# Show That W Is In The Subspace Of 4 Spanned By V1, V2, And V3, Where These Vectors Are Defined As Follows.

**Show That W Is In The Subspace Of 4 Spanned By V1, V2, And V3, Where These Vectors Are Defined As Follows.**

Determining whether a vector W resides within a particular subspace of $\mathbb{R}^4$ involves understanding the foundational concepts of linear algebra, specifically subspace criteria, span, and linear combinations. When a subspace is spanned by a set of vectors, any vector within that subspace can be expressed as a linear combination of those vectors. This article aims to guide you through the process of demonstrating that a given vector W belongs to the subspace of $\mathbb{R}^4$ spanned by vectors $V_1$, $V_2$, and $V_3$, whose explicit definitions are provided.

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## Understanding Subspaces and Spanning Sets

### What Is a Subspace?

A subspace of a vector space, such as $\mathbb{R}^4$, is a subset that itself satisfies the properties of a vector space. Specifically, a subspace W of $\mathbb{R}^4$ must satisfy:
- The zero vector is in W.
- W is closed under vector addition.
- W is closed under scalar multiplication.

In practical terms, a subspace can be thought of as a "smaller" space within a larger one, generated by linear combinations of certain vectors.

### What Does It Mean to Span a Subspace?

A set of vectors $\{V_1, V_2, V_3\}$ in $\mathbb{R}^4$ spans a subspace if every vector W in that subspace can be written as a linear combination:
$$ W = c_1 V_1 + c_2 V_2 + c_3 V_3 $$
where $c_1, c_2, c_3$ are scalars.

The set $\{V_1, V_2, V_3\}$ is called a spanning set for the subspace. Our goal is to verify that the specific vector W can be expressed in this form.

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# Defining the Vectors $V_1$, $V_2$, $V_3$, and W

Before proceeding, let's clarify the definitions of the vectors involved. Suppose the vectors are given as follows:

- $V_1 = (v_{11}, v_{12}, v_{13}, v_{14})$
- $V_2 = (v_{21}, v_{22}, v_{23}, v_{24})$
- $V_3 = (v_{31}, v_{32}, v_{33}, v_{34})$
- $W = (w_1, w_2, w_3, w_4)$

(Note: Replace these placeholder components with the actual vector components provided in your problem.)

The main task is to determine whether there exist scalars $c_1, c_2, c_3$ such that:

$$W = c_1 V_1 + c_2 V_2 + c_3 V_3$$

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# Methodology for Showing W Is in the Subspace

To verify W belongs to the subspace spanned by $V_1$, $V_2$, and $V_3$, follow these steps:

## Step 1: Set Up the Linear Equation

Express W as a linear combination:

$$(w_1, w_2, w_3, w_4) = c_1 (v_{11}, v_{12}, v_{13}, v_{14}) + c_2 (v_{21}, v_{22}, v_{23}, v_{24}) + c_3 (v_{31}, v_{32}, v_{33}, v_{34})$$

This leads to the system of equations:

$$
\begin{cases}
w_1 = c_1 v_{11} + c_2 v_{21} + c_3 v_{31} \\

w_2 = c_1 v_{12} + c_2 v_{22} + c_3 v_{32} \\
w_3 = c_1 v_{13} + c_2 v_{23} + c_3 v_{33} \\
w_4 = c_1 v_{14} + c_2 v_{24} + c_3 v_{34}
\end{cases}
\]

## Step 2: Formulate the System in Matrix Form

Express the equations in matrix form:

\[
\begin{bmatrix}
v_{11} & v_{21} & v_{31} \\
v_{12} & v_{22} & v_{32} \\
v_{13} & v_{23} & v_{33} \\
v_{14} & v_{24} & v_{34}
\end{bmatrix}
\begin{bmatrix}
c_1 \\ c_2 \\ c_3
\end{bmatrix}
=
\begin{bmatrix}
w_1 \\ w_2 \\ w_3 \\ w_4
\end{bmatrix}
\]

Let’s denote the matrix of vectors as \( A \) and the vector W as \( \mathbf{w} \):

\[
A = \begin{bmatrix}
v_{11} & v_{21} & v_{31} \\
v_{12} & v_{22} & v_{32} \\
v_{13} & v_{23} & v_{33} \\
v_{14} & v_{24} & v_{34}
\end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix}
w_1 \\ w_2 \\ w_3 \\ w_4
\end{bmatrix}
\]

### Step 3: Solve the System

Since the matrix A is 4x3, it's generally an overdetermined system, but because we are testing for membership in a subspace, the key is whether the system is consistent.

- Use methods such as Gaussian elimination or matrix rank to check for solutions.
- If a solution exists for $c_1, c_2, c_3$, then W lies in the span of $V_1$, $V_2$, $V_3$.

### Step 4: Verify the Existence of a Solution

- Calculate the augmented matrix $[A | \mathbf{w}]$.
- Row-reduce to echelon form.
- Check for consistency: no contradictory equations (e.g., 0 = non-zero).

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## Practical Example Demonstration

Suppose the vectors are explicitly given as:

- $V_1 = (1, 2, 3, 4)$
- $V_2 = (0, 1, 0, 1)$
- $V_3 = (2, 0, 1, 3)$
- $W = (3, 4, 5, 6)$

Following the steps:

1. Write the system:

$$
\begin{cases}
3 = c_1 \times 1 + c_2 \times 0 + c_3 \times 2 \\
4 = c_1 \times 2 + c_2 \times 1 + c_3 \times 0 \\
5 = c_1 \times 3 + c_2 \times 0 + c_3 \times 1 \\
6 = c_1 \times 4 + c_2 \times 1 + c_3 \times 3
\end{cases}
$$

2. Form the matrix and augmented vector:

$$

A = \begin{bmatrix}
1 & 0 & 2 \\
2 & 1 & 0 \\
3 & 0 & 1 \\
4 & 1 & 3
\end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix}
3 \\ 4 \\ 5 \\ 6
\end{bmatrix}
\]

3. Solve using Gaussian elimination:

- From the first equation:

\[ c_1 + 2 c_3 = 3 \quad (1) \]

- From the second:

\[ 2 c_1 + c_2 = 4 \quad (2) \]

- From the third:

\[ 3 c_1 + c_3 = 5 \quad (3) \]

- From the fourth:

\[ 4 c_1 + c_2 + 3 c_3 = 6 \quad (4) \]

Solve equations (1) and (3):

- From (3):

\[ c_3 = 5 - 3 c_1 \]

Substitute into (1):

\[ c_1 + 2 (5 - 3 c_1) = 3 \]
\[ c_1 + 10 - 6 c_1 = 3 \]
\[ -5 c_1 = -7 \]
\[ c_1 = \frac{7}{5} \]

Now, find \( c_3 \):

$$c_3 = 5 - 3 \times \frac{7}{5} = 5 - \frac{21}{5} = \frac{25}{5} - \frac{21}{5} = \frac{4}{5}$$

Next, find $c_2$ from (2

# Frequently Asked Questions

## What does it mean for a vector W to be in the subspace spanned by vectors V1, V2, and V3?

It means that the vector W can be expressed as a linear combination of V1, V2, and V3, i.e., W = aV1 + bV2 + cV3 for some scalars a, b, and c.

## How can we verify if a vector W is in the subspace spanned by V1, V2, and V3?

By setting up the equation W = aV1 + bV2 + cV3 and solving for a, b, and c. If a solution exists, then W lies in the subspace.

## What is the importance of the vectors V1, V2, and V3 in demonstrating that W is in the subspace?

They form a basis (or part of a basis) for the subspace, and expressing W as their linear combination confirms its membership in that subspace.

## How do the definitions of V1, V2, and V3 influence the process of showing W is in their span?

Knowing the explicit forms of V1, V2, and V3 allows us to set up equations and solve for the scalars needed to express W as their combination.

## What role do linear equations play in establishing that W is in the subspace spanned by V1, V2, and V3?

They form the system of equations derived from W = aV1 + bV2 + cV3, which we solve to determine whether W can be written as a linear combination of the vectors.

## Can a vector W be in the span of V1, V2, and V3 if it cannot be written as

### a linear combination of them?

No, if W cannot be expressed as such a linear combination, then W is not in the subspace spanned by V1, V2, and V3.

### What is the significance of the vectors V1, V2, and V3 being linearly independent when showing W is in their span?

If V1, V2, and V3 are linearly independent, they form a basis for their span, making it easier to determine whether W lies in that subspace and to find its coordinates.

### How does knowing the explicit coordinates of W and the basis vectors V1, V2, and V3 help in showing W is in their span?

By expressing W and V1, V2, V3 in coordinate form, we can set up and solve a straightforward linear system to verify W's membership in the subspace.

### If vectors V1, V2, and V3 are given explicitly, what is the step-by-step process to demonstrate that W is in their span?

First, write W and V1, V2, V3 in coordinate form; second, set up the linear system W = aV1 + bV2 + cV3; third, solve the system for a, b, and c; finally, verify if a solution exists. If it does, W is in their span.

## Additional Resources

Subspace: Unveiling the Foundations of Vector Spaces and Their Intricate Structures

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Introduction

In the realm of linear algebra, the concept of subspaces forms the backbone of understanding how vectors relate, combine, and define the structure of higher-dimensional spaces. When examining a particular vector $\mathbf{W}$, a common challenge encountered is determining whether it resides within a specific subspace—particularly, one spanned by a set of known vectors such as $\mathbf{V}_1, \mathbf{V}_2,$ and $\mathbf{V}_3$. This problem is not just a theoretical exercise; it has practical implications in fields ranging from computer graphics and data science to engineering and physics.

In this article, we will delve into the process of proving that a vector $\mathbf{W}$ belongs to the subspace spanned by $\mathbf{V}_1, \mathbf{V}_2,$ and $\mathbf{V}_3$. We will explore the underlying principles, step-by-step methods, and the significance of such an analysis, all while adopting an expert tone to provide clarity and depth.

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Understanding the Foundation: Vector Spaces and Subspaces

What is a Vector Space?

Before we examine the specifics of the problem, it's essential to revisit what constitutes a vector space. A vector space over a field (typically the real numbers, $\mathbb{R}$) is a set of vectors equipped with two operations: vector addition and scalar multiplication, satisfying certain axioms such as associativity, commutativity, and distributivity.

Subspaces Defined

A subspace is a subset of a vector space that is itself a vector space under the same operations. For a subset $U \subseteq V$ to be a subspace, it must satisfy:

- The zero vector $\mathbf{0}$ is in $U$.
- Closure under vector addition: If $\mathbf{u}_1, \mathbf{u}_2 \in U$, then $\mathbf{u}_1 + \mathbf{u}_2 \in U$.
- Closure under scalar multiplication: If $\mathbf{u} \in U$ and $c \in \mathbb{R}$, then $c\mathbf{u} \in U$.

Spanning Sets and Subspace Generation

A set of vectors $\{\mathbf{V}_1, \mathbf{V}_2, \mathbf{V}_3 \}$ spans a subspace $W$ if every vector in $W$ can be expressed as a linear combination of these vectors:

$$
\mathbf{w} \in W \quad \Rightarrow \quad \mathbf{w} = a_1 \mathbf{V}_1 + a_2 \mathbf{V}_2 + a_3 \mathbf{V}_3,
$$

where $a_1, a_2, a_3 \in \mathbb{R}$.

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The Core Problem: Showing $\mathbf{W}$ Is in the Subspace Spanned by $\mathbf{V}_1, \mathbf{V}_2, \mathbf{V}_3$

Restating the Goal

Suppose we are given:

- Vectors $\mathbf{V}_1, \mathbf{V}_2, \mathbf{V}_3$, which span a subspace $W$.
- A vector $\mathbf{W}$.

The task is to demonstrate that:

$$
\mathbf{W} \in W \quad \Longleftrightarrow \quad \mathbf{W} = a_1 \mathbf{V}_1 + a_2 \mathbf{V}_2 + a_3 \mathbf{V}_3,
$$

for some scalars $a_1, a_2, a_3$.

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Step-by-Step Approach to the Proof

To establish whether $\mathbf{W}$ resides within the span of $\mathbf{V}_1, \mathbf{V}_2, \mathbf{V}_3$, we follow a systematic process rooted in linear algebra principles:

1. Express the Vectors Explicitly

Begin by writing each vector in component form. For instance, if the vectors are in $\mathbb{R}^n$, they can be expressed as:

$$
\mathbf{V}_1 = (v_{11}, v_{12}, \ldots, v_{1n}),
$$
$$
\mathbf{V}_2 = (v_{21}, v_{22}, \ldots, v_{2n}),
$$
$$
\mathbf{V}_3 = (v_{31}, v_{32}, \ldots, v_{3n}),
$$
$$
\mathbf{W} = (w_1, w_2, \ldots, w_n).
$$

This explicit form is crucial for setting up the linear system.

2. Formulate the Linear System

Since $\mathbf{W}$ is a linear combination of the $\mathbf{V}_i$, the following holds:

$$
a_1 \mathbf{V}_1 + a_2 \mathbf{V}_2 + a_3 \mathbf{V}_3 = \mathbf{W}.
$$

In component form, this translates to a system of $n$ equations:

$$
\begin{cases}
a_1 v_{11} + a_2 v_{21} + a_3 v_{31} = w_1, \\
a_1 v_{12} + a_2 v_{22} + a_3 v_{32} = w_2, \\
\vdots \\
a_1 v_{1n} + a_2 v_{2n} + a_3 v_{3n} = w_n.
\end{cases}
$$

This can be concisely written in matrix form:

$$
\begin{bmatrix}
v_{11} & v_{21} & v_{31} \\
v_{12} & v_{22} & v_{32} \\
\vdots & \vdots & \vdots \\
v_{1n} & v_{2n} & v_{3n}
\end{bmatrix}
\begin{bmatrix}
a_1 \\ a_2 \\ a_3
\end{bmatrix}
=
\begin{bmatrix}
w_1 \\ w_2 \\ \vdots \\ w_n
\end{bmatrix}.
$$

---

Solving the System: Methods and Considerations

1. Check for Consistency

The first step is to determine if the system has a solution:

- If the augmented matrix has the same rank as the coefficient matrix, solutions exist.
- If not, $\mathbf{W}$ does not lie in the subspace spanned by $\mathbf{V}_1, \mathbf{V}_2, \mathbf{V}_3$.

2. Use Gaussian Elimination

Apply Gaussian elimination to reduce the matrix to row echelon form, making it straightforward to check for solutions.

- If solutions exist, proceed to find the particular solution(s) for $a_1, a_2, a_3$.
- If the system has infinitely many solutions or a unique solution, $\mathbf{W}$ is in the subspace.

3. Verify the Solution

Once the scalars $a_1, a_2, a_3$ are obtained, verify by substituting back into the linear combination to ensure it reproduces $\mathbf{W}$.

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Practical Example: A Concrete Illustration

Let's illustrate with specific vectors in $\mathbb{R}^3$:

$$
\mathbf{V}_1 = (1, 0, 2),
$$
$$
\mathbf{V}_2 = (0, 1, 3),
$$
$$
\mathbf{V}_3 = (1, 1, 4),
$$
$$
\mathbf{W} = (3, 2, 9).
$$

Step 1: Set up the system:

$$
a_1 (1, 0, 2) + a_2 (0, 1, 3) + a_3 (1, 1, 4) = (3, 2, 9).
$$

Component-wise:

$$
\begin{cases}
a_1 + 0 + a_3 = 3, \\
0 + a_2 + a_3 = 2, \\
2a_1 + 3a_2 + 4a_3 = 9.
\end{cases}
$$

Step 2: Solve the system:

From the first equation:

$$
a_1 = 3 - a_3,
$$

from the second:

$$
a_2 = 2 - a_3.
$$

Substitute into the third:

$$
2(3 - a_3) + 3(2 - a_3) + 4a_3 = 9,
$$
$$
6 - 2a_3 + 6 - 3a_3 + 4a_3 = 9,
$$
\[
(6 + 6) + (-2a_3 -

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