A Sample Of A Monatomic Ideal Gas Occupies 5.00L At Atmospheric Pressure And 300K (point A In Fig. P21.63).

A Sample Of A Monatomic Ideal Gas Occupies 5.00L At Atmospheric Pressure And 300K (point A In Fig. P21.63)

Understanding the behavior of gases under different conditions is fundamental in thermodynamics and physical chemistry. In this article, we explore a specific scenario involving a monatomic ideal gas that occupies a volume of 5.00 liters at atmospheric pressure and a temperature of 300K, referring to it as point A in Fig. P21.63. We will analyze the thermodynamic properties, apply the ideal gas law, and examine how various processes such as isothermal, isobaric, adiabatic, and isochoric transformations affect the state of this gas. This comprehensive overview aims to provide clarity on how to approach problems involving ideal gases and their thermodynamic states.

Understanding the Initial Conditions of the Gas

Basic Properties at Point A

The initial state of the gas is characterized by:

- Volume (V_1): 5.00 liters
- Pressure (P_1): Atmospheric pressure, approximately 1 atm (101.3 kPa)
- Temperature (T_1): 300K
- Type of Gas: Monatomic ideal gas (e.g., helium, neon, argon)

Since it is a monatomic ideal gas, we can use the ideal gas law to compute the number of moles, internal energy, and other thermodynamic parameters.

Applying the Ideal Gas Law

The ideal gas law is expressed as:

$$PV = nRT$$

Where:
- P = pressure
- V = volume
- n = number of moles
- R = universal gas constant (8.314 J/mol·K)
- T = temperature in Kelvin

Calculating the number of moles:

$$
n = \frac{PV}{RT}
$$

Converting volume to cubic meters for SI consistency:

\[

$$
V = 5.00\, \text{L} = 5.00 \times 10^{-3}\, \text{m}^3
$$

Substituting the known values:

$$
n = \frac{(101.3 \times 10^3\, \text{Pa}) \times (5.00 \times 10^{-3}\, \text{m}^3)}{8.314\, \text{J/mol·K} \times 300\, \text{K}} \approx 0.203\, \text{mol}
$$

This calculation provides the foundational data needed for thermodynamic analysis.

Thermodynamic Properties of a Monatomic Ideal Gas

Internal Energy (U)

For a monatomic ideal gas, the internal energy depends solely on temperature:

$$
U = \frac{3}{2} nRT
$$

Calculating:

$$
U = \frac{3}{2} \times 0.203\, \text{mol} \times 8.314\, \text{J/mol·K} \times 300\, \text{K} \approx 756\, \text{J}
$$

This value indicates the total internal energy stored in the gas at point A.

Enthalpy (H)

Similarly, the enthalpy for an ideal gas is:

$$
H = U + PV = nRT \quad (\text{since } PV = nRT)
$$

Calculating:

$$
H = 0.203 \times 8.314 \times 300 \approx 507\, \text{J}
$$

These thermodynamic quantities are essential when analyzing various processes involving the gas.

Analyzing Thermodynamic Processes Starting From Point A

Understanding how the gas responds to different types of processes allows for designing and predicting thermodynamic cycles.

Isothermal Process (Constant Temperature)

In an isothermal process:

- The temperature remains at 300K
- The internal energy stays constant ($\Delta U = 0$)
- Work done by the gas:

$$
W = nRT \ln \frac{V_f}{V_i}
$$

- Heat transferred into or out of the gas:

$$
Q = W
$$

If the volume changes from 5.00L to, say, 10.00L, the work done by the gas is:

$$
W = 0.203 \times 8.314 \times 300 \times \ln 2 \approx 142\, \text{J}
$$

The process involves heat exchange with surroundings to maintain constant temperature.

Isobaric Process (Constant Pressure)

In an isobaric process:

- The pressure remains at 1 atm
- The work done:

$$
W = P \Delta V
$$

- Change in internal energy:

$$
\Delta U = \frac{3}{2} n R \Delta T
$$

- Heat added:

$$
Q = \Delta U + W
$$

For example, if the volume increases from 5.00L to 10.00L at constant pressure, the temperature rises proportionally, and the heat added can be calculated accordingly.

Adiabatic Process (No Heat Exchange)

In an adiabatic process:

- $Q = 0$

- The relation between pressure and volume:

$$
PV^\gamma = \text{constant}
$$

where $\gamma = \frac{C_p}{C_v} = \frac{5}{3}$ for monatomic gases.

- Final temperature:

$$
T_f = T_i \left(\frac{V_i}{V_f}\right)^{\gamma - 1}
$$

Suppose the gas expands adiabatically from 5.00L to 10.00L:

$$
T_f = 300 \times \left(\frac{5}{10}\right)^{\frac{5}{3} - 1} \approx 300 \times (0.5)^{2/3} \approx 300 \times 0.63 \approx 189\, \text{K}
$$

The internal energy decreases accordingly.

Isochoric Process (Constant Volume)

In an isochoric process:

- Volume remains constant
- No work is done ($W=0$)
- The temperature change results in internal energy change:

$$
\Delta U = \frac{3}{2} n R \Delta T
$$

- Heat added:

$$
Q = \Delta U
$$

If the temperature increases from 300K to 600K at constant volume, the internal energy doubles, and the heat supplied is approximately 756 J.

Practical Applications and Significance

Thermodynamic Cycle Design

Understanding the behavior of a monatomic ideal gas under different process conditions is vital for designing engines, refrigerators, and other thermodynamic systems. For example:

- Carnot cycles involve isothermal and adiabatic processes
- Stirling engines utilize isothermal and isochoric processes

Real-World Implications

While ideal gases are theoretical models, they provide a foundational understanding applicable to real gases under many conditions. This knowledge assists in:

- Predicting gas behavior in laboratory and industrial settings
- Developing efficient energy conversion systems
- Analyzing atmospheric and environmental phenomena

Conclusion

Analyzing a monatomic ideal gas that occupies 5.00 liters at atmospheric pressure and 300K offers valuable insights into thermodynamics. By applying the ideal gas law, calculating internal energy and enthalpy, and understanding various process types—such as isothermal, isobaric, adiabatic, and isochoric—we can predict how the gas responds to different transformations. These principles are essential in fields ranging from engineering to environmental science, illustrating the importance of thermodynamics in understanding the physical world.

Remember: The key to mastering gas behavior lies in understanding the fundamental relationships and how they govern the state changes during various thermodynamic processes.

Frequently Asked Questions

What is the volume of the monatomic ideal gas sample at point A?

The volume of the monatomic ideal gas sample at point A is 5.00 liters.

What are the initial temperature and pressure conditions of the gas sample?

The initial temperature is 300 K and the pressure is atmospheric pressure (approximately 1 atm).

How can the number of moles of gas in the sample be calculated?

Using the ideal gas law, $n = PV / RT$, where P is pressure, V is volume, R is the ideal gas constant, and T is temperature.

What is the significance of point A in the context of the gas's thermodynamic process?

Point A represents the initial state of the gas before any thermodynamic process such as expansion or compression occurs.

How does the ideal gas law relate to this sample's initial conditions?

The ideal gas law ($PV = nRT$) relates pressure, volume, and temperature, allowing calculation of unknown properties given the known initial conditions.

If the gas undergoes an isothermal process, how would its volume change at

constant temperature?

In an isothermal process, the volume would change inversely with pressure, maintaining constant temperature, following PV = constant.

What is the significance of the gas being monatomic in this context?

A monatomic ideal gas has specific heat capacities and thermodynamic properties that differ from diatomic or polyatomic gases, affecting calculations of internal energy and heat transfer.

How would you calculate the internal energy of the gas at point A?

For a monatomic ideal gas, internal energy $U = (3/2)nRT$; once n is known, U can be calculated using the initial temperature and the ideal gas constant.

What potential thermodynamic processes could the gas undergo starting from point A?

The gas could undergo processes such as isothermal expansion/compression, adiabatic expansion/compression, or isobaric heating/cooling.

How does changing the temperature affect the behavior of the monatomic ideal gas?

Increasing temperature raises the average kinetic energy of the particles, leading to increased pressure at constant volume or increased volume at constant pressure in thermodynamic processes.

Additional Resources

A Sample Of A Monatomic Ideal Gas Occupies 5.00L At Atmospheric Pressure And 300K (Point A In Fig. P21.63)

Understanding the behavior of gases under various conditions is a cornerstone of thermodynamics and statistical mechanics. The scenario involving a monatomic ideal gas occupying a specific volume at a given temperature and pressure provides a rich context for exploring fundamental principles, equations of state, and the properties of ideal gases. In this article, we delve into the detailed analysis of such a system, examining how its physical parameters relate, what theoretical models predict, and the implications for real-world applications.

Introduction to the System and Its Significance

The problem statement describes a monatomic ideal gas contained within a 5.00-liter volume at atmospheric pressure (approximately 1 atm) and a temperature of 300 K. This setup is classic in physics and chemistry for illustrating the ideal gas law and related concepts. It serves as a foundational example that helps students and practitioners understand the thermodynamic behavior of gases, how to derive various properties, and how theoretical models compare with real gases.

This particular configuration, often visualized as point A in a phase diagram or a process diagram, acts as a baseline for analyzing processes such as isothermal expansion or compression, adiabatic changes, and heat transfer. The simplicity of a monatomic ideal gas—comprising atoms like helium, neon, or argon—allows for straightforward calculations due to the absence of molecular complexities such as vibrational or rotational modes.

Fundamental Principles and Assumptions

The Ideal Gas Law

The core relation governing the system is the ideal gas law:

$$ PV = nRT $$

where:
- P = pressure,
- V = volume,
- n = number of moles,
- R = universal gas constant,
- T = temperature.

For a monatomic ideal gas, the behavior adheres closely to this law, especially at low pressures and high temperatures where interactions between particles are negligible.

Assumptions of the Ideal Gas Model

- Particles are point masses with no volume.
- No intermolecular forces act between particles.
- Collisions are perfectly elastic.
- The gas is in thermodynamic equilibrium.

These assumptions simplify analysis but introduce limitations when comparing real gases, especially at high pressures or low temperatures.

Calculations and Derivations

Given data:
- Volume $V = 5.00\, \text{L} = 5.00 \times 10^{-3}\, \text{m}^3$,
- Temperature $T = 300\, \text{K}$,
- Pressure $P = 1\, \text{atm} \approx 1.013 \times 10^5\, \text{Pa}$.

Number of Moles of Gas

Applying the ideal gas law:

$$ n = \frac{PV}{RT} $$

Using $R = 8.314\, \text{J mol}^{-1}\text{K}^{-1}$:

$$ n = \frac{(1.013 \times 10^5\, \text{Pa})(5.00 \times 10^{-3}\, \text{m}^3)}{8.314\, \text{J mol}^{-1}\text{K}^{-1} \times 300\, \text{K}} $$

$$ n \approx \frac{506.5}{2494.2} \approx 0.203\, \text{mol} $$

This indicates the system contains approximately 0.203 moles of monatomic gas.

Mass and Molar Mass Considerations

For a typical monatomic gas like helium (He), with molar mass $M = 4.00\, \text{g/mol}$:

- Total mass:

$$m = n \times M = 0.203\, \text{mol} \times 4.00\, \text{g/mol} \approx 0.812\, \text{g}$$

Similarly, for argon (Ar) with $M \approx 39.95\, \text{g/mol}$:

$$m \approx 0.203 \times 39.95\, \text{g} \approx 8.11\, \text{g}$$

This highlights how the same molar amount corresponds to different mass depending on the gas type.

Thermodynamic Properties and Behavior

Internal Energy

For a monatomic ideal gas, the internal energy U depends only on temperature:

$$U = \frac{3}{2} nRT$$

At 300 K:

$$U = \frac{3}{2} \times 0.203\, \text{mol} \times 8.314\, \text{J mol}^{-1}\text{K}^{-1} \times 300\, \text{K}$$

$$U \approx 0.203 \times 12.471 \times 300 \approx 0.203 \times 3741.3 \approx 759.7\, \text{J}$$

This energy represents the total kinetic energy of the gas particles, assuming ideality.

Average Kinetic Energy and Molecular Speed

The average kinetic energy per molecule:

$$\langle KE \rangle = \frac{3}{2} k_B T$$

where k_B is Boltzmann's constant:

$$k_B = 1.381 \times 10^{-23}\, \text{J/K}$$

Calculating:

$$\langle KE \rangle = \frac{3}{2} \times 1.381 \times 10^{-23} \times 300 \approx 6.21 \times 10^{-21}\, \text{J}$$

The root-mean-square (rms) speed:

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

Expressed in SI units (mass per molecule):

$$M_{kg} = \frac{M_{g}}{1000}$$

For helium:

$$ M_{kg} = 4.00 \times 10^{-3}\, \text{kg/mol} $$

Number of molecules:

$$ N = n \times N_A = 0.203 \times 6.022 \times 10^{23} \approx 1.22 \times 10^{23} $$

RMS speed:

$$ v_{rms} = \sqrt{\frac{3 \times 8.314 \times 300}{4.00 \times 10^{-3}}} $$

$$ v_{rms} \approx \sqrt{\frac{7473.6}{4.00 \times 10^{-3}}} $$

$$ v_{rms} \approx \sqrt{1.868 \times 10^{6}} \approx 1366\, \text{m/s} $$

This high molecular speed illustrates the rapid motion of gas particles.

Features, Pros, and Cons of the Ideal Gas Model in This Context

Features:
- Simplifies calculations by neglecting particle volume and intermolecular forces.
- Provides accurate predictions at low pressures and high temperatures.
- Facilitates understanding of thermodynamic processes such as heat capacity, work, and energy transfer.

Pros:
- Straightforward mathematical framework.
- Useful for designing and analyzing systems like internal combustion engines, atmospheric phenomena, and vacuum systems.
- Enables derivation of key thermodynamic quantities with ease.

Cons:
- Fails to account for real gas interactions at high pressures or low temperatures.
- Neglects molecular size, which becomes significant under certain conditions.
- Cannot describe phase transitions like condensation or solidification.

Applications and Broader Implications

Analyzing a monatomic ideal gas under the given conditions helps in multiple domains:

- Educational Context: Serves as a fundamental example for students learning thermodynamics and kinetic theory.
- Engineering: Aids in the design of gas-based systems, including turbines, compressors, and heat engines.
- Physics Research: Provides a baseline for understanding deviations in real gases and developing more complex models like the Van der Waals equation.
- Environmental Science: Helps model atmospheric behavior, where gases like helium and neon are present in trace amounts.

Limitations and Real-World Considerations

While the ideal gas law offers a powerful tool for approximations, real gases often deviate from ideal behavior. For example:

- At high pressures, gas particles are closer, and intermolecular forces become non-negligible.
- At low temperatures, attractive forces can lead to condensation, which the ideal gas law cannot predict.
- Molecular size effects can influence pressure and volume relationships.

In such cases, corrections like the Van der Waals equation or other real gas models are necessary for accurate predictions.

Conclusion

The scenario of a monatomic ideal gas occupying 5.00 L at atmospheric pressure and 300 K encapsulates essential principles of thermodynamics and statistical mechanics. Through calculations of particle number, internal energy, and molecular speeds, we see how theoretical models translate into tangible physical parameters. While the ideal gas law provides a remarkably accurate framework under many conditions, awareness of its limitations is crucial for advanced applications

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