

A Supply Voltage Given By $V(t) = 100 \sin(\omega t)$ V Is Connected Across An Impedance And Provides An Instantaneous Current

Understanding the Relationship Between Supply Voltage and Impedance

A Supply Voltage Given By $V(t) = 100 \sin(\omega t)$ V Is Connected Across An Impedance And Provides An Instantaneous Current highlights a fundamental concept in electrical engineering: how voltage and impedance interact to determine current flow in a circuit. When a voltage source is applied across an impedance, the resulting current is governed by Ohm's Law and the characteristics of the impedance itself. This article will explore this relationship in depth, examining how supply voltage influences current, the nature of impedance, and the implications for circuit design and analysis.

Fundamentals of Voltage, Current, and Impedance

What Is Supply Voltage?

Supply voltage, often denoted as V , is the potential difference provided by a power source to drive current through a circuit. In our context, the voltage is specified as a function of time: $V(t) = 100 \sin(\omega t)$ volts, indicating a sinusoidal alternating voltage with an amplitude of 100 volts.

Understanding Impedance in Circuits

Impedance (Z) is a comprehensive measure that combines resistance (R), inductive reactance (X_L), and capacitive reactance (X_C) to describe how an impedance element opposes the flow of alternating current (AC). It is a complex quantity expressed as:

$$Z = R + jX$$

where:

- R = Resistance (ohms, Ω)
- X = Reactance (ohms, Ω)
- j = imaginary unit

The impedance determines both the magnitude and phase of the current in response to a voltage.

The Role of Impedance in Circuit Behavior

Impedance affects:

- The amplitude of the current
- The phase difference between voltage and current
- Power consumption and distribution

In AC circuits, impedance is crucial in tuning circuits, filters, and power transmission systems.

Instantaneous Current When Voltage Is Applied Across Impedance

Applying Ohm's Law in AC Circuits

Ohm's Law extends to AC circuits as:

$$I(t) = V(t) / Z$$

where:

- $I(t)$ is the instantaneous current
- $V(t)$ is the instantaneous voltage
- Z is the impedance magnitude

Given a sinusoidal voltage $V(t) = V_m \sin(\omega t)$, the current $I(t)$ will also be sinusoidal but may lag or lead depending on the impedance's reactive components.

Calculating Instantaneous Current

Suppose $V(t) = 100 \sin(\omega t)$ V, and the impedance Z has a magnitude $|Z|$ and phase angle θ . Then:

$$I(t) = (V_m / |Z|) \sin(\omega t - \theta)$$

where:

- $V_m = 100$ V (amplitude)
- $|Z|$ = magnitude of impedance
- θ = phase angle between voltage and current

The phase angle θ is determined by:

$$\tan(\theta) = X / R$$

This phase difference means the current may not be instantaneously in phase with the voltage, especially when reactive components are significant.

Impact of Impedance Components on Instantaneous Current

Resistive Impedance (Pure Resistance)

When impedance is purely resistive ($Z = R$):

- The current is in phase with the voltage.
- Instantaneous current follows the voltage waveform directly:

- $I(t) = (V(t)) / R$
- Peak current: $I_m = V_m / R$

Inductive Impedance (Inductive Reactance)

When impedance includes inductance ($Z = R + jX_L$):

- The current lags the voltage by angle θ .
- The magnitude of current:

- $I_m = V_m / \sqrt{R^2 + X_L^2}$
- Phase angle: $\theta = \arctangent (X_L / R)$

Capacitive Impedance (Capacitive Reactance)

When impedance includes capacitance ($Z = R - jX_C$):

- The current leads the voltage by angle θ .
- Similar calculations apply:

- $I_m = V_m / \sqrt{R^2 + X_C^2}$
- Phase angle: $\theta = \arctangent (X_C / R)$

Practical Applications and Considerations

Designing Circuits with Specified Voltage and Impedance

Understanding how supply voltage interacts with impedance allows engineers to:

- Calculate expected current flow
- Design components to achieve desired current levels
- Manage power distribution efficiently

Maximizing Power Transfer

Maximum power transfer occurs when the impedance of the load matches the source impedance, considering reactive components:

- $Z_{\text{load}} = Z_{\text{source}}$

This matching ensures optimal energy transfer from the source to the load.

Implications for Circuit Stability and Safety

Knowing the instantaneous current helps in:

- Preventing overload conditions
- Designing protective devices like circuit breakers
- Ensuring stable operation of sensitive electronics

Summary of Key Points

- A voltage source, such as $V(t) = 100 \sin(\omega t)$ volts, drives current through an impedance.
- The instantaneous current depends on the magnitude and phase of the impedance.
- Ohm's Law for AC circuits relates voltage and current via impedance: $I(t) = V(t) / Z$.
- The reactive components of impedance cause phase shifts, affecting the timing of current relative to voltage.
- Proper circuit design considers impedance characteristics to optimize performance and safety.

Conclusion

Understanding how an applied voltage interacts with circuit impedance to produce instantaneous current is fundamental in electrical engineering. Whether dealing with simple resistive loads or complex RLC circuits, the principles outlined here enable engineers and technicians to analyze, design, and troubleshoot electrical systems effectively. Recognizing the role of impedance in shaping current waveforms ensures that electrical devices operate safely, efficiently, and reliably across various applications.

Keywords: supply voltage, impedance, instantaneous current, AC circuits, Ohm's Law, reactive components, phase angle, power transfer, circuit design, electrical engineering

Frequently Asked Questions

What is the significance of the supply voltage function $V(t) = 100 \cos(\omega t)$ V in analyzing AC circuits?

The function $V(t) = 100 \cos(\omega t)$ V represents a sinusoidal AC voltage source with a peak voltage of 100 V. It helps in analyzing the instantaneous current and power in the circuit by modeling real-world AC supply behavior.

How do you determine the instantaneous current when a sinusoidal voltage is applied across a given impedance?

The instantaneous current $i(t)$ can be found by applying Ohm's law in the time domain: $i(t) = V(t) / Z$, where Z is the impedance. For sinusoidal sources, this often involves using complex impedance and phasor analysis to find the sinusoidal current waveform.

What is the role of impedance in calculating the instantaneous current for a sinusoidal voltage source?

Impedance Z , which includes resistance and reactance, determines the amplitude and phase of the current relative to the voltage. It directly influences the magnitude of the instantaneous current via Ohm's law and affects how the current waveform relates to the voltage waveform.

How does the phase difference between voltage and impedance affect the instantaneous current?

The phase difference, determined by the impedance's reactive components, causes the current to either lead or lag the voltage. This phase shift influences the shape and timing of the instantaneous current waveform relative to the voltage.

Can the instantaneous current be expressed mathematically for the given voltage $V(t) = 100 \cos(\omega t)$ V?

Yes, if the impedance is known and represented as $Z = R + jX$, the instantaneous current can be expressed as $i(t) = (V_{\text{peak}} / |Z|) \cos(\omega t - \theta)$, where θ is the phase angle of Z .

What are the practical applications of understanding the instantaneous current for a sinusoidal voltage source?

Understanding instantaneous current helps in designing and analyzing AC power systems, ensuring components are rated for current surges, and optimizing power transfer efficiency in electrical and electronic circuits.

How does changing the impedance affect the maximum instantaneous current in the circuit?

Increasing impedance decreases the maximum current amplitude, while decreasing impedance increases it. The maximum instantaneous current is proportional to $V_{\text{peak}} / |Z|$, so impedance directly controls current magnitude.

What is the significance of the peak voltage of 100 V in the context of instantaneous current calculations?

The peak voltage of 100 V determines the maximum possible instantaneous voltage and consequently the maximum instantaneous current (assuming impedance remains constant), which is crucial for designing circuits to handle peak current levels safely.

How would the instantaneous current waveform change if the supply voltage frequency ω increases?

An increase in ω affects the impedance's reactance (especially for inductors and capacitors), altering both the magnitude and phase of the current

waveform. This results in a different shape and timing of the instantaneous current, impacting circuit behavior.

Additional Resources

A Supply Voltage Given By $v(t) = 100 \text{ V}$ Is Connected Across An Impedance And Provides An Instantaneous Current

Introduction

In the realm of electrical engineering, understanding how voltages and currents behave in various circuit configurations is fundamental. The statement "A supply voltage given by $v(t) = 100 \text{ V}$ is connected across an impedance and provides an instantaneous current" encapsulates a scenario that warrants detailed investigation. This article explores the intricacies of such a setup, examining the theoretical underpinnings, practical implications, and potential applications. The focus is on dissecting the behavior of the circuit, especially the nature of the instantaneous current response to a specified voltage source, with an emphasis on the mathematical and physical principles involved.

Theoretical Foundations

The Voltage Source: $v(t) = 100 \text{ V}$

The notation $v(t) = 100 \text{ V}$ suggests a time-dependent voltage source, possibly a function of time t . Although the expression appears incomplete, it can be interpreted as a voltage source with a magnitude of 100 V, possibly varying with time or representing a specific waveform. For the purpose of this investigation, we consider two common cases:

- Constant DC Voltage: $v(t) = 100 \text{ V}$
- Time-Varying Voltage: $v(t) = 100 \cdot f(t)$, where $f(t)$ is a function describing the waveform (e.g., sinusoidal, step, exponential).

Understanding whether the voltage is steady or transient significantly influences the current response.

Connecting to an Impedance

The impedance Z in the circuit can be complex-valued, representing the combined effects of resistance R , inductance L , and capacitance C :

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right),$$

\]

where ω is the angular frequency. The nature of Z determines how the voltage translates into current.

In the simplest case, if the impedance is purely resistive ($Z = R$), Ohm's law applies directly:

$$i(t) = \frac{v(t)}{R}.$$

However, for reactive components, the current response becomes more complex, especially when considering transient or instantaneous behaviors.

Analyzing Instantaneous Current

The core of this investigation lies in understanding what is meant by "instantaneous current" in this context. Typically, this refers to the current at a specific moment in time, denoted as $i(t)$. The analysis depends heavily on the nature of the voltage source and the circuit elements.

Case 1: Steady DC Voltage Source

Suppose $v(t) = 100\text{ V}$ (a constant). In a purely resistive circuit, the current is also constant:

$$i(t) = \frac{100\text{ V}}{R}.$$

The current is "instantaneous" in the sense that it remains constant (steady-state), and at any given instant, it is $100/R$ Amperes.

Case 2: Time-Varying Voltage Source

If the voltage varies with time, for example:

$$v(t) = 100 \sin(\omega t),$$

then the current response depends on the impedance:

$$i(t) = \frac{v(t)}{|Z|} \cos(\phi),$$

where ϕ is the phase angle between voltage and current.

Transient and Instantaneous Behavior in Circuit Response

Step Response and Impulse Response

Understanding how circuits respond to abrupt changes or specific waveforms is crucial. When a voltage suddenly changes, the current response is governed by the circuit's differential equations, especially for reactive elements.

- Step Input Response: When a voltage suddenly jumps from 0 to 100 V, the current initially reacts instantaneously, especially in circuits with ideal inductors or capacitors.
- Impulsive Behavior: In cases involving ideal components, the current may exhibit an impulsive response, characterized by instantaneous changes in current or voltage.

The Role of Ideal Components

- Ideal Resistor: No instantaneous change; current follows voltage directly.
- Ideal Inductor: Opposes instantaneous change in current; current cannot change instantaneously.
- Ideal Capacitor: Voltage across the capacitor cannot change instantaneously; current can exhibit an impulsive nature.

This distinction is vital in understanding the "instantaneous current" provided when connecting a voltage source to an impedance.

Mathematical Modeling and Analysis

Differential Equation Formulation

The general form for the current $i(t)$ in an RLC circuit with a voltage source $v(t)$:

$$L \frac{di(t)}{dt} + R i(t) + \frac{1}{C} \int i(t) dt = v(t).$$

For purely resistive circuits, the equation simplifies to:

$$i(t) = \frac{v(t)}{R}.$$

In reactive circuits, solving these equations yields the transient and steady-state responses.

Instantaneous Power and Current

Power at any instant is given by:

$$p(t) = v(t) \times i(t).$$

In the context of an idealized voltage source and impedance, the instantaneous power and current can be characterized precisely, especially during transients.

Practical Considerations and Implications

Real-World Circuit Behavior

In practical circuits, idealized instantaneous changes are physically impossible due to parasitic inductances and capacitances. However, theoretical models often assume ideal components to understand fundamental behaviors.

- Switching Events: When a voltage source is suddenly connected, the initial current may be constrained by inductance, preventing an infinite instantaneous current.
- Transient Overshoot: Real circuits may exhibit overshoot or undershoot during switching, affecting the "instantaneous" current.

Applications and Relevance

Understanding instantaneous current responses has practical applications in:

- Power electronics: Switching power supplies and inverters.
- Signal processing: Transient analysis of filters.
- Circuit protection: Surge current management.
- Electromagnetic compatibility: Managing transient currents to prevent interference.

Cases of Special Interest

Idealized Instantaneous Current in Theoretical Models

In ideal circuit theory, when a perfect voltage source is suddenly connected to a purely reactive impedance (like an inductor), the current response is characterized by a delta function or an impulsive behavior:

$$i(t) \sim \delta(t),$$

\]

indicating an instantaneous surge in current, which is a mathematical abstraction rather than a physical reality.

Limitations and Assumptions

- Ideal Components: Assumption of zero parasitic inductance or capacitance.
- Instantaneous Switching: Modeled as an ideal step function, ignoring real-world switching times.
- Neglecting Parasitics: Ignoring physical limitations that prevent truly instantaneous changes.

Summary and Conclusions

The analysis of "A supply voltage given by $V = 100 \text{ V}$ is connected across an impedance and provides an instantaneous current" reveals the layered complexity of circuit behavior during transient events. While idealized models suggest that connecting a voltage source directly to an impedance can produce an instantaneous current surge—especially in the case of ideal inductors or capacitors—actual physical systems are constrained by parasitic elements that prevent truly instantaneous changes.

In practical applications, understanding the nature of the source waveform, the impedance characteristics, and the circuit elements is crucial for predicting current behavior. Transient analysis becomes central to designing circuits that can handle sudden voltage changes without damage or unintended oscillations.

This investigation underscores the importance of theoretical models in guiding real-world circuit design, while also acknowledging their limitations and the necessity of considering non-idealities. As technology advances, more sophisticated models and simulation tools continue to improve our capacity to predict and control instantaneous currents in complex electrical systems.

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Note: This article aims to provide a comprehensive review of the phenomena associated with instantaneous current response in electrical circuits, serving as a resource for students, educators, and professionals seeking to deepen their understanding of transient circuit behavior.

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