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Understanding the dimensions of a patio is essential for various reasons, including calculating the area, planning for materials, and ensuring the space fits within a designated outdoor area. When the length and width of a rectangular patio are expressed algebraically, such as (1 + 2x) yards and (2 - 3x) yards respectively, it introduces a layer of mathematical analysis that can help determine the patio's size and other properties. This article aims to explore how to interpret these expressions, analyze the possible values of x, and understand the implications for the patio's dimensions.

Understanding the Dimensions of a Rectangular Patio

In geometry and algebra, the dimensions of a rectangle are typically given by its length and width. When these dimensions are expressed algebraically, they depend on a variable, often denoted as x. Here, the patio's length and width are:

- Length: (1 + 2x) yards
- Width: (2 - 3x) yards

This notation indicates that both measurements vary based on the value of x, which can be any real number within a certain range to ensure the dimensions make sense practically.

Interpreting the Algebraic Expressions

Understanding what the expressions (1 + 2x) and (2 - 3x) represent is crucial. These expressions are linear functions of x, and their values depend on x's value.

What does x represent?

- x is a real number that can be adjusted to find valid dimensions.
- The value of x affects both the length and width, potentially making the patio larger or smaller.

Constraints on x

- Since physical dimensions cannot be negative, both the length and width must be positive:
- $(1 + 2x) > 0$
- $(2 - 3x) > 0$

By solving these inequalities, we can identify the range of x that produces valid, physically possible patio dimensions.

Determining the Range of x for Valid Dimensions

To find the acceptable values of x, we analyze the inequalities:

Length Constraint: $(1 + 2x) > 0$

- $1 + 2x > 0$
- $2x > -1$
- $x > -1/2$

Width Constraint: $(2 - 3x) > 0$

- $2 - 3x > 0$
- $-3x > -2$
- $x < 2/3$

Combining these, the possible values of x are:

- $x > -1/2$
- $x < 2/3$

Therefore, the domain of x for which the dimensions are positive is:

$(-1/2, 2/3)$

Implications of the Range of x

Within this range, the patio's length and width vary linearly with x. Let's explore what happens at specific points:

At $x = -1/2$

- Length: $1 + 2(-1/2) = 1 - 1 = 0$ yards (degenerate, no length)
- Width: $2 - 3(-1/2) = 2 + 1.5 = 3.5$ yards

Since length is zero, x cannot be exactly -1/2 for a valid patio.

At x approaching -1/2 from above

- Length approaches zero from above
- Width approaches 3.5 yards

At $x = 0$

- Length: $1 + 0 = 1$ yard
- Width: $2 - 0 = 2$ yards

At x approaching 2/3 from below

- Length: $1 + 2(2/3) = 1 + 4/3 = 1 + 1.333... \approx 2.333$ yards
- Width: $2 - 3(2/3) = 2 - 2 = 0$ yards

Again, width approaches zero but remains positive for $x < 2/3$.

At $x = 2/3$

- Width: 0 yards (degenerate)
- Length: $1 + 2(2/3) \approx 2.333$ yards

This indicates the maximum x value for a valid rectangle is just less than 2/3.

Calculating the Area of the Patio

The area A (in square yards) of the patio depends on x:

$A(x) = (\text{length}) \times (\text{width}) = (1 + 2x) \times (2 - 3x)$

Expanding this:

$A(x) = (1)(2) + (1)(-3x) + (2x)(2) + (2x)(-3x)$

Simplify:

$A(x) = 2 - 3x + 4x - 6x^2$

Combine like terms:

$A(x) = 2 + x - 6x^2$

Analyzing the Area Function

The quadratic function:

$A(x) = -6x^2 + x + 2$

has a downward opening parabola (since coefficient of x^2 is negative). To find the maximum area:

Vertex of the parabola

- x-coordinate of vertex: $x = -b/(2a)$
- Here, $a = -6$, $b = 1$

Calculate:

$x = -1 / (2 -6) = -1 / (-12) = 1/12$

Check if $x = 1/12$ falls within the valid domain $(-1/2, 2/3)$:

- $1/12 \approx 0.0833$
- Since $-0.5 < 0.0833 < 0.6667$, the maximum area occurs within the valid range.

Maximum area

- $A(1/12) = -6(1/12)^2 + (1/12) + 2$

Calculate:

$(1/12)^2 = 1/144$

$A(1/12) = -6(1/144) + 1/12 + 2$
$= -6/144 + 1/12 + 2$
$= -1/24 + 1/12 + 2$

Express all with denominator 24:

$-1/24 + 2/24 + 48/24 = (-1 + 2 + 48) / 24 = 49/24 \approx 2.04$

Thus, the maximum area of approximately 2.04 square yards occurs at $x = 1/12$.

Practical Considerations in Patio Design

When designing a patio with algebraic dimensions, understanding the relationships between variables helps optimize space and materials.

Ensuring Valid Dimensions

- Always choose x within the interval $(-1/2, 2/3)$ to avoid degenerate (zero or negative) dimensions.
- The optimal x for maximum area is $1/12$, which provides the largest possible patio area while maintaining positive length and width.

Material Planning

- Knowing the maximum area helps estimate the amount of material needed.
- For example, if you plan for the maximum area (≈2.04 sq yards), you can prepare appropriate quantities of paving stones or concrete.

Cost Optimization

- Adjusting x slightly below $1/12$ can slightly reduce the area but might be necessary if constraints or aesthetic preferences favor different proportions.

Common Errors to Avoid

When working with algebraic expressions for dimensions, it's important to avoid certain pitfalls:

- Ignoring constraints: Failing to consider that dimensions cannot be

negative.
- Overlooking the domain of x: Choosing values outside (-1/2, 2/3) makes dimensions invalid.
- Miscalculating inequalities: Incorrectly solving inequalities can lead to invalid ranges.
- Neglecting the maximum or minimum points: Not analyzing the quadratic area function can lead to suboptimal designs.

Summary and Key Takeaways

- The dimensions of the patio are given by (1 + 2x) yards for length and (2 - 3x) yards for width.
- Valid values of x are within the interval (-1/2, 2/3) to ensure positive dimensions.
- The area function A(x) = -6x^2 + x + 2 is quadratic with a maximum at x = 1/12.
- The maximum possible area is approximately 2.04 square yards, achieved at x = 1/12.
- Proper understanding of algebraic expressions and inequalities is essential for effective patio design.

Final Thoughts

Using algebra to analyze and optimize the dimensions of a patio provides valuable insights into space planning and resource management. By carefully selecting the variable x within the valid range, you can maximize the patio's area while ensuring the dimensions are practical and aesthetically pleasing. Whether you're designing a new outdoor space or interpreting mathematical expressions in real-world scenarios, a solid grasp of these concepts is essential for successful planning and

Frequently Asked Questions

What are the expressions for the length and width of the patio?

The length is (1 + 2x) yards and the width is (2 - 3x) yards.

How do you determine if the patio has positive dimensions?

By ensuring both (1 + 2x) > 0 and (2 - 3x) > 0, which leads to inequalities 2x > -1 and -3x < -2, or x > -0.5 and x > 2/3 respectively.

What is the possible range of x for the patio to have valid dimensions?

x must be greater than 2/3 to satisfy both inequalities simultaneously, so x > 2/3.

How would you find the area of the patio in terms of x?

The area A is given by (1 + 2x)(2 - 3x).

What is the simplified form of the area expression?

Expanding the product gives A = 2 + 4x - 3x - 6x^2 = 2 + x - 6x^2.

How can we determine the maximum area of the patio?

By treating the area as a quadratic function of x, A = -6x^2 + x + 2, and finding its vertex using x = -b/2a, where a = -6 and b = 1.

What is the significance of the domain restrictions on x in this problem?

They ensure the patio's dimensions are positive, making the area and other measurements meaningful and physically possible.

Additional Resources

A Rectangular Patio Has A Length Of (1 + 2x) Yards And A Width Of (2 - 3x) Yards. Which Correctly Describes?

Introduction

Mathematics often presents us with real-world applications that demand precise analysis and understanding. One such scenario involves a rectangular patio with dimensions expressed in algebraic form, specifically as functions of a variable \(x \). The problem states: A rectangular patio has a length of \((1 + 2x)\) yards and a width of \((2 - 3x)\) yards. The core question is: Which statements correctly describe this patio? This investigation aims to dissect the problem thoroughly, exploring the algebraic relationships, constraints, and geometric implications, ultimately providing a comprehensive understanding suitable for review and academic discussion.

Understanding the Problem Statement

The problem involves a rectangle with:

- Length L: $1 + 2x$ yards
- Width W: $2 - 3x$ yards

Here, x is an algebraic variable that could represent a parameter influencing the dimensions. The key is to determine the valid values of x and interpret what the expressions imply about the patio's physical characteristics.

Fundamental Concepts

Before delving into the problem specifics, it’s essential to review some foundational concepts:

- Dimensions of a rectangle: Both length and width are positive real numbers.
- Algebraic expressions: The dimensions depend on the value of x.
- Constraints: The physical feasibility requires that both L and W are greater than zero.

Analyzing Constraints on x

Positivity Conditions

Since the dimensions represent physical measurements, they must be positive:

$$
\begin{cases}
1 + 2x > 0 \\
2 - 3x > 0
\end{cases}
$$

Solving these inequalities:

1. For the length:

$$
1 + 2x > 0 \Rightarrow 2x > -1 \Rightarrow x > -\frac{1}{2}
$$

2. For the width:

$$
2 - 3x > 0 \Rightarrow -3x > -2 \Rightarrow 3x < 2 \Rightarrow x < \frac{2}{3}
$$

Combined constraint:

$$
-\frac{1}{2} < x < \frac{2}{3}
$$

This range ensures that the dimensions are positive, making the patio physically realizable.

Geometric and Physical Interpretations

Validity of Dimensions

- If x is outside the interval $-\frac{1}{2} < x < \frac{2}{3}$, then at least one dimension becomes zero or negative, which is impossible for a physical patio.
- If x is within this interval, the patio's length and width are positive, and the dimensions vary depending on x.

Behavior of Dimensions as x Varies

- When x approaches $-\frac{1}{2}$ from above, the length approaches:

$$
L = 1 + 2(-\frac{1}{2}) = 1 - 1 = 0
$$

which is the limiting case where the length tends to zero.

- When x approaches $\frac{2}{3}$ from below, the width approaches:

$$
W = 2 - 3 \times \frac{2}{3} = 2 - 2 = 0
$$

similarly tending to zero.

Area of the Patio

The area A of the rectangular patio is given by:

$$
A = L \times W = (1 + 2x)(2 - 3x)
$$

Expanding:

$$
A = (1)(2) + (1)(-3x) + (2x)(2) + (2x)(-3x) = 2 - 3x + 4x - 6x^2
$$

Simplify:

$$
A = 2 + x - 6x^2
$$

This quadratic function describes how the area changes with x.

Analyzing the Area Function

Vertex of the Quadratic

The quadratic:

$$
A(x) = -6x^2 + x + 2
$$

has:

- Leading coefficient: $-6 < 0$, so the parabola opens downward.
- Vertex x-coordinate:

$$
x_v = -\frac{b}{2a} = -\frac{1}{2 \times (-6)} = -\frac{1}{-12} = \frac{1}{12}
$$

which lies within the interval $-\frac{1}{2} < x < \frac{2}{3}$, indicating the maximum area occurs at $x = \frac{1}{12}$.

Maximum Area:

$$
A_{max} = A\left(\frac{1}{12} \right) = -6 \left(\frac{1}{12} \right)^2 + \frac{1}{12} + 2
$$

Calculate:

$$
-6 \times \frac{1}{144} + \frac{1}{12} + 2 = -\frac{6}{144} + \frac{1}{12} + 2 = -\frac{1}{24} + \frac{1}{12} + 2
$$

Express all with denominator 24:

$$
-\frac{1}{24} + \frac{2}{24} + \frac{48}{24} = \frac{-1 + 2 + 48}{24} = \frac{49}{24}
$$

Thus:

$$
A_{max} = \frac{49}{24} \text{ square yards}
$$

indicating the largest possible area of the patio within the constraints.

Which Statements Are Correct?

Based on the thorough analysis, the following are accurate descriptions:

1. The dimensions depend linearly on x, with positivity constraints requiring x to be between $-\frac{1}{2}$ and $\frac{2}{3}$.

2. The maximum area of the patio occurs at $x = \frac{1}{12}$, with an area of $\frac{49}{24}$ square yards.

3. For x in the interval $-\frac{1}{2} < x < \frac{2}{3}$, both the length and width are positive, making the patio physically valid.

4. As x approaches the endpoints of the interval, either the length or width approaches zero, meaning the patio becomes degenerate (line segment).

Additional Considerations

Practical Implications

- The parameter x can be viewed as a design variable, adjusting the dimensions of the patio.
- The constraints ensure the patio remains structurally sound and physically feasible.
- The maximum area point signifies the optimal design within the given

algebraic constraints.

Limitations and Assumptions

- The analysis assumes x is a real number within the specified interval.
- The model does not account for other factors like land shape, material constraints, or aesthetic considerations.
- The algebraic expressions are linear or quadratic, simplifying real-world complexities.

Summary and Conclusion

This detailed investigation reveals that the dimensions of the patio are governed by linear expressions in x, with feasible values confined to the interval $-\frac{1}{2} < x < \frac{2}{3}$. The area function is quadratic, attaining a maximum at $x = \frac{1}{12}$, leading to an optimal patio size. The physical validity hinges on maintaining positive dimensions, which directly translates into inequalities on x.

In conclusion:

- The expressions accurately model the patio's dimensions, provided x remains within the specified bounds.
- The maximum area is achieved at a specific x, illustrating the utility of algebraic analysis in design optimization.
- The problem exemplifies the interplay between algebra, geometry, and practical constraints, emphasizing the importance of thorough mathematical reasoning.

This comprehensive analysis can serve as a foundation for further exploration or real-world application, demonstrating how algebraic modeling informs design decisions and optimizes spatial layouts.

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