A Right Circular Cone Is Generated By Revolving The Region Bounded By Y=3x/4,y=3, And X=0 About The Y-axis.

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Understanding the formation and properties of three-dimensional objects such as cones is fundamental in calculus and geometry. When a specific region in the xy-plane is revolved around an axis, it generates a solid with a unique volume and surface characteristics. In this article, we explore the process of generating a right circular cone by revolving a bounded region around the y-axis, analyze the related equations, and delve into the applications, calculations, and properties of this geometric figure.

Understanding the Region Bounded by the Curves and Lines

Before discussing the revolution process, it's essential to understand the region in the xy-plane that forms the basis of the solid.

Equations and Boundaries

- Line 1: y = (3/4) x
- Line 2: y = 3
- Line 3: x = 0 (the y-axis)

These equations define a bounded region:

- The line y = (3/4) x is a straight line passing through the origin with a slope of 3/4.
- The line y = 3 is horizontal, serving as an upper boundary.
- The line x = 0 is the y-axis, serving as the left boundary.

Visualization of the Region

Imagine plotting these:

- The line y = (3/4) x starts at the origin and rises with slope 3/4.
- The horizontal line y = 3 intersects the line y = (3/4) x at a point where:

$y = 3 = (3/4)\ x$
$x = (4/3)\ 3 = 4$

- The region bounded is thus a triangle with vertices at:

- (0, 0) (origin)
- (4, 3) (intersection point of y=3 and y=(3/4)x)
- (0, 3) (on the y-axis at y=3)

This triangular region is the domain that, when revolved about the y-axis, generates a right circular cone.

Revolution of the Region About the Y-Axis

The process involves rotating the bounded region around the y-axis to create a three-dimensional solid.

The Concept of Solid of Revolution

When a two-dimensional region is revolved about an axis, the resulting solid can be analyzed using calculus techniques such as the disk or washer method to compute its volume. The key points include:

- Axis of revolution: y-axis (x=0)
- Shape of the cross-section: Circular, because every point at a fixed y-coordinate traces a circle when revolved.

Why the Y-Axis?

Revolving around the y-axis is suitable because:

- The region is bounded on the left by x=0.
- The boundary functions are expressed in terms of y and x, making the problem symmetric around the y-axis.
- The resulting shape is a cone with its apex at the origin.

Deriving the Equation of the Cone

Understanding the cone's geometry requires analyzing the equations and the resulting solid.

Expressing x as a function of y

From the line $y = (3/4) x$, we get:

$x = (4/3) y$

This relation indicates that at a given height y, the radius r of the cross-sectional circle (due to revolution) is:

$r(y) = x = (4/3) y$

Because the boundary $x=0$ corresponds to the cone's tip at the origin, and the line $y=3$ corresponds to the base of the cone at height $y=3$ with radius $r(3) = (4/3)3=4$.

Geometric Interpretation

- The cone's height is from $y=0$ to $y=3$.
- The radius at any height y is proportional to y, given by $r(y) = (4/3) y$.
- The base of the cone is at $y=3$ with radius 4.
- The apex is at $y=0$ with radius 0.

This confirms that the shape generated is a right circular cone with:

- Apex at: $(x=0, y=0)$
- Base at: $y=3$, radius = 4

Calculating the Volume of the Cone

Using calculus, specifically the disk method, we can derive the volume of the cone generated by this revolution.

Volume Formula for a Solid of Revolution

The general formula for volume using the disk method when revolving around the y-axis is:

$$V = \pi \int_{a}^{b} [r(y)]^2 dy$$

where:

- $r(y)$ is the radius of the cross-sectional disk at height y.
- a and b are the bounds along the y-axis.

Applying the Formula

Given:

- $r(y) = \frac{4}{3} y$
- y ranges from 0 to 3

The volume becomes:

$$V = \pi \int_{0}^{3} \left(\frac{4}{3} y \right)^2 dy$$

Calculating the integral:

$$V = \pi \int_{0}^{3} \frac{16}{9} y^2 dy$$

$$V = \pi \times \frac{16}{9} \int_{0}^{3} y^2 dy$$

$$V = \frac{16\pi}{9} \left[\frac{y^3}{3} \right]_0^3$$

$$V = \frac{16\pi}{9} \times \frac{3^3}{3}$$

$$V = \frac{16\pi}{9} \times \frac{27}{3}$$

$$V = \frac{16\pi}{9} \times 9$$

$$V = 16\pi$$

Therefore, the volume of the cone is 16π cubic units.

Properties and Dimensions of the Derived Cone

Understanding the cone's dimensions and properties provides insight into its geometric features.

Key Dimensions

- Height (h): 3 units
- Base radius (r): 4 units

Slant Height and Surface Area

- Slant height (l):

$$ l = \sqrt{r^2 + h^2} = \sqrt{4^2 + 3^2} = \sqrt{16 + 9} = \sqrt{25} = 5 $$

- Lateral surface area (A):

$$ A = \pi r l = \pi \times 4 \times 5 = 20 \pi $$

- Total surface area (including base):

$$ A_{total} = A + \text{area of base} = 20 \pi + \pi r^2 = 20 \pi + 16 \pi = 36 \pi $$

Mathematical Applications and Significance

The process of generating and analyzing such a cone has numerous applications in science, engineering, and mathematics.

Applications

- Engineering design: Cone-shaped objects like funnels, lampshades, and nozzles.
- Manufacturing: Designing conical parts with precise volume and surface area specifications.
- Physics: Understanding rotational bodies and moment of inertia.
- Mathematics education: Visualizing and calculating volumes of revolution to reinforce integral calculus concepts.

Significance in Calculus

- Demonstrates how to compute volumes of complex solids using integral calculus.
- Illustrates the relationship between the region in the xy-plane and the resulting three-dimensional shape.
- Offers insight into the geometric interpretation of integrals.

Summary and Key Takeaways

- The bounded region defined by $y = (3/4) x$, $y = 3$, and $x=0$, when revolved about the y-axis, forms a right circular cone.
- The cone's dimensions are a height of 3 units and a base radius of 4 units.
- The volume of the cone is 16π cubic units, calculated using the disk method.
- The slant height is 5 units, and the total surface area is 36π.
- Understanding these derivations enhances comprehension of solids of revolution and their applications in various fields.

Conclusion

The process of generating a right circular cone by revolving a bounded region around the y-axis exemplifies the interplay between algebra, geometry, and calculus. By analyzing the boundary equations, deriving the cone's dimensions, and calculating its volume, one gains a comprehensive understanding of the geometric and mathematical principles involved. This knowledge serves as a foundation for more advanced studies in three-dimensional calculus and geometric modeling, making it a vital aspect of mathematical education and practical applications.

Keywords: right circular cone, volume of revolution, calculus, geometry, bounded region, y-axis revolution, disk method, cone dimensions, surface area, mathematical applications.

Frequently Asked Questions

How do you find the volume of the cone generated by revolving the region bounded by y=3x/4, y=3, and x=0 about the y-axis?

To find the volume, set up the integral using the shell method. Express x in terms of y: x = (4/3) y. The volume V = 2π ∫ from y=0 to y=3 of x(y) y dy = 2π $\int_0^3$ (4/3 y) y dy. Simplify and evaluate the integral to find the volume.

What are the key steps to determine the dimensions of the resulting cone in this problem?

First, identify the region bounded by y=3x/4, y=3, and x=0. When revolving around the y-axis, the radius at a given y is x = (4/3) y. The height of the cone is from y=0 to y=3, and the radius at the top is x = (4/3) 3 = 4. These dimensions define the cone's size.

Why is the shell method appropriate for calculating the volume of this cone?

The shell method is appropriate because the region is revolved around the y-axis, and the shells are vertical, making it straightforward to integrate with respect to y. It simplifies the calculation by summing cylindrical shells formed at different y-values.

How does the equation y=3x/4 help in defining the bounds for

the integration when calculating the volume?

Rearranged, x = (4/3) y, which expresses x in terms of y. Since y varies from 0 to 3, x varies from 0 to 4 at y=3. These bounds determine the limits of integration for the volume calculation using the shell method.

What is the significance of the line y=3 in generating the cone, and how does it influence the shape?

The line y=3 serves as the upper boundary of the region being revolved. When revolved about the y-axis, it defines the maximum radius of the cone at the top, ensuring the cone's height is 3 units and its maximum radius is 4 units at y=3.

Additional Resources

A Right Circular Cone Is Generated By Revolving The Region Bounded By Y=3x/4, y=3, And X=0 About The Y-axis

The process of generating three-dimensional solids through the revolution of planar regions is a fundamental concept in calculus and solid geometry. In particular, the creation of a right circular cone by revolving a specific bounded region around an axis exemplifies the practical application of integral calculus and geometric reasoning. This article aims to provide a comprehensive, detailed, and analytical exploration of how the given region, bounded by the lines $y = \frac{3x}{4}$, $y=3$, and $x=0$, generates a right circular cone when revolved about the y-axis. We will delve into the geometric interpretation, the calculus involved in deriving the volume, the properties of the resulting cone, and the broader context of such problems in mathematical analysis.

Understanding the Geometric Region

Defining the Boundaries

The region in question is bounded by three curves and lines:

1. The line $y = \frac{3x}{4}$: This is a straight line passing through the origin with a slope of $\frac{3}{4}$.
2. The horizontal line $y = 3$: This acts as an upper boundary for the region.
3. The vertical boundary $x=0$: The y-axis itself, serving as the leftmost boundary.

Visualizing the Region:

- The line $y = \frac{3x}{4}$ intersects the y-axis at $x=0$, $y=0$.
- When $y=3$, solving for x yields $x = \frac{4}{3} \times 3 = 4$.
- The region thus is a triangular area with vertices at $(0,0)$, $(4,3)$, and the origin $(0,0)$.

Summary of Region:

- It is bounded below by the x-axis (since $y=0$ is implicitly the lower boundary when $y=\frac{3x}{4}$ crosses the x-axis).
- It is bounded on the right by the line $y=3$, which intersects $y=\frac{3x}{4}$ at $x=4$.
- It is bounded on the left by the y-axis ($x=0$).

Revolution About the Y-Axis: From Region to Solid

The Concept of Solid of Revolution

Revolving a planar region about an axis creates a three-dimensional solid. The shape and volume depend on the boundaries of the region and the axis of revolution. In this case, the region bounded by the three curves is revolved about the y-axis, producing a right circular cone.

Why a cone?:
Because the region is triangular with one vertex at the origin and the boundary along a straight line, revolving this triangular region around the y-axis generates a conical shape. The line $y=\frac{3x}{4}$ defines a sloped boundary that, when revolved, forms the slant of the cone.

Key features of the generated cone:

- The apex of the cone is at the origin $(0,0)$.
- The base of the cone corresponds to the boundary at $y=3$.
- The radius of the cone at any height y depends on the distance from the y-axis to the boundary x, which, due to the region's boundaries, varies linearly with y.

Mathematical Derivation of the Volume

Choosing the Method of Integration

To compute the volume of the solid obtained by revolving the region around the y-axis, calculus offers several methods:

- Disk/washer method: Suitable when the cross-sections perpendicular to the axis are disks or washers.
- Cylindrical shells method: Often more convenient when revolving around the y-axis and when the region is described in terms of x.

Given the region's description, the cylindrical shells method is particularly effective because:

- The region is bounded on the left by $x=0$.
- It extends horizontally from $x=0$ to $x=4$.
- The shells are generated by revolving vertical slices around the y-axis.

Choosing Shells:

- Each shell is formed by a vertical strip at x with thickness dx.
- The height of each shell is from $y=0$ to $y=\frac{3x}{4}$ (for the part below $y=3$).
- The radius of the shell is x, the distance from the y-axis.

Setting Up the Integral

The volume of the solid using the shell method:

$$
V = 2\pi \int_{x=0}^{x=4} (\text{radius}) \times (\text{height}) \, dx
$$

where:

- Radius: x, the distance from the y-axis.
- Height: y, the extent of the region at each x, which is from $y=0$ up to $y=\frac{3x}{4}$.

Thus:

$$
V = 2 \pi \int_{0}^{4} x \times \left(\frac{3x}{4} \right) \, dx
$$

Simplify the integrand:

$$
V = 2 \pi \int_{0}^{4} x \times \frac{3x}{4} \, dx = 2 \pi \int_{0}^{4} \frac{3x^2}{4} \, dx
$$

Pull constants out:

$$
V = 2 \pi \times \frac{3}{4} \int_{0}^{4} x^2 \, dx = \frac{3 \pi}{2} \int_{0}^{4} x^2 \, dx
$$

Calculate the integral:

$$

$$
\int_{0}^{4} x^2 \, dx = \left[\frac{x^3}{3} \right]_0^4 = \frac{4^3}{3} - 0 = \frac{64}{3}
$$

Insert back into the volume expression:

$$
V = \frac{3 \pi}{2} \times \frac{64}{3} = \frac{3 \pi \times 64}{2 \times 3} = \frac{64 \pi}{2} = 32 \pi
$$

Final Result:

$$
\boxed{
\text{The volume of the generated cone is } 32 \pi \text{ cubic units}
}
$$

Properties of the Resulting Cone

Dimensions and Geometric Features

- Apex: Located at the origin $(0,0)$, the cone's vertex.
- Base: Located at $y=3$, with a radius equal to the maximum x-value at $y=3$, which is $x=4$.
- Height: The distance along the y-axis from the apex to the base is 3 units.
- Base Radius: At $y=3$, the boundary line $y=\frac{3x}{4}$ gives $x=4$, so the radius of the base is 4 units.

Summary of Dimensions:

Feature	Value
Cone height	3 units
Base radius	4 units
Volume	32π cubic units

Note: The cone is a right circular cone because its axis (the y-axis) passes through its apex and center of the base, with the base lying in the plane $y=3$.

Broader Context and Applications

Significance in Calculus and Geometry

The derivation exemplifies the power of integral calculus in solving volume problems involving complex regions. Revolving a region bounded by linear and constant functions about an axis is a common problem type in calculus courses, often serving as an introduction to volume calculations via shells and disks.

Applications include:

- Engineering: Designing conical components such as funnels, nozzles, or conical tanks.
- Physics: Calculating mass, center of mass, or moments of inertia for conical objects.
- Mathematics: Understanding the relationship between planar regions and their three-dimensional counterparts.

Extensions and Variations

- Different axes of revolution: revolving around the x-axis or other lines can lead to different solids.
- Other boundary curves: replacing \(y=\frac{3x}{4} \) with nonlinear functions introduces more complex shapes.
- Multiple regions: combining several bounded regions to form composite solids.

Conclusion

Revolving the region bounded by \(y=\frac{3x}{4} \

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