = 26 - W \) derived from the perimeter of 52 feet provides a powerful tool for understanding and calculating the dimensions of rectangles under specific constraints. Whether for academic exercises or real-world applications, this relationship forms a core concept in geometric problem-solving.

Frequently Asked Questions

How do you express the length L of a rectangle in terms of its width W if the perimeter is 52 feet?

Since the perimeter P = 2(L + W), and P = 52, then L = (52/2) - W, which simplifies to L = 26 - W.

What is the formula for the perimeter of a rectangle?

The perimeter of a rectangle is given by P = 2(L + W), where L is the length and W is the width.

If the width W of a rectangle is 8 feet, what is its length L given the perimeter is 52 feet?

Using L = 26 - W, substituting W = 8, L = 26 - 8 = 18 feet.

How can you use the expression for L in terms of W to find W if the rectangle's perimeter is 52 feet?

From L = 26 - W and perimeter P = 2(L + W) = 52, substitute L to get 2(26 - W + W) = 52, which simplifies to 2 26 = 52, confirming the relationship. To find W for a specific L, rearrange the formula accordingly.

What is the significance of expressing L in terms of W in solving rectangle problems?

Expressing L in terms of W allows you to analyze how changing the width affects the length and to solve for specific dimensions given constraints like perimeter.

Using the expression L = 26 - W, how can you determine the possible dimensions of the rectangle?

Since both L and W must be positive, W must be less than 26, and L = 26 - W. Both dimensions are valid as long as W > 0 and L > 0, i.e., W < 26.

How do you use the expression for L in terms of W to find the dimensions when given a different perimeter?

Replace the perimeter value in P = 2(L + W), express L as 26 - W, and solve for W to find the corresponding dimensions.

Can the length L be equal to the width W in this problem? Why or why not?

Yes, if W = 13 feet, then L = 26 - 13 = 13 feet, making the rectangle a square with perimeter 52 feet. So, the rectangle can be a square with sides of 13 feet.

Additional Resources

Understanding the Perimeter of a Rectangle and Deriving Its Length in Terms of Width

When exploring geometric figures, rectangles stand out due to their simplicity and widespread application in real-world contexts—from architecture and interior design to manufacturing and everyday problem-solving. A fundamental property of rectangles is the relationship between their length, width, and perimeter. In this detailed review, we will analyze the problem: "A rectangle has a perimeter of 52 feet," and then systematically develop expressions for its length in terms of its width, as well as use this relationship for further computations.

Introduction to the Rectangle's Perimeter

The perimeter of a rectangle is the total distance around its boundary. It encompasses the sum of all four sides. Since rectangles have opposite sides equal in length, calculating the perimeter simplifies to adding twice the length and twice the width.

Key Concept:
- Perimeter (P) of a rectangle = 2 × (Length + Width)

In our case, the perimeter is given as 52 feet, which provides us with an equation involving the rectangle's length and width.

Expressing Length (L) in Terms of Width (W): Deriving the Formula

Step 1: Write the Perimeter Equation

Given the perimeter:

$$ P = 2(L + W) $$

Substituting the known perimeter:

$$ 52 = 2(L + W) $$

Step 2: Isolate the Sum of Length and Width

Divide both sides by 2:

$$ \frac{52}{2} = L + W $$

$$ 26 = L + W $$

Step 3: Express Length in Terms of Width

Rearranged, the equation becomes:

$$ L = 26 - W $$

This gives us the first critical relationship:

> L in terms of W:
>
> L = 26 - W

This formula allows us to determine the length of the rectangle once the width is known, or vice versa. It highlights the inverse relationship between length and width for a fixed perimeter of 52 feet.

Implications of the Relationship: Constraints and Practical Considerations

Understanding that $L = 26 - W$ is not only a mathematical fact but also has real-world implications. Since lengths cannot be negative, the dimensions must satisfy:

$$W > 0$$

$$L > 0$$

Applying the formula:

$$26 - W > 0 \Rightarrow W < 26$$

Similarly:

$$L = 26 - W > 0 \Rightarrow W < 26$$

Thus, the width must be less than 26 feet for the length to be positive.

Range of possible dimensions:

- $0 < W < 26$ feet
- Correspondingly, $0 < L < 26$ feet

This range ensures the rectangle is physically meaningful with positive dimensions.

Using the Relationship to Explore Specific Dimensions

Once the general expression $L = 26 - W$ is established, it opens avenues for various specific calculations or constraints, including:

- Finding the dimensions that maximize area
- Determining the dimensions for a given area
- Exploring the effects of changing dimensions on perimeter and area

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Maximizing Area with Fixed Perimeter

Often, problems involve optimizing the rectangle's area given a fixed perimeter. Using the relationship:

$$L = 26 - W$$

The area A of the rectangle is:

$$A = L \times W$$

Substituting for L:

$$A = (26 - W) \times W$$

$$A = 26W - W^2$$

This quadratic function describes the area in terms of the width W.

Finding the maximum area:

- The quadratic $A(W) = -W^2 + 26W$ opens downward (coefficient of W^2 is negative), indicating a maximum point at its vertex.
- The vertex of $A(W)$ occurs at:

$$ W = -\frac{b}{2a} $$

where $a = -1$, $b = 26$:

$$ W = -\frac{26}{2 \times (-1)} = -\frac{26}{-2} = 13 $$

- Corresponding length:

$$ L = 26 - W = 26 - 13 = 13 $$

Conclusion:
- The rectangle has maximum area when width = 13 feet and length = 13 feet.
- The maximum area is:

$$ A_{max} = 13 \times 13 = 169 \text{ square feet} $$

This result conforms with the principle that, for a given perimeter, a square maximizes the area among rectangles.

Using the Expression for Practical Problem Solving

The formula $L = 26 - W$ is instrumental in solving various real-world problems, such as:

- Designing a garden or a fencing project where the total fencing length (perimeter) is fixed.

- Calculating dimensions for manufacturing items with specific perimeter constraints.
- Optimizing material usage when constructing rectangular enclosures.

Let's analyze some of these applications.

Example 1: Given a specific width, find the length

Suppose, in a construction project, the width of a rectangular flower bed is 8 feet. What is the length?

Using the formula:

$$ L = 26 - W $$

$$ L = 26 - 8 = 18 \text{ feet} $$

Result: The length should be 18 feet to achieve a perimeter of 52 feet.

Example 2: Find the dimensions for a square with perimeter 52 feet

Since a square is a special rectangle with all sides equal, the perimeter:

$$ P = 4 \times \text{side} $$

Set:

$$ 4 \times s = 52 \Rightarrow s = \frac{52}{4} = 13 \text{ feet} $$

This aligns with our earlier maximum area calculation, confirming that a square with sides of 13 feet has a perimeter of 52 feet and maximizes area among rectangles with the same perimeter.

Graphical Representation and Analysis

Visualizing the relationship between length and width helps deepen understanding.

- Plotting $L = 26 - W$ yields a straight line with intercepts at:

- When $W = 0$, $L = 26$
- When $W = 26$, $L = 0$

- The feasible region for dimensions lies within:

$$ 0 < W < 26 $$
$$ 0 < L < 26 $$

- The maximum area point at $W = 13$, $L = 13$, appears at the center of this feasible region.

This graphical perspective emphasizes the symmetry and the optimal point for maximum area.

Summary and Key Takeaways

- The perimeter of a rectangle relates directly to its length and width via the formula:

$$ P = 2(L + W) $$

- For a perimeter of 52 feet, the relationship simplifies to:

$$ L + W = 26 $$

- The key formula derived:

$$ \boxed{L = 26 - W} $$

which allows calculations of length for any given width within the feasible range.

- The constraints for the dimensions:

$$ 0 < W < 26 \quad \text{and} \quad L = 26 - W $$

- The maximum area of the rectangle occurs when:

$$ W = L = 13 \text{ feet} $$

yielding an area of 169 square feet.

- This analysis demonstrates how algebraic expressions facilitate problem-solving in geometry and real-world applications. The relationship between dimensions guides optimal design, efficient material utilization, and understanding the geometric properties of rectangles.

Final Remarks

Understanding how to express the length of a rectangle in terms of its width when given a fixed perimeter is a fundamental skill in geometry. It combines algebraic manipulation with geometric intuition, providing a foundation for more advanced topics such as optimization problems, coordinate geometry, and even calculus-based analysis.

The method illustrated—deriving an explicit formula from a given perimeter—can be extended to various shapes and constraints, making it a versatile tool in both academic settings and practical scenarios. Recognizing the importance of these relationships enhances problem-solving efficiency and encourages a deeper appreciation for the interconnectedness of geometric properties.

In conclusion, knowing that a rectangle with a perimeter of 52 feet satisfies the relationship \(L = 26 - W\) empowers students and professionals alike to analyze, design, and optimize rectangular structures with clarity and confidence.

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