A Rectangular Box Is To Be Made From A Piece Of Cardboard 24 Cm Long And 9 Cm Wide By Cutting Out Identical

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Creating a rectangular box from a flat piece of cardboard is a common and practical problem that combines geometric reasoning with practical craftsmanship. Whether you're a student working on a math project, an artisan crafting custom storage solutions, or a hobbyist looking to recycle materials creatively, understanding how to transform a flat rectangular sheet into a three-dimensional box is invaluable. In this article, we will explore the step-by-step process of designing and constructing a rectangular box from a 24 cm by 9 cm piece of cardboard by cutting out identical squares and folding the remaining flaps to form a sturdy container.

Understanding the Problem: Making a Box from a Rectangular Cardboard Sheet

Before diving into the construction process, it is essential to understand the problem's parameters:

- The original piece of cardboard measures 24 cm in length and 9 cm in width.
- The goal is to create an open-topped rectangular box.
- The method involves cutting out square sections from each corner of the rectangle.
- The size of these squares must be identical on all four corners.
- After cutting, folding up the sides will form the sides of the box.

This problem is a classic example used in geometry to illustrate concepts of area, volume, and the effects of folding and cutting on three-dimensional shapes.

Determining the Size of the Cut-Out Squares

The key to constructing the box lies in selecting the size of the squares to cut out from each corner. This size is crucial because:

- It directly influences the height of the resulting box.
- It affects the volume and stability of the box.
- It must be small enough to allow the sides to fold up without overlapping or collapsing.

Variables to consider:

- Let the side length of each square cut-out be x cm.
- The new dimensions of the box after folding will be:

- Length: $(24 - 2x)$ cm
- Width: $(9 - 2x)$ cm
- Height: x cm

Constraints:

- Since the squares are cut out from each corner, the dimensions after cutting must be positive:

- $24 - 2x > 0 \rightarrow x < 12$ cm
- $9 - 2x > 0 \rightarrow x < 4.5$ cm

Therefore, the maximum possible value of x is less than 4.5 cm to ensure the box has a base.

Calculating the Volume of the Box

The most common reason for creating such a box is to maximize its volume. The volume V can be expressed as:

V = (length after cut) × (width after cut) × (height)

Substituting the expressions:

$$V(x) = (24 - 2x)(9 - 2x)(x)$$

Expanding this:

$$V(x) = x \times (24 - 2x) \times (9 - 2x)$$

First, multiply the two binomials:

$$(24 - 2x)(9 - 2x) = 24 \times 9 - 24 \times 2x - 2x \times 9 + 2x \times 2x$$

$$= 216 - 48x - 18x + 4x^2$$

$$= 216 - 66x + 4x^2$$

Now, multiply by x:

$$V(x) = x \times [216 - 66x + 4x^2]$$

$$= 216x - 66x^2 + 4x^3$$

The volume function:

$V(x) = 4x^3 - 66x^2 + 216x$

Maximizing the Volume: Finding the Optimal Cut Size

To maximize the volume, differentiate V(x) with respect to x and find the critical points:

$V'(x) = 12x^2 - 132x + 216$

Set V'(x) = 0 to find the critical points:

$12x^2 - 132x + 216 = 0$

Divide through by 12:

$x^2 - 11x + 18 = 0$

Solve using the quadratic formula:

$x = [11 \pm \sqrt{(11^2 - 4 \times 1 \times 18)}] / 2$

Calculate discriminant:

$D = 121 - 72 = 49$

Compute solutions:

$x = [11 \pm 7] / 2$

1. $x = (11 + 7) / 2 = 18 / 2 = 9$

2. $x = (11 - 7) / 2 = 4 / 2 = 2$

Recall the earlier constraints: $x < 4.5$ cm, so $x = 9$ cm is invalid. The feasible critical point is $x = 2$ cm.

Verifying the Maximum Volume at x = 2 cm

Evaluate the volume at x = 2 cm:

$V(2) = 4(2)^3 - 66(2)^2 + 216(2)$

Calculate step-by-step:

- $4(8) = 32$
- $66(4) = 264$
- $216(2) = 432$

Now, substitute:

$V(2) = 32 - 264 + 432 = (32 + 432) - 264 = 464 - 264 = 200\ cm^3$

This is the maximum volume achievable with the given dimensions and constraints.

Constructing the Box Step-by-Step

Once the optimal square size $x = 2$ cm is determined, proceed with the physical construction:

Materials Needed:

- A piece of cardboard measuring 24 cm by 9 cm
- A ruler
- A pencil
- Scissors
- Tape or glue (optional, for extra stability)

Step-by-step process:

1. Mark the Cut-Outs:

- Using the ruler, measure 2 cm inward from each corner along the length and width.
- Mark the points at these measurements to define the squares to cut.

2. Cut Out the Squares:

- Carefully cut out 2 cm by 2 cm squares from each corner.

3. Fold Up the Sides:

- Fold along the lines where the squares were cut out to create the sides of the box.
- Bring the sides upward to form the box shape.

4. Secure the Edges:

- Use tape or glue to secure the sides if needed, ensuring the box maintains its shape.

5. Check the Dimensions:

- Confirm that the base measures $(24 - 2\times2) = 20$ cm in length and $(9 - 2\times2) = 5$ cm in width.

- The height should be 2 cm.

Result:

Your box should now be a sturdy, open-topped container with maximum volume of approximately 200 cm^3.

Applications and Variations

Constructing boxes from cardboard is not only an academic exercise but also has numerous practical applications:

- Creating storage containers
- Packaging design
- DIY craft projects
- Learning about optimization and geometry

Variations to explore:

- Making boxes with different base dimensions
- Including a lid for the box
- Using different materials like paper or plastic
- Designing boxes with handles or special features

Tips for successful construction:

- Use precise measurements for accuracy.
- Ensure cuts are clean and straight.
- Fold carefully along the lines to maintain shape.
- Experiment with different square sizes to see how volume changes.

Conclusion

Transforming a flat piece of cardboard into a functional rectangular box involves understanding geometric relationships, calculating optimal dimensions, and applying practical skills. By carefully analyzing the problem, deriving the volume function, and finding the critical point, you can maximize the space within your box. This process exemplifies how mathematics and craftsmanship work hand-in-hand to create efficient and effective designs. Whether for educational purposes or practical projects, mastering this method enhances problem-solving skills and encourages creative thinking in everyday applications.

Meta Description: Learn how to make a rectangular box from a 24 cm by 9 cm cardboard sheet by cutting out identical squares. Discover the steps to maximize volume and build your own sturdy container.

Frequently Asked Questions

How do you determine the size of the squares to cut out to make a rectangular box from a 24 cm by 9 cm cardboard sheet?

You set a variable 'x' for the side length of the squares to cut out from each corner. The dimensions of the box after folding will be (24 - 2x) cm in length, (9 - 2x) cm in width, and x cm in height. To find the optimal size of 'x', you can maximize the volume function V = x(24 - 2x)(9 - 2x).

What is the maximum volume of the box that can be made from the given cardboard sheet?

By setting up the volume function V = x(24 - 2x)(9 - 2x) and finding its maximum through calculus or graphing, you can determine the value of 'x' that maximizes volume. The approximate maximum volume occurs when x ≈ 1.5 cm, resulting in a volume of around 223.5 cubic centimeters.

How do you derive the formula for the volume of the box made from the cardboard?

The volume V is calculated as length × width × height. After cutting out squares of side 'x', the dimensions become (24 - 2x) and (9 - 2x) for the base, with height 'x'. Therefore, V = x(24 - 2x)(9 - 2x).

What are the constraints on 'x' when cutting squares from the cardboard?

Since the squares are cut from each corner, 'x' must be positive and less than half of the shortest side to avoid negative dimensions. Therefore, 0 < x < 4.5 cm, but realistically limited by the width of 9 cm, so 0 < x < 4.5 cm, with the practical maximum being less than 4.5 cm.

Can the maximum volume be found without calculus methods?

Yes, by analyzing the volume function as a quadratic or cubic polynomial and using methods like completing the square or plotting the function, you can approximate the maximum volume without calculus. Alternatively, trial and error with different 'x' values can give a close estimate.

Why is it important to cut out identical squares from each

corner?

Cutting out identical squares ensures the box folds symmetrically, resulting in a rectangular box with uniform height and stable structure. It helps in maximizing volume while maintaining the shape's symmetry.

What practical considerations should be kept in mind when making such a box?

Consider the thickness of the cardboard, which may reduce the internal dimensions, and ensure the cut-out squares are precise for proper folding. Also, avoid cutting too large squares that leave insufficient material for the sides of the box.

Is there an alternative method to maximize the volume besides calculus?

Yes, you can use numeric methods like graphing the volume function and identifying its maximum point, or perform iterative calculations with different values of 'x' to find the approximate maximum volume.

Additional Resources

A Rectangular Box Is To Be Made From A Piece Of Cardboard 24 Cm Long And 9 Cm Wide By Cutting Out Identical squares or rectangles from the corners, then folding up the sides to form a box — this classic problem combines practical craftsmanship with fundamental geometric principles. This task not only tests one's understanding of spatial reasoning but also offers a hands-on approach to learning concepts such as volume, surface area, and optimization. In this article, we will explore the process of creating such a box, analyze various methods, discuss the mathematical underpinnings, and evaluate the practical aspects of this activity.

Understanding the Basic Concept

The task involves transforming a flat rectangular piece of cardboard measuring 24 cm by 9 cm into a three-dimensional box by cutting out identical squares or rectangles from each corner. Once the cutouts are made, the sides are folded upward and secured (either by tape, glue, or simply by the fold) to create an open-top box.

Key steps include:

- Deciding on the size of the cutouts.
- Making precise cuts to ensure the box is symmetrical.
- Folding the sides accurately to maintain the box’s shape.
- Calculating the resulting dimensions and volume.

This activity combines practical skills with mathematical analysis, making it an excellent educational tool for students learning about geometry and measurement.

Mathematical Foundations

The problem is fundamentally rooted in geometric and algebraic principles. Let’s define variables and derive formulas to understand the process deeply.

Variables and Dimensions

- Original dimensions of the cardboard: length $L = 24$ cm, width $W = 9$ cm.
- Size of the cutout squares: x cm (assuming squares for simplicity).
- Resulting dimensions of the box:
- Length: $L' = L - 2x$
- Width: $W' = W - 2x$
- Height: $h = x$

The volume V of the box can then be expressed as:

$$V(x) = (L - 2x) \times (W - 2x) \times x$$

which simplifies to:

$$V(x) = (24 - 2x)(9 - 2x) \times x$$

Expanding:

$$V(x) = (216 - 48x - 18x + 4x^2) \times x$$
$$V(x) = (216 - 66x + 4x^2) \times x$$
$$V(x) = 216x - 66x^2 + 4x^3$$

This cubic function models how the volume varies with the size of the cutouts.

Optimization: Finding the Maximum Volume

A common goal is to determine the size of the cutouts x that maximizes the volume of the box. This involves calculus — taking the derivative of $V(x)$ with respect to x and finding critical points.

Calculus Approach

Differentiate $V(x)$:

$$V'(x) = 216 - 132x + 12x^2$$

Set the derivative to zero to find critical points:

$$12x^2 - 132x + 216 = 0$$

Divide through by 12:

$$x^2 - 11x + 18 = 0$$

Factor:

$$(x - 9)(x - 2) = 0$$

Solutions:

$$x = 2 \quad \text{or} \quad x = 9$$

Since the cutout size cannot be larger than half the smaller dimension (which is 4.5 cm), $x = 9$ cm is invalid. The feasible solution is $x = 2$ cm.

Interpretation:

- Cutting out 2 cm squares from each corner yields the maximum volume.
- Corresponding dimensions of the box:

- Length: $24 - 2 \times 2 = 20$ cm
- Width: $9 - 2 \times 2 = 5$ cm
- Height: 2 cm

- Maximum volume:

$$V(2) = 216(2) - 66(2)^2 + 4(2)^3 = 432 - 264 + 32 = 200 \ \text{cm}^3$$

This mathematical analysis guides practical decision-making, especially in manufacturing or packaging designs.

Practical Considerations and Variations

While the mathematical approach provides an ideal solution, real-world applications require attention to detail and considerations of materials, tools, and constraints.

Choosing the Cutout Shape

- Squares: Simplest to cut and fold, ensuring symmetry.
- Rectangles: Can be used for specific design needs; however, they complicate the calculation.
- Pros and Cons:
- Squares: Easy to measure and cut; predictable folding.
- Rectangles: May allow for larger volume in specific designs but are harder to cut precisely.

Tools and Techniques

- Sharp scissors or utility knives for clean cuts.
- Ruler and pencil for accurate measurements.
- Folding tools or straight edges for crisp folds.
- Adhesives if needed for added strength.

Material Constraints

- Thinner cardboard may tear; thicker materials are harder to fold.
- Edge reinforcement might be necessary for durability.
- Consider the weight capacity if the box is used for storage.

Design Variations and Features

- Adding flaps or lids for closure.
- Incorporating handles or reinforcement strips.
- Experimenting with different cutout sizes for aesthetic or functional purposes.

Pros and Cons of the Cutting and Folding Method

Pros:

- Educational value: Enhances understanding of geometry, measurement, and optimization.
- Cost-effective: Uses minimal material and simple tools.
- Customizable: Can be tailored for specific dimensions and purposes.
- Hands-on learning: Engages learners through tactile experience.

Cons:

- Precision required: Small errors in measurement or cutting can affect the final shape.
- Material limitations: Not suitable for heavy-duty or delicate materials.

- Limited complexity: Only basic shapes are feasible without advanced techniques.

Applications and Real-World Uses

Creating boxes from cardboard is more than an academic exercise; it has numerous practical applications:

- Packaging Design: Optimizing box dimensions for maximum volume or minimal material.
- Prototyping: Testing container shapes before manufacturing.
- Art and Craft: Custom containers for gifts or decorations.
- Educational Demonstrations: Teaching concepts of surface area, volume, and optimization.

Furthermore, understanding how to manipulate simple materials efficiently underpins many manufacturing and logistical processes.

Conclusion and Final Thoughts

Transforming a piece of cardboard measuring 24 cm by 9 cm into an optimal rectangular box through cutting and folding is a classic problem that beautifully combines practical skills with mathematical reasoning. By carefully choosing the size of the corner cutouts — particularly around 2 cm for this specific case — one can maximize the volume of the resulting box. This activity exemplifies how geometric principles can be applied to real-world scenarios, fostering critical thinking and problem-solving skills.

Whether for educational purposes, packaging solutions, or creative projects, understanding the underlying mathematics and practical considerations ensures efficient and effective results. The process highlights the importance of precision, planning, and experimentation, encouraging learners and practitioners alike to explore the fascinating relationships between shape, size, and function in everyday objects.

In summary, this task demonstrates that even simple activities like making a cardboard box can serve as valuable lessons in geometry, optimization, and craftsmanship, offering both theoretical insights and tangible results.

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