Solve The Given System Of Differential Equations By Systematic Elimination. 2 Dx/dt- 6x + Dy/dt = E^tdx/dt

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When tackling systems of differential equations, one of the most effective methods is systematic elimination. This technique simplifies complex systems by eliminating variables step-by-step, allowing us to solve for one variable at a time and ultimately find the solution to the entire system. In this article, we will explore how to solve the given system of differential equations using systematic elimination, providing detailed explanations, step-by-step procedures, and practical insights.

Understanding the Given System of Differential Equations

Before diving into the solution process, it’s essential to understand the structure and components of the system:

1. Equation: 2 Dx/dt - 6x + Dy/dt = E^t dx/dt
2. Variables involved: x(t) and y(t)
3. Derivatives: dx/dt and dy/dt
4. Parameters: constants like 2, -6, and the exponential term E^t

The goal is to find explicit functions x(t) and y(t) that satisfy this system.

Rewriting the Equation for Clarity

Let’s examine the given differential equation:

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2 dx/dt - 6x + dy/dt = e^t  dx/dt
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Bringing all terms to one side:

$$2 \, dx/dt - 6x + dy/dt - e^t \, dx/dt = 0$$

Group derivatives of dx/dt:

$$(2 \, dx/dt - e^t \, dx/dt) + dy/dt - 6x = 0$$

Factor dx/dt:

$$(2 - e^t) \, dx/dt + dy/dt - 6x = 0$$

This form is more manageable and sets the stage for elimination.

Step-by-Step Solution Using Systematic Elimination

While the original system involves only one equation, for systematic elimination, we typically require at least two equations. If the problem provides only one equation, it suggests that the system might be part of a larger set, or we may need to introduce additional equations or variables to proceed.

Assumption: To demonstrate the elimination method comprehensively, assume the system includes a second differential equation involving x and y. For example, suppose:

$$dy/dt = m x + n y$$

where m and n are constants.

Alternatively, if only one equation is given, we can interpret it as the combined form of a larger system or derive an auxiliary relation.

For this explanation, let’s consider the following system:

1. (Original Equation): $(2 - e^t) \, dx/dt + dy/dt = 6x$
2. Auxiliary Equation: $dy/dt = m x + n y$

Our goal is to eliminate y or dy/dt to find an equation involving only x and its derivatives.

Step 1: Express dy/dt from the second equation

From the auxiliary equation:

$$dy/dt = m x + n y$$

Step 2: Substitute dy/dt into the first equation

Replace dy/dt in the first equation:

$$(2 - e^t) dx/dt + m x + n y = 6x$$

Rearranged as:

$$(2 - e^t) dx/dt + n y = 6x - m x$$

Step 3: Express y in terms of x and dy/dt

From the second equation:

$$dy/dt = m x + n y$$

Rearranged as:

$$n y = dy/dt - m x$$

So,

$$y = (dy/dt - m x) / n$$

Substitute this into the previous equation to eliminate y:

$$(2 - e^t) dx/dt + n [(dy/dt - m x) / n] = 6x - m x$$

Simplify:

$$(2 - e^t) dx/dt + dy/dt - m x = 6x - m x$$

Now, note that (6x - m x) on the right is simply (6 - m) x:

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(2 - e^t) dx/dt + dy/dt - m x = (6 - m) x
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Step 4: Use the relation for dy/dt to eliminate dy/dt

From earlier, dy/dt = m x + n y. But since y is expressed in terms of dy/dt, this suggests a differential equation for x alone.

Alternatively, if the system is more straightforward, and only the single equation is available, we can treat it as a differential equation in x(t) and attempt to solve it directly.

Step 5: Solving the simplified differential equation

Recall the simplified form:

```
(2 - e^t) dx/dt + dy/dt = 6x
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Suppose we consider the original equation as a relation between dx/dt and y, and aim to express everything in terms of x and its derivatives.

Alternatively, if the original problem is solely:

```
2 dx/dt - 6x + dy/dt = e^t  dx/dt
```

We can treat dy/dt as an unknown and express it in terms of x and dx/dt.

Rearranged:

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dy/dt = e^t dx/dt - 2 dx/dt + 6x
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Thus,

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dy/dt = (e^t - 2) dx/dt + 6x
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Eliminating y to Derive a Second-Order Differential Equation

Given the relation for dy/dt, if we differentiate y(t):

$$\frac{dy}{dt} = (e^t - 2) \frac{dx}{dt} + 6x$$

then,

$$\frac{d^2 y}{dt^2} = \frac{d}{dt} \left[(e^t - 2) \frac{dx}{dt} + 6x \right]$$

Applying the product rule:

$$\frac{d^2 y}{dt^2} = \frac{d}{dt} (e^t - 2) \frac{dx}{dt} + (e^t - 2) \frac{d^2 x}{dt^2} + 6 \frac{dx}{dt}$$

Calculate derivatives:

$$\frac{d}{dt} (e^t - 2) = e^t$$

So,

$$\frac{d^2 y}{dt^2} = e^t \frac{dx}{dt} + (e^t - 2) \frac{d^2 x}{dt^2} + 6 \frac{dx}{dt}$$

Now, if the original goal is to eliminate y entirely, and express everything in terms of x and its derivatives, this second-order differential equation can be used to find x(t), and subsequently y(t).

Summary of the Systematic Elimination Approach

In conclusion, solving a system of differential equations via systematic elimination generally involves:

- Rewriting equations to express derivatives of one variable in terms of others.
- Substituting these expressions into other equations to eliminate variables.
- Deriving a higher-order differential equation involving a single variable.
- Solving the resulting differential equation using standard methods.
- Back-substituting to find the eliminated variables.

Key Techniques Used in Systematic Elimination

- Expressing derivatives: Isolating dy/dt or dx/dt from one equation.
- Substitution: Replacing derivatives or variables with expressions from other equations.
- Differentiation: Differentiating relations to generate higher-order equations.

- Higher-order equations: Reducing the system to a single differential equation for one variable.
- Back-substitution: Solving for one variable and then finding others.

Practical Tips for Solving Systems of Differential Equations

- Always verify the form and order of the equations before starting.
- Simplify equations by factoring or grouping similar terms.
- Use substitution wisely to reduce the number of variables.
- When derivatives are involved in exponential or other functions, consider substitution methods or integrating factors.
- Keep track of initial conditions if provided to determine particular solutions.

Conclusion

Solving a system of differential equations by systematic elimination requires careful manipulation of derivatives and algebraic expressions. The process involves expressing one derivative or variable in terms of others, substituting, differentiating as needed, and reducing the system to a single differential equation. Once this is achieved, standard solution techniques can be applied, such as integrating factors, characteristic equations, or variation of parameters.

The example provided demonstrates how to approach such problems methodically, emphasizing the importance of clarity, algebraic manipulation, and strategic substitution. Whether dealing with linear or nonlinear systems, the elimination method remains a powerful tool in the mathematician’s toolkit, enabling the resolution of complex differential systems efficiently.

Note: The actual solution for specific initial conditions or parameter values may involve additional steps, including solving the resulting differential equations explicitly.

Frequently Asked Questions

What is the main goal when solving a system of

differential equations by systematic elimination?

The main goal is to eliminate one variable to reduce the system to a single differential equation, making it easier to solve for the remaining variable.

How do you identify the coefficients to use for elimination in the given system?

Identify the coefficients of the derivatives in each equation and manipulate the equations (by multiplying or dividing) to align these coefficients for elimination, such as making the derivatives' coefficients equal or opposites.

What steps are involved in solving the system 2 dx/dt - 6x + dy/dt = e^t and another related equation?

First, express one derivative in terms of the other, then multiply equations to align coefficients, add or subtract equations to eliminate one variable, and finally solve the resulting differential equation.

Can you explain how to eliminate dy/dt from the given system to find an equation solely in terms of x?

Yes, by manipulating the equations so that the coefficients of dy/dt cancel out, typically by multiplying one equation by a suitable factor and subtracting, leaving an equation only in terms of x and its derivatives.

What role does substitution play after elimination in solving the system?

Once one variable is eliminated, substitution is used to solve the resulting differential equation for the remaining variable, then back-substitute to find the other variable.

Is the systematic elimination method applicable to nonlinear systems of differential equations?

Systematic elimination is primarily effective for linear systems; nonlinear systems often require different methods like substitution, integrating factors, or numerical approaches.

How do you verify the solution obtained after elimination?

Substitute the solutions for x and y back into the original system equations to check if both equations are satisfied, confirming the correctness of the solutions.

What are common challenges faced when solving systems of differential equations by elimination?

Challenges include correctly aligning coefficients, dealing with complex expressions after elimination, and ensuring the solutions satisfy all original equations.

Can you briefly summarize the process of solving 2 dx/dt - 6x + dy/dt = e^t using systematic elimination?

Yes. First, isolate dy/dt from the first equation, then substitute into the second or manipulate equations to eliminate dy/dt, resulting in a differential equation in x alone. Solve for x(t), then back-substitute to find y(t).

Additional Resources

Solve The Given System Of Differential Equations By Systematic Elimination. 2 Dx/dt- 6x + Dy/dt = E^tdx/dt

In the realm of differential equations, solving systems efficiently and accurately is a fundamental skill that finds applications across engineering, physics, and applied mathematics. Today, we delve into a particular system of differential equations:

2 Dx/dt - 6x + Dy/dt = E^t dx/dt

This equation presents a fascinating challenge: it combines derivatives of multiple variables with exponential functions, demanding a systematic approach to eliminate variables and solve for the unknowns. Our goal is to understand how to approach such a system methodically—using the technique known as systematic elimination. Through clear explanations and step-by-step procedures, we'll explore how to simplify and solve this complex system, providing insights that can be applied to similar problems in mathematical modeling and analysis.

Understanding the System of Differential Equations

Before diving into the solution process, it's crucial to interpret the given equation correctly and understand its structure. The equation can be written as:

2 dx/dt - 6x + dy/dt = e^t dx/dt

At first glance, the equation involves derivatives of two functions, x(t) and y(t), as well as exponential functions of t. It appears to be a single equation involving both dx/dt and dy/dt, with coefficients and exponential terms.

Key observations:

- The equation involves derivatives of x and y:
- dx/dt appears twice, once multiplied by 2 and once multiplied by e^t.
- dy/dt appears once, multiplied by 1.
- The equation is linear in derivatives, which suggests that if we have another equation involving these derivatives, we can form a system suitable for elimination.

Important note: The problem statement mentions "Solve the Given System of Differential Equations," implying that there's likely an implicit system structure. For clarity, we will treat the given as part of a system involving two equations:

1. 2 dx/dt - 6x + dy/dt = e^t dx/dt
2. (Possibly another related equation; if not explicitly provided, we may consider an auxiliary relation or assume the given as a combined form).

In the absence of a second explicit equation, the most natural approach is to interpret the given as a single equation involving two unknown functions and their derivatives, and to manipulate it to eliminate one derivative or variable. Alternatively, if the problem is to handle this as a single equation with derivatives, the key is to express all derivatives in a common form, then eliminate one variable or derivative to solve for the other.

Rearranging the Equation for Clarity

The first step in systematic elimination is to rewrite the equation in a more manageable form, isolating derivatives and grouping like terms.

Given:

2 dx/dt - 6x + dy/dt = e^t dx/dt

Let's bring all derivatives to one side:

2 dx/dt - e^t dx/dt + dy/dt = 6x

Factor the dx/dt terms:

(2 - e^t) dx/dt + dy/dt = 6x

Now, the equation is:

(2 - e^t) dx/dt + dy/dt = 6x

This form makes it clear that the derivatives are linearly combined, and the coefficients depend on t (through e^t).

Formulating the System for Systematic Elimination

To apply elimination, it's helpful to consider the derivatives as variables:

Let’s define:

- $X(t) = x(t)$
- $Y(t) = y(t)$

Our goal is to eliminate one of the derivatives or variables, leading to an equation solely in terms of one variable and its derivatives.

Suppose we have an auxiliary relation or an additional equation involving Y and its derivatives (which is typical in such systems). If not explicitly given, perhaps the problem is to manipulate the existing equation to express one derivative in terms of the other.

Step 1: Express dy/dt in terms of dx/dt and x(t):

From the rearranged equation:

dy/dt = 6x - (2 - e^t) dx/dt

This relation links dy/dt to dx/dt and x(t). To eliminate y(t), we need to differentiate or integrate as appropriate, or to find a second relation involving y(t).

However, if the problem restricts us to this single equation, the key is to eliminate either dx/dt or y(t).

Applying Systematic Elimination Technique

The core idea of systematic elimination is to:

- Obtain two equations involving the derivatives of the variables.
- Use algebraic manipulation to eliminate one of the derivatives, resulting in a differential equation in a single variable.

In our case, since only one equation is provided, and it involves both derivatives, we can proceed as follows:

Method 1: Differentiate the equation to generate a second relation

Differentiate the entire equation with respect to t:

d/dt [(2 - e^t) dx/dt + dy/dt] = d/dt [6x]

Let's compute each term:

- $d/dt [(2 - e^t) dx/dt] = d/dt (2 - e^t) dx/dt + (2 - e^t) d^2x/dt^2$

Since $d/dt (2 - e^t) = - e^t$

So,

$d/dt [(2 - e^t) dx/dt] = - e^t dx/dt + (2 - e^t) d^2x/dt^2$

- $d/dt [dy/dt] = d^2 y/dt^2$

- $d/dt [6x] = 6 dx/dt$

Putting it all together:

$- e^t dx/dt + (2 - e^t) d^2x/dt^2 + d^2 y/dt^2 = 6 dx/dt$

Now, rearranged:

$(2 - e^t) d^2x/dt^2 + d^2 y/dt^2 = 6 dx/dt + e^t dx/dt = (6 + e^t) dx/dt$

This second relation involves second derivatives of x and y. If we can express y(t) or its derivatives in terms of x(t) and its derivatives, we could eliminate y altogether.

Method 2: Express y(t) in terms of x(t)

From earlier:

$dy/dt = 6x - (2 - e^t) dx/dt$

Integrate or differentiate as needed to eliminate y.

Step-by-Step Solution Process

Given the above, here is a systematic approach:

Step 1: Express dy/dt in terms of x(t) and dx/dt

$dy/dt = 6x - (2 - e^t) dx/dt$

Step 2: Differentiate dy/dt to find $d^2 y/dt^2$

$d^2 y/dt^2 = d/dt [6x - (2 - e^t) dx/dt]$

Calculate:

$d^2 y/dt^2 = 6 dx/dt - d/dt [(2 - e^t) dx/dt]$

Again, as before:

$d/dt [(2 - e^t) dx/dt] = - e^t dx/dt + (2 - e^t) d^2 x/dt^2$

Therefore,

$d^2 y/dt^2 = 6 dx/dt - [- e^t dx/dt + (2 - e^t) d^2 x/dt^2] = 6 dx/dt + e^t dx/dt - (2 - e^t) d^2 x/dt^2$

Now, substitute this into the second derivative expression from earlier:

$(2 - e^t) d^2 x/dt^2 + d^2 y/dt^2 = (6 + e^t) dx/dt$

Plugging in $d^2 y/dt^2$:

$(2 - e^t) d^2 x/dt^2 + [6 dx/dt + e^t dx/dt - (2 - e^t) d^2 x/dt^2] = (6 + e^t) dx/dt$

Notice that $(2 - e^t) d^2 x/dt^2$ cancels with $- (2 - e^t) d^2 x/dt^2$, leaving:

$6 dx/dt + e^t dx/dt = (6 + e^t) dx/dt$

Simplify the left:

$(6 + e^t) dx/dt = (6 + e^t) dx/dt$

Thus, the relation is consistent, confirming the internal consistency of the derivatives.

Step 3: Formulate a single differential equation in x(t)

Now, revisit the expression for dy/dt:

$dy/dt = 6x - (2 - e^t) dx/dt$

Suppose we aim to find a differential equation solely in x(t). Let's consider the original relation:

$(2 - e^t) dx/dt + dy/dt = 6x$

Substituting dy/dt:

(2 - e^t) dx/dt + [6x

[Solve The Given System Of Differential Equations By Systematic Elimination 2 Dx Dt 6x Dy Dt E Tdx Dt]

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