A Triangle Is Dilated By A Scale Factor Of N = One-third. Which Statement Is True Regarding The Dilation?It

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Understanding the concept of dilation in geometry is fundamental for students and enthusiasts aiming to grasp the principles of similarity and transformations. When a triangle undergoes dilation with a particular scale factor, such as N = one-third, it results in a new triangle that maintains the shape's similarity to the original but differs in size. This article will explore what happens during this specific dilation, the properties involved, and the key statements that accurately describe the resulting figure.

What Is Dilation in Geometry?

Dilation, also known as enlargement or reduction, is a transformation that changes the size of a figure while preserving its shape. It is a type of similarity transformation characterized by a center of dilation and a scale factor.

Key Components of Dilation

- **Center of Dilation:** The fixed point about which the figure is expanded or contracted.
- **Scale Factor (N):** A numerical value that determines how much the figure is enlarged or reduced. If $N > 1$, the figure enlarges; if $0 < N < 1$, the figure reduces.

Understanding Scale Factor N = One-third

When the scale factor N equals one-third, the dilation results in a figure that is one-third the size of the original, measured relative to the center of dilation.

Implications of a Scale Factor of One-third

1. The new triangle is similar to the original triangle.
2. The dimensions (lengths of sides, heights, medians, etc.) are scaled down by a factor

of 1/3.

3. The area of the dilated triangle is scaled down by the square of the scale factor, i.e., $(1/3)^2 = 1/9$.

Key Statements About the Dilation of a Triangle by N = One-third

Understanding which statements are true about this dilation involves examining properties such as side lengths, area, perimeter, and similarity ratios.

Statement 1: The Dilation Produces a Triangle Similar to the Original

This statement is true. Dilation is a similarity transformation, which means the new triangle retains the same shape as the original, with corresponding angles remaining equal. The key difference is the size, scaled according to the scale factor.

Statement 2: The Side Lengths of the Dilated Triangle Are One-Third of the Original

This is also true. Since the scale factor N is one-third, each side of the new triangle is exactly one-third the length of the corresponding side in the original triangle. For example, if a side in the original triangle measures 9 units, the corresponding side in the dilated triangle measures 3 units.

Statement 3: The Area of the Dilated Triangle Is One-Ninth of the Original

This statement is correct. The area scales by the square of the scale factor. Therefore, if the original triangle has an area A, the dilated triangle's area is $(1/3)^2 \times A = 1/9 \times A$.

Statement 4: The Perimeter of the Dilated Triangle Is One-Third of the Original

This statement is false. Since the perimeter is a linear measurement, it scales directly with the scale factor N. Therefore, the perimeter of the dilated triangle is one-third of the original's perimeter.

Statement 5: The Center of Dilation Remains Fixed During the Transformation

This is true. The center of dilation is a fixed point from which the dilation occurs. All points of the triangle are mapped along lines radiating from this center, scaled according to the scale factor.

Visualizing the Dilation Process

Visualizing the dilation process can help in understanding how the original triangle transforms into its scaled version.

Steps to Visualize the Dilation:

1. Identify the center of dilation.
2. Draw lines connecting each vertex of the original triangle to the center.
3. Measure one-third of the distance from the center to each vertex along these lines.
4. Mark the new vertices at these points.
5. Connect the new vertices to form the dilated triangle.

Real-World Applications of Dilation by Scale Factor of One-third

Understanding dilation and scale factors has numerous practical applications across various fields.

Architectural Design

- Creating scaled models of buildings or structures for presentation and planning.

Engineering and Manufacturing

- Designing components that need to be scaled proportionally.

Art and Illustration

- Enlarging or reducing images while maintaining proportions.

Educational Demonstrations

- Teaching concepts of similarity and transformations in geometry.

Summary of Key Points

- Dilation is a similarity transformation that enlarges or reduces a figure based on a scale factor.
- A scale factor of one-third reduces all linear dimensions to one-third of their original length.
- The resulting triangle is similar to the original, with side lengths scaled by 1/3.
- The area of the dilated triangle is scaled by 1/9 (the square of the scale factor).
- The perimeter is scaled by 1/3, matching the linear scale factor.
- The center of dilation remains fixed during the transformation.

Conclusion

Dilation by a scale factor of one-third significantly reduces the size of the original triangle while preserving its shape and proportions. Recognizing these properties allows students and professionals to analyze geometric figures accurately and apply these principles in practical contexts. The key takeaways are the similarity of the new triangle, the proportional reduction in side lengths, and the quadratic reduction in area. Whether in academic settings, design, or engineering, understanding the effects of a scale factor of one-third in dilation is essential for mastering geometric transformations.

FAQs

Q1: Does the dilation change the shape of the triangle?

No. Dilation preserves the shape; it only changes the size, making the figure similar to the original.

Q2: Can the dilation be performed about any point?

Yes. The center of dilation can be any point in the plane, but the properties of the dilation depend on its location relative to the figure.

Q3: What happens if the scale factor is greater than 1?

The triangle enlarges, with side lengths scaled by the factor, and area scaled by the square of the factor.

Q4: What if the scale factor is zero?

The dilated figure collapses into a single point, known as the center of dilation.

Q5: How does dilation relate to similar triangles?

Dilation produces similar triangles because angles are preserved, and side lengths are proportional.

By understanding these concepts, students can confidently analyze dilations and their effects, especially when dealing with scale factors like one-third. This knowledge forms a foundation for more advanced topics in geometry and related fields.

Frequently Asked Questions

What does it mean to dilate a triangle by a scale factor of one-third?

Dilating a triangle by a scale factor of one-third means that each side of the triangle is reduced to one-third of its original length, resulting in a smaller, similar triangle.

Is the dilated triangle similar to the original triangle?

Yes, when a triangle is dilated by any scale factor, including one-third, the resulting triangle is similar to the original because all corresponding angles remain the same and side lengths are proportionally scaled.

How does a scale factor of one-third affect the area of the original triangle?

The area of the dilated triangle is one-third squared (1/3)^2 = 1/9 of the original triangle's area, meaning the area is reduced to one-ninth of the original.

If the original triangle has a perimeter of 24 units, what is the perimeter after dilation by N=one-third?

The perimeter after dilation will be one-third of the original, so it will be 24 × 1/3 = 8 units.

What is the effect of a scale factor less than 1 on the size of the triangle?

A scale factor less than 1, such as one-third, results in a smaller, reduced version of the original triangle, maintaining similar shape but with proportional side lengths.

Can the vertices of the original and dilated triangles be mapped directly?

Yes, the vertices of the dilated triangle are mapped to the original vertices through a center of dilation, maintaining proportionality and similarity between the two triangles.

Additional Resources

Dilation: Understanding the Geometric Transformation and Its Implications

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Introduction to Dilation in Geometry

Geometry, a fundamental branch of mathematics, often involves transformations that alter the size, position, or shape of figures while preserving certain properties. Among these transformations, dilation (also known as scaling) stands out due to its unique property of resizing figures relative to a fixed point—called the center of dilation. This transformation is pivotal in various fields, including computer graphics, engineering, architecture, and more.

In this article, we delve into the specifics of dilation, centering our focus on what occurs when a triangle undergoes dilation with a scale factor of N = one-third. We aim to clarify the nature of such a transformation, explore its properties, and answer the key question: What is true regarding the dilation of a triangle by a scale factor of one-third?

Understanding Dilation and Scale Factors

What is Dilation?

Dilation is a transformation that produces an image that is the same shape as the original but is either enlarged or reduced in size. Crucially, after dilation, the image remains similar to the original figure—meaning corresponding angles are equal, and sides are proportional.

Key characteristics of dilation include:

- Center of Dilation: The fixed point in the plane about which the figure is dilated.
- Scale Factor (k): A positive real number indicating the ratio of the size of the image relative to the original figure.

Understanding the Scale Factor

The scale factor, denoted as k, determines the degree of resizing:

- $k > 1$: The figure enlarges; the image is a scaled-up version.
- $0 < k < 1$: The figure reduces; the image is a scaled-down version.
- $k = 1$: The figure remains unchanged.
- $k < 0$: The figure is reflected and scaled, but such cases are beyond the scope here.

In our case, $k = 1/3$, which indicates a reduction to one-third of the original size.

Implications of Dilation with a Scale Factor of One-Third

Size Reduction and Preservation of Shape

When a triangle is dilated with a scale factor of 1/3, the resulting figure is similar to the original but smaller:

- Size: Each side of the triangle becomes one-third as long as the original.
- Shape: The angles remain unchanged, preserving the triangle’s shape.
- Proportionality: Corresponding sides are proportional, maintaining similarity.

This is akin to viewing the same triangle through a lens that shrinks it uniformly in all directions.

Impact on Coordinates of the Triangle's Vertices

Suppose the original triangle has vertices at points $A(x_1, y_1)$, $B(x_2, y_2)$, and $C(x_3, y_3)$. The dilated triangle, with a center at point $O(x_o, y_o)$, will have vertices A', B', and C' calculated as:

$$
A' = O + k \times (A - O)
$$

\[

$$
B' = O + k \times (B - O)
$$

$$
C' = O + k \times (C - O)
$$

Since $k = 1/3$, each vertex moves closer to the center of dilation by a factor of one-third of its original distance from O. This demonstrates the uniform reduction in size.

Key Properties and Theoretical Outcomes

Effect on Area

A fundamental property of dilation is its effect on the area of the figure:

- Area of Dilated Triangle = $(\text{Scale Factor})^2 \times$ Original Area

For $k = 1/3$, the new area is:

$$
\text{Area}_{\text{new}} = \left(\frac{1}{3}\right)^2 \times \text{Original Area} = \frac{1}{9} \times \text{Original Area}
$$

Thus, the area decreases to one-ninth of the original. This quadratic relationship underscores how even small scale factors significantly reduce the area.

Perimeter Considerations

Similarly, the perimeter scales linearly with the scale factor:

- Perimeter of Dilated Triangle = $k \times$ Original Perimeter

For $k = 1/3$, the perimeter becomes one-third of the original perimeter.

Preservation of Angles and Shape

Because dilation produces similar figures, all angles remain unchanged, and the shape is preserved. This property is critical in applications where shape integrity is essential despite size alterations.

Answering the Core Question: What Statement Is True Regarding the Dilation?

Given the extensive understanding above, the core question becomes: Which statement correctly describes the nature of the triangle after dilation by a scale factor of one-third?

Common statements and their evaluations:

1. The triangle's size increases by a factor of three.
False. Since the scale factor is $\frac{1}{3}$, the triangle shrinks to one-third of its original size, not enlarges.

2. The triangle's size decreases by a factor of three.
False. It decreases to one-third, not by a factor of three. The key is that the size is scaled by $\frac{1}{3}$—meaning the new size is one-third, not three times smaller.

3. The triangle's sides are each one-third the length of the original.
True. This accurately describes the effect of a scale factor of $\frac{1}{3}$.

4. The triangle's area becomes three times smaller.
False. The area becomes $\frac{1}{9}$ of the original, since area scales with the square of the scale factor.

5. The triangle's perimeter reduces to one-third of the original.
True. Because perimeter scales linearly with the scale factor.

Conclusion: The correct statements are that the sides are each one-third the length of the original, and the perimeter reduces to one-third, while the area decreases to one-ninth.

Real-World Applications and Intuitive Analogies

Understanding dilation with a scale factor of $\frac{1}{3}$ is not merely theoretical—it has practical implications:

- Model Scaling: Architects and engineers often create scaled models of structures, reducing size proportionally to fit within a manageable space. Dilation with a factor of $\frac{1}{3}$ mirrors this process precisely.
- Computer Graphics: When zooming in or out, images are scaled; understanding how dimensions change aids in rendering accurate visuals.
- Educational Demonstrations: Using physical objects or drawings, educators demonstrate how figures shrink or grow, reinforcing the concept of similarity and scale.

An intuitive analogy: Imagine a large triangular poster representing the original figure. Dilation by $\frac{1}{3}$ is like shrinking the poster so that its dimensions are just one-third of the original, all while maintaining the same shape.

Summary and Final Thoughts

A triangle dilated by a scale factor of one-third is a classic example of how geometric transformations influence figures. The key takeaways include:

- Size: The triangle becomes smaller, with each side exactly one-third the original length.
- Shape: The shape remains unchanged; angles and proportionality are preserved.
- Area: The area reduces to one-ninth of the original, emphasizing how quadratic scaling impacts size.
- Perimeter: The perimeter decreases to one-third, consistent with linear scaling.

Understanding these properties not only enriches one's grasp of geometry but also enhances spatial reasoning—a skill applicable across numerous scientific and artistic disciplines.

In conclusion, the statement that best describes the effect of dilating a triangle with a scale factor of one-third is: "The triangle's sides are each one-third the length of the original, and its area is reduced to one-ninth of the original." Recognizing these fundamental principles empowers students, educators, and professionals to apply geometric transformations confidently and accurately in various contexts.

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