The Equation X - 4x + Y2-2y = 16 Describes A Circle. What Is The Y-coordinate Of The Center Of The Circle?

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Understanding the geometric significance of equations is fundamental in algebra and coordinate geometry. When an equation represents a circle, it often provides essential details about its center and radius. In this article, we will analyze the given equation, transform it into the standard form of a circle, and determine the y-coordinate of its center. This process involves algebraic manipulations, completing the square, and understanding the geometric implications of the equation.

Understanding the Given Equation

The initial step is to interpret the provided equation:

Equation: $X - 4x + Y2 - 2y = 16$

At first glance, this equation appears to contain some notation inconsistencies or typographical errors, particularly with the variables. Let's clarify the notation:

- The variable X appears, but then similarly, -4x is present. It’s likely that X and x are intended to be the same variable.
- The term Y2 probably represents Y^2 (Y squared).
- The term -2y is straightforward, involving the variable y.

Assuming the variables are consistent and the notation is as follows:

Corrected Equation: $x - 4x + y^2 - 2y = 16$

Alternatively, if the initial notation was intended differently, it might be:

$x - 4x + y^2 - 2y = 16$

which simplifies to:

$(-3x) + y^2 - 2y = 16$

However, this form suggests the equation is not symmetric in x and y and might not directly represent a circle unless further manipulation reveals so.

Alternatively, perhaps the original equation is:

$x^2 - 4x + y^2 - 2y = 16$

This is a common form for circle equations, involving quadratic terms.

Given the context of the question, which is about a circle, the most logical assumption is that the original equation was intended as:

$x^2 - 4x + y^2 - 2y = 16$

This form aligns with the standard form of a circle equation:

$(x - h)^2 + (y - k)^2 = r^2$

where (h, k) is the center, and r is the radius.

Therefore, for the purpose of this article, we will proceed with the equation:

$x^2 - 4x + y^2 - 2y = 16$

Transforming the Equation into Standard Form

To identify the center and radius of the circle, we need to rewrite the given quadratic equation into the standard form:

$(x - h)^2 + (y - k)^2 = r^2$

This involves completing the square for both x and y terms.

Step 1: Group x and y terms

Rewrite the equation as:

$(x^2 - 4x) + (y^2 - 2y) = 16$

Step 2: Complete the square for x terms

- Take the coefficient of x: -4
- Half of -4 is -2

- Square -2: $(-2)^2 = 4$

Add and subtract 4 within the x terms:

$(x^2 - 4x + 4) - 4 + (y^2 - 2y) = 16$

Similarly, complete the square for y:

- Coefficient of y: -2
- Half of -2 is -1
- Square -1: 1

Add and subtract 1:

$(x^2 - 4x + 4) - 4 + (y^2 - 2y + 1) - 1 = 16$

Now, rewrite as:

$(x - 2)^2 - 4 + (y - 1)^2 - 1 = 16$

Step 3: Simplify the equation

Combine the constants:

$(x - 2)^2 + (y - 1)^2 - 4 - 1 = 16$

which simplifies to:

$(x - 2)^2 + (y - 1)^2 = 16 + 5$

$(x - 2)^2 + (y - 1)^2 = 21$

Identifying the Center and Radius of the Circle

The equation in standard form:

$(x - 2)^2 + (y - 1)^2 = 21$

provides immediate information:

- The center of the circle is at $(h, k) = (2, 1)$
- The radius r is $\sqrt{21}$

This confirms that the original equation indeed describes a circle with a specific center and radius.

The Y-Coordinate of the Center of the Circle

From the standard form, the center's coordinates are:

- x-coordinate: 2
- y-coordinate: 1

Therefore, the y-coordinate of the circle's center is 1.

Additional Insights into Circle Equations

Understanding the structure of circle equations is essential in coordinate geometry. Let's explore some key concepts related to the standard form and how to interpret various parameters.

Standard Equation of a Circle

The general form:

$(x - h)^2 + (y - k)^2 = r^2$

- (h, k): Coordinates of the center
- r: Radius of the circle

This form makes it straightforward to identify the circle's position and size.

Implications of the Standard Form

- The center is located exactly at (h, k).
- The radius is the square root of the constant on the right side of the equation.
- If the equation is not in this form, algebraic manipulation (completing the square) can be used to convert it.

Practical Applications of Circle Equations

Understanding the center and radius of a circle has numerous practical applications across various fields:

- **Engineering:** Designing circular components and structures.
- **Navigation:** Determining the position of objects relative to a fixed point.
- **Computer Graphics:** Rendering circles and curved shapes.
- **Physics:** Analyzing orbital paths and circular motion.

In all these applications, accurate identification of the circle's center and radius is crucial.

Summary and Key Takeaways

- The given quadratic equation (assumed to be x^2 - 4x + y^2 - 2y = 16) describes a circle.
- Completing the square for both x and y terms rewrites it into standard form.
- The standard form reveals the circle's center at (2, 1) and radius √21.
- The y-coordinate of the circle’s center is 1.

Final Notes

When analyzing equations that describe circles, always look to rewrite them into the standard form by completing the square. This process makes it easy to identify the key features of the circle, especially the center and radius. Recognizing the structure and applying algebraic techniques are vital skills in geometry and related disciplines.

In conclusion, the y-coordinate of the center of the circle described by the equation is 1. This insight helps in understanding the geometric placement of the circle within the coordinate plane and can serve as a foundation for further geometric or algebraic analysis.

Frequently Asked Questions

What is the general form of the equation describing a circle in the given problem?

The given equation is in the form X - 4x + Y^2 - 2y = 16, which can be rearranged to identify the circle's center and radius.

How do you find the y-coordinate of the center of the circle from the given equation?

By rewriting the equation in the standard form of a circle (completing the square for y), the y-coordinate of the center is the value that completes the square for the y-term.

What steps are involved in converting the given equation into standard form to identify the center?

First, rearrange the equation to group y-terms, then complete the square for y, and similarly for x if needed, to find the center coordinates.

What is the y-coordinate of the center of the circle described by the equation?

The y-coordinate of the center is 1, obtained by completing the square for the y-terms.

Why is completing the square necessary in identifying the center of the circle from the equation?

Completing the square transforms the equation into standard form, revealing the center's coordinates directly.

Additional Resources

The Equation X - 4x + Y2-2y = 16 Describes A Circle. What Is The Y-coordinate Of The Center Of The Circle?

Understanding the fundamental properties of geometric figures is a cornerstone of algebra and coordinate geometry. Among these figures, the circle holds a special place due to its symmetry and applications across various fields—from engineering to computer graphics. When presented with an equation that describes a circle, one of the key tasks is to identify its center and radius. This article delves into a specific equation, clarifying how to interpret it and, most importantly, how to find the y-coordinate of the circle's center.

Deciphering the Equation: From the Given Form to Standard Form

The initial challenge with the equation X - 4x + Y2 - 2y = 16 lies in understanding its structure. At first glance, it appears somewhat unconventional, possibly due to typographical inconsistencies. To analyze it effectively, we need to interpret and rewrite it in a recognizable form.

Step 1: Clarify the Equation

Given the notation, it’s reasonable to assume that:

- X and Y are variables, but the capitalizations suggest possible typographical errors or alternative notation.
- The term Y2 likely represents Y^2, i.e., Y squared.
- Similarly, the presence of X and x suggests a mix-up; perhaps the original intended equation is:

$x^2 - 4x + y^2 - 2y = 16$

This form is common in coordinate geometry when describing a circle.

Step 2: Recognize the Standard Form of a Circle

A circle's equation in standard form is:

$(x - h)^2 + (y - k)^2 = r^2$

where:

- (h, k) is the center of the circle.
- r is the radius.

Our goal is to manipulate the given equation into this form to identify the center’s coordinates, specifically the y-coordinate.

Rewriting the Equation in Standard Form

Assuming the corrected version:

$x^2 - 4x + y^2 - 2y = 16$

we proceed with completing the square for both x and y terms.

Step 1: Group x and y terms

$(x^2 - 4x) + (y^2 - 2y) = 16$

Step 2: Complete the square for each group

- For $x^2 - 4x$:

Add and subtract \((\frac{-4}{2})^2 = 4\)

\[
x^2 - 4x + 4 - 4
\]
which simplifies to:
\[
(x - 2)^2 - 4
\]

- For $y^2 - 2y$:

Add and subtract \((\frac{-2}{2})^2 = 1\)

\[
y^2 - 2y + 1 - 1
\]
which simplifies to:
\[
(y - 1)^2 - 1
\]

Step 3: Rewrite the entire equation

\[
(x - 2)^2 - 4 + (y - 1)^2 - 1 = 16
\]

Combine constants:

\[
(x - 2)^2 + (y - 1)^2 - 5 = 16
\]

Add 5 to both sides:

\[
(x - 2)^2 + (y - 1)^2 = 21
\]

Result:

The equation in standard form:

\[

$$
(x - 2)^2 + (y - 1)^2 = 21
$$

Identifying the Center and Radius of the Circle

From the standard form:

$$
(x - h)^2 + (y - k)^2 = r^2
$$

we observe:

- Center (h, k): (2, 1)
- Radius (r): $\sqrt{21}$

The critical part of the question is to determine the y-coordinate of the center. From the above, it's straightforward: k = 1.

Understanding the Significance of the Center's Coordinates

The center of a circle provides vital geometric information:

- It is the point equidistant from all points on the circle.
- Its coordinates define the position of the circle relative to the coordinate axes.
- The y-coordinate, in particular, indicates the vertical position of the circle's center.

In our case, the y-coordinate is 1, which means the circle is centered one unit above the x-axis.

Why Is the Y-coordinate Important?

Knowing the y-coordinate of the center allows for several practical and theoretical applications:

- Graphical Representation: When plotting the circle, knowing the center's y-coordinate helps accurately place the circle on the coordinate plane.
- Distance Calculations: If analyzing the circle's position relative to other figures or points, the center's y-coordinate is essential.
- Transformations: Shifting or translating the circle vertically depends on knowing the y-coordinate.
- Problem Solving: Many geometric problems involve the circle's center; for example, determining the circle's position relative to other shapes or points.

Additional Insights and Common Mistakes

While the process appears straightforward, several common pitfalls can confuse students and practitioners:

- Misinterpreting the Equation: Ensuring the correct form (standard vs. general) is vital. The initial confusion over symbols like Y2 underscores the importance of clarity.

- Completing the Square: Forgetting to add and subtract the same value when completing the square can lead to incorrect conclusions.

- Ignoring Constants: Failing to move constants to the right side of the equation after completing the square can cause miscalculations.

- Sign Errors: When completing the square, watch out for signs; for example, the coefficient of y is negative, but the process remains the same.

Summary and Final Remarks

By carefully analyzing the given equation, correcting typographical errors, and completing the square, we identified the circle's center as (2, 1) and its radius as \(\sqrt{21}\). The y-coordinate of the center is therefore 1.

Understanding how to transform the general or expanded form of a circle's equation into its standard form is a fundamental skill in algebra and geometry. It not only allows for precise identification of the circle's key features but also enhances problem-solving efficiency across many mathematical contexts.

In conclusion, the y-coordinate of the center of the circle described by the equation $x^2 - 4x + y^2 - 2y = 16$ is 1. This insight demonstrates the power of algebraic manipulation and the importance of mastering the techniques of completing the square and standard form conversion—tools that are indispensable in the realm of coordinate geometry.

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