Select The Correct Answer.Consider Function G.g(x)= 5/x-1 +2What Is The Average Rate Of Change Of Function

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Understanding the concept of the average rate of change of a function is fundamental in mathematics, especially in calculus and algebra. It provides insight into how a function behaves between two points, offering a snapshot of its overall trend. In this article, we will explore the function $G.g(x) = \frac{5}{x-1} + 2$, analyze how to compute its average rate of change over an interval, and guide you through the process step-by-step to select the correct answer among multiple choices.

What Is the Average Rate Of Change?

The average rate of change of a function between two points measures how much the function's output changes relative to a change in input over a specific interval. Think of it as the slope of the secant line connecting two points on the graph of the function. Mathematically, for a function $f(x)$, the average rate of change over the interval $[x_1, x_2]$ is:

$$
\text{Average Rate of Change} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}
$$

This formula essentially measures the average "speed" of the function’s change between two points.

Key Points:

- It provides an overall measure of change, unlike the instantaneous rate of change which is given by the derivative at a single point.
- It is useful for understanding the general trend of the function over an interval.
- The units of the average rate of change are typically units of the output variable per units of the input variable.

Analyzing the Given Function $G.g(x) = \frac{5}{x - 1} + 2$

Before calculating the average rate of change, it’s essential to understand the structure and behavior of the given function:

- Type of Function: Rational function, with a vertical asymptote at $x=1$ because the denominator becomes zero.
- Behavior: As x approaches 1 from the left, $G.g(x)$ tends to negative infinity; from the right, it tends to positive infinity.
- Range: All real numbers except possibly the values where the function is undefined or approaches asymptotes.

Step 1: Choose the Interval $[x_1, x_2]$

To find the average rate of change, we need specific values for x_1 and x_2. Typically, these are given in the problem options, or we select meaningful points in the domain, avoiding the asymptote at $x=1$.

For example, suppose the interval is $[2, 4]$:

- $x_1 = 2$
- $x_2 = 4$

Alternatively, if options suggest different intervals, choose those accordingly.

Step 2: Calculate $G.g(x_1)$ and $G.g(x_2)$

Calculate the function values at the selected points:

$$
G.g(2) = \frac{5}{2 - 1} + 2 = \frac{5}{1} + 2 = 5 + 2 = 7
$$

$$
G.g(4) = \frac{5}{4 - 1} + 2 = \frac{5}{3} + 2 = \frac{5}{3} + \frac{6}{3} = \frac{11}{3} \approx 3.6667
$$

Step 3: Compute the Average Rate of Change

Using the formula:

$$
\frac{G.g(4) - G.g(2)}{4 - 2} = \frac{\frac{11}{3} - 7}{2}
$$

Express 7 as a fraction with denominator 3:

$$
7 = \frac{21}{3}
$$

Subtract:

$$
\frac{11}{3} - \frac{21}{3} = -\frac{10}{3}
$$

Divide by 2:

$$
\frac{-\frac{10}{3}}{2} = -\frac{10}{3} \times \frac{1}{2} = -\frac{10}{6} = -\frac{5}{3}
$$

Result:

$$
\boxed{\text{Average Rate of Change} = -\frac{5}{3}}
$$

This indicates that, on average, over the interval $[2, 4]$, the function decreases at a rate of $\frac{5}{3}$ units per unit increase in x.

Interpreting the Result and Choosing the Correct Answer

Depending on the multiple-choice options provided, the result of $-\frac{5}{3}$ should guide your selection. Common options might include fractions, decimal approximations, or descriptive terms.

Example options:

- A) $\frac{5}{3}$
- B) $-\frac{5}{3}$
- C) 0
- D) 2

In this case, the correct choice would be B) $-\frac{5}{3}$, reflecting the negative average rate of change over the interval.

Additional Considerations When Calculating Average Rate of Change

- Interval selection: Always ensure that the interval chosen does not include the function's asymptotes or undefined points.
- Multiple intervals: Sometimes, questions may ask for the average rate of change over different intervals; repeat the process accordingly.
- Graphical interpretation: Plotting the function can provide visual insight into whether the function is increasing or decreasing over the interval, aligning with the calculated rate.
- Units and context: In real-world applications, consider what the rate of change represents in context (e.g., speed, growth rate).

Practice Problems for Better Understanding

1. For the function $G.g(x) = \frac{5}{x - 1} + 2$, compute the average rate of change between $x=2$ and $x=3$.

2. How does the average rate of change differ if you choose $x=0.5$ and $x=2$, considering the asymptote at $x=1$?

3. Graph $G.g(x)$ and identify the intervals where the function is increasing or decreasing.

Conclusion

Calculating the average rate of change is a vital skill in understanding the behavior of functions, particularly rational functions like $G.g(x) = \frac{5}{x-1} + 2$. By selecting appropriate intervals, calculating the function values at those points, and applying the basic formula, you can determine whether the function is increasing or decreasing over that interval and quantify its average rate of change.

Remember, always consider the function's domain and asymptotes to avoid undefined points, and verify your calculations carefully. Whether for academic assessments or practical applications, mastering this concept enhances your overall mathematical proficiency.

Summary

- The average rate of change measures how a function changes over an interval.
- For $G.g(x) = \frac{5}{x-1} + 2$, choose intervals avoiding asymptotes.
- Calculate the function at endpoints, subtract, and divide by the interval length.
- The result indicates the overall trend: positive for increasing, negative for decreasing.
- Practice with different intervals to solidify understanding.

Remember: Always double-check your calculations and interpret the results within the context of the problem to make accurate conclusions.

Frequently Asked Questions

What is the function G.g(x) given as 5/x - 1 + 2?

The function G.g(x) is defined as G.g(x) = −5/x - 1 + 2.

How do you interpret the average rate of change of a function over an interval?

The average rate of change over an interval is the difference in the function's values at the endpoints divided by the length of the interval, representing the average slope between those points.

What is the formula for the average rate of change of G.g(x) over an interval [a, b]?

The average rate of change of G.g(x) over [a, b] is − [G.g(b) - G.g(a)] / (b - a).

Calculate the average rate of change of G.g(x) between x=2 and x=4.

First, G.g(2) = 5/2 - 1 + 2 = 2.5 - 1 + 2 = 3.5; G.g(4) = 5/4 - 1 + 2 = 1.25 - 1 + 2 = 2.25; thus, the average rate of change = (2.25 - 3.5) / (4 - 2) = (-1.25) / 2 = -0.625.

Is the average rate of change of the function positive or negative between x=2 and x=4?

The average rate of change is negative (-0.625) between x=2 and x=4.

What does the sign of the average rate of change indicate

about the function's behavior over the interval?

A negative average rate of change indicates that the function is decreasing over the interval.

Additional Resources

Select The Correct Answer. Consider Function G.g(x) = 5/(x - 1) + 2. What Is The Average Rate Of Change Of The Function?

Understanding the average rate of change of a function is fundamental in calculus, as it provides insight into how a function behaves over a specific interval. In this article, we examine the function \(G.g(x) = \frac{5}{x - 1} + 2 \) and explore how to determine its average rate of change over a given interval. We will break down the concept, outline the mathematical process, and guide you through the steps needed to arrive at the correct answer.

What Is the Average Rate of Change?

Before diving into the specifics of the function \(G.g(x) \), it’s essential to understand what the average rate of change (AROC) represents in mathematics.

Definition

The average rate of change of a function \(f(x) \) over an interval \([a, b]\) is a measure of how much the function’s output changes on average per unit increase in the input over that interval. It is mathematically expressed as:

\[
\text{Average Rate of Change} = \frac{f(b) - f(a)}{b - a}
\]

This formula resembles the slope of the secant line connecting the points \((a, f(a))\) and \((b, f(b))\) on the graph of the function.

Significance

- Slope Interpretation: It provides an average slope between two points.
- Comparing Intervals: Helps compare how the function behaves over different segments.
- Foundation for Derivatives: As the interval shrinks, the average rate of change approaches the instantaneous rate of change, which is the derivative.

Analyzing the Function \(G.g(x) = \frac{5}{x - 1} + 2 \)

The function under consideration combines a rational component and a constant:

\[
G.g(x) = \frac{5}{x - 1} + 2

\]

This function exhibits interesting behavior due to the rational component, especially near the vertical asymptote at \(x = 1 \). For the purpose of calculating the average rate of change, the interval chosen must avoid the asymptote to ensure the function is defined at both endpoints.

Characteristics of the Function

- Domain: All real numbers except \(x = 1 \).
- Asymptote: Vertical line at \(x = 1 \).
- Behavior: As \(x \to 1^{+} \), \(G.g(x) \to +\infty \); as \(x \to 1^{-} \), \(G.g(x) \to -\infty \).
- Range: All real numbers except potentially some values depending on \(x \).

Understanding these features is crucial because they influence the choice of interval when calculating the average rate of change.

Selecting an Interval for Calculation

The average rate of change depends on the interval \([a, b]\). To grasp the concept clearly, consider the following points:

- The interval should be within the domain of \(G.g(x) \), i.e., avoid \(x = 1 \).
- The interval should be specified for a meaningful calculation, such as from \(x = 2 \) to \(x = 4 \).

Suppose we choose the interval \([2, 4]\). This choice is practical because both points are within the domain, and the function is well-defined there.

Step-by-Step Calculation

Given the interval \([a, b] = [2, 4]\), the average rate of change is:

\[
\frac{G.g(4) - G.g(2)}{4 - 2}
\]

Let's compute \(G.g(2) \) and \(G.g(4) \):

Calculating \(G.g(2) \)

\[
G.g(2) = \frac{5}{2 - 1} + 2 = \frac{5}{1} + 2 = 5 + 2 = 7
\]

Calculating \(G.g(4) \)

\[
G.g(4) = \frac{5}{4 - 1} + 2 = \frac{5}{3} + 2 \approx 1.6667 + 2 = 3.6667

\]

Now, plug these into the average rate of change formula:

$$
\frac{3.6667 - 7}{4 - 2} = \frac{-3.3333}{2} = -1.6667
$$

Thus, over the interval $[2, 4]$, the average rate of change of the function is approximately -1.6667.

Interpreting the Result

The negative value indicates that, on average, the function decreases as x moves from 2 to 4. This aligns with the behavior of the function, as the rational component diminishes in value over this interval.

Generalization to Other Intervals

The process demonstrated can be applied to any interval within the domain:

1. Choose an interval $[a, b]$ that does not include the asymptote at $x = 1$.
2. Calculate the function’s values at a and b.
3. Apply the average rate of change formula.

This approach provides meaningful insights into how the function behaves over different ranges.

Common Mistakes and Misconceptions

When calculating the average rate of change, students often encounter pitfalls:

- Including the asymptote: The function is undefined at $x=1$, so be cautious to select intervals that avoid this point.
- Incorrect function evaluation: Ensure accurate substitution into the function, especially with fractions.
- Misinterpreting the result: Remember that a negative average rate of change indicates a decreasing trend, while a positive indicates an increasing trend.

Practical Applications of Average Rate of Change

Understanding the average rate of change has broad applications:

- Physics: Calculating average velocity over a time interval.
- Economics: Determining average growth rates of investments or markets.
- Biology: Measuring average growth rates of populations.

In each case, the concept helps quantify how a quantity changes over a specific period or interval.

Summary and Final Thoughts

In the context of the function \(G.g(x) = \frac{5}{x - 1} + 2 \), calculating the average rate of change involves selecting an appropriate interval within the domain, computing the function values at the interval's endpoints, and applying the basic formula:

\[
\frac{G.g(b) - G.g(a)}{b - a}
\]

By choosing intervals away from asymptotes and carefully performing calculations, one can accurately gauge how the function behaves over different ranges.

In summary:

- The average rate of change provides a snapshot of the function's overall trend between two points.
- For \(G.g(x) \), the rational component influences the function's behavior significantly near \(x=1 \).
- Proper interval selection and careful computation are essential to obtaining meaningful results.

Understanding these principles empowers students and professionals alike to analyze functions effectively, enhancing their grasp of calculus concepts and their real-world applications.

Final Note

While the specific calculation demonstrated used the interval \([2, 4]\), the same methodology applies broadly. Whether analyzing stock prices, physical motion, or biological growth, the average rate of change remains a vital tool for understanding change over time or space.

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