The Function F Is Given By F(x)=4x^3x^4. On What Intervals Is The Graph Of Ff Concave Up?(A) (-infinity,0)

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Understanding the concavity of a function is essential in calculus as it provides insight into the graph's shape, indicating where the function is curving upward or downward. In this article, we will analyze the function $F(x) = 4x^3x^4$, explore its derivatives, and determine the intervals where the graph of F is concave up, with a focus on the interval $(-\infty, 0)$.

Understanding the Function $F(x) = 4x^3x^4$

Simplifying the Function

The given function appears as $F(x) = 4x^3x^4$. Recognizing that this is a product of two powers of x, we can simplify it:

$$
F(x) = 4 \times x^3 \times x^4 = 4 \times x^{3+4} = 4x^7
$$

Thus, the function simplifies to:

$$
F(x) = 4x^7
$$

This simplification makes analyzing the function's properties much easier.

Analyzing the Function $F(x) = 4x^7$

First Derivative $F'(x)$

The first derivative indicates the slope of the tangent to the graph at any point x, revealing where the function is increasing or decreasing.

$$
F'(x) = \frac{d}{dx} (4x^7) = 4 \times 7 x^{6} = 28x^{6}
$$

Since x^6 is always non-negative, and multiplied by 28 (a positive constant), the first derivative $F'(x)$ is:

$$
F'(x) = 28x^{6} \geq 0, \quad \forall x \in \mathbb{R}
$$

This means the function $F(x)$ is non-decreasing everywhere. It is flat at $x=0$ (since $F'(0) = 0$), and increasing elsewhere.

Second Derivative $F''(x)$

The second derivative tells us about the concavity of the function:

$$
F''(x) = \frac{d}{dx} (28x^{6}) = 28 \times 6 x^{5} = 168x^{5}
$$

The sign of $F''(x)$ determines the concavity:
- $F''(x) > 0$ implies the graph is concave up.
- $F''(x) < 0$ implies the graph is concave down.

Given $F''(x) = 168x^{5}$, the sign depends on x^{5}:

- For $x > 0$, $x^{5} > 0$, so $F''(x) > 0$.
- For $x < 0$, $x^{5} < 0$, so $F''(x) < 0$.
- At $x=0$, $F''(0) = 0$.

Determining Concavity of $F(x) = 4x^7$

Concave Up and Concave Down Regions

- The function is concave up where $F''(x) > 0$, which occurs for $x > 0$.
- The function is concave down where $F''(x) < 0$, which occurs for $x < 0$.
- At $x=0$, the second derivative is zero, indicating potential an inflection point, but further analysis is required to confirm.

Focus on the Interval $(-\infty, 0)$

Since we're interested in the interval $(-\infty, 0)$:

- For all $x < 0$, $F''(x) = 168 x^{5} < 0$.
- Therefore, the graph of F is concave down on $(-\infty, 0)$.

Conclusion: On the interval $(-\infty, 0)$, the graph of F is concave down, not concave up.

Summary of Concavity for $F(x) = 4x^7$

Interval	Sign of $F''(x)$	Concavity	Description
$(-\infty, 0)$	Negative ($168x^{5}<0$)	Concave down	The graph curves downward
$(0, +\infty)$	Positive ($168x^{5}>0$)	Concave up	The graph curves upward
$x=0$	Zero ($F''(0)=0$)	Inflection point	The curvature changes at zero

Implications for Graph Shape and Behavior

Behavior on $(-\infty, 0)$

Since the second derivative is negative in this interval, the graph is concave down. This indicates that the function's slope is decreasing as x approaches zero from the left, and the graph is curving downward.

Behavior on $(0, +\infty)$

In this region, the graph is concave up, with the slope increasing as x increases. The graph transitions from concave down to concave up at $x=0$, which is an inflection point.

Additional Considerations and Visualizing the Graph

Plotting the Function $F(x) = 4x^7$

When plotting $F(x) = 4x^7$, the key features to observe include:

- The point at $x=0$, where the function crosses the origin.
- The behavior as $x \to -\infty$, where $F(x) \to -\infty$.
- The behavior as $x \to +\infty$, where $F(x) \to +\infty$.
- The shape change at $x=0$ where the concavity switches.

Inflection Point at $x=0$

The second derivative test shows that $x=0$ is a point of inflection, where the concavity changes from down to up. The function transitions from concave down on the negative side to concave up on the positive side.

Summary and Final Answer

Based on the detailed analysis:

- The simplified form of the given function is $F(x) = 4x^7$.
- The second derivative $F''(x) = 168x^{5}$ determines the concavity:
- For $x < 0$, $F''(x) < 0$, so the graph is concave down.
- For $x > 0$, $F''(x) > 0$, so the graph is concave up.
- At $x=0$, the second derivative is zero, indicating an inflection point.

Answer to the question:
On the interval $(-\infty, 0)$, the graph of F is concave down. Therefore, it is not concave up on $(-\infty, 0)$.

If the question asks specifically for intervals where F is concave up, that would be for $x > 0$.

Additional Resources for Calculus Students

- Understanding Concavity and Inflection Points: Mastering the second derivative test is crucial for analyzing the shape of graphs.
- Graphing Polynomial Functions: Practice plotting functions like x^7 and their transformations to build intuition.
- Application in Real-World Contexts: Concavity indicates acceleration in physical models, such as velocity and displacement graphs.

Conclusion

In conclusion, the function $F(x) = 4x^7$ is concave down on $(-\infty, 0)$ and concave up on $(0, \infty)$. The interval $(-\infty, 0)$ specifically exhibits downward curvature, highlighting the importance of the second derivative in analyzing the behavior of functions. Understanding these principles enhances our ability to interpret the graphical behavior of functions and their applications across various fields in mathematics and science.

Frequently Asked Questions

What is the given function F(x) in the problem?

The function is $F(x) = 4x^3 x^4$.

How can I simplify the function $F(x) = 4x^3 x^4$?

Since both are powers of x, you can combine them: $F(x) = 4x^{3+4} = 4x^7$.

What is the first step to find where the graph of F(x) is concave up?

First, find the second derivative of F(x) to analyze its concavity.

What is the second derivative of $F(x) = 4x^7$?

$F'(x) = 28x^6$, and $F''(x) = 168x^5$.

On which intervals is the function concave up based on the second derivative?

Since $F''(x) = 168x^5$, it is positive when $x > 0$, so the graph is concave up on $(0, \infty)$.

What does the given answer (A) (-infinity, 0) imply about the concavity?

It suggests that the graph is concave up on $(-\infty, 0)$, but based on the second derivative, it is actually concave up on $(0, \infty)$.

What is the correct interval where the graph of F(x) is concave up?

The graph of $F(x) = 4x^7$ is concave up on $(0, \infty)$.

Additional Resources

Understanding the Concavity of the Function F(x) = 4x^3x^4

When analyzing the behavior of the function F(x) = 4x^3x^4, particularly in the context of concavity, it's essential to understand how the function curves and where its graph is "concave up" or "concave down." In this guide, we'll explore the concept of concavity, how to determine the intervals where the function is concave up, and provide a step-by-step process for analyzing F(x) = 4x^3x^4.

Breaking Down the Function F(x) = 4x^3x^4

Before diving into concavity, let's simplify the function:

- Original Function:

F(x) = 4x^3x^4

- Simplification:

Since both factors involve powers of x, combine them:

F(x) = 4 x^3 x^4 = 4 x^(3 + 4) = 4x^7

This simplified form makes the analysis of derivatives more straightforward. The function is now:

F(x) = 4x^7

Understanding Concavity and Its Significance

Concavity relates to the curvature of the graph of a function:

- Concave Up: The graph opens upward, like a cup. The second derivative is positive (F''(x) > 0).
- Concave Down: The graph opens downward, like an upside-down cup. The second derivative is negative (F''(x) < 0).

Determining where the function is concave up involves calculating the second derivative and analyzing its sign over different intervals.

Step-by-Step Analysis of F(x) = 4x^7

Step 1: Find the First Derivative F'(x)

Given:

$F(x) = 4x^7$

Differentiating:

$F'(x) = d/dx [4x^7] = 4 \; 7x^6 = 28x^6$

Step 2: Find the Second Derivative F''(x)

Differentiating F'(x):

$F''(x) = d/dx [28x^6] = 28 \; 6x^5 = 168x^5$

Step 3: Analyze the Second Derivative F''(x)

The sign of F''(x) determines concavity:

- $F''(x) = 168x^5$

Since 168 is positive, the sign depends solely on x^5:

- For $x > 0$: $x^5 > 0 \rightarrow F''(x) > 0$
- For $x < 0$: $x^5 < 0 \rightarrow F''(x) < 0$
- For $x = 0$: $F''(0) = 0$

Step 4: Identify Intervals Where $F''(x) > 0$

- $F''(x) > 0$ when $x > 0$.
- Therefore, the graph of $F(x) = 4x^7$ is concave up on $(0, \infty)$.

Step 5: Confirm the Concavity Behavior

- For $x < 0$: $F''(x) < 0$, so the graph is concave down.
- At $x = 0$: $F''(x) = 0$, indicating potential inflection point.

Conclusion: Intervals Where F Is Concave Up

Based on the second derivative test, the function $F(x) = 4x^7$ is concave up precisely on the interval:

$(0, \infty)$

This means that for all positive values of x, the graph of the function opens upward, resembling the shape of a cup.

Addressing the Given Multiple Choice Option: $(-\infty, 0)$

The question references the interval $(-\infty, 0)$, which is where the function is concave down, given our analysis. The original problem appears to ask for the intervals where the graph is concave up, and the choice provided is:

> (A) $(-\infty, 0)$

However, based on the mathematical derivation, the correct interval for concave up is $(0, \infty)$, not $(-\infty, 0)$.

Summary of Key Points

- The function $F(x) = 4x^7$ simplifies from the original expression.
- Its derivatives are:
- $F'(x) = 28x^6$
- $F''(x) = 168x^5$
- The second derivative indicates the concavity:
- $F''(x) > 0$ when $x > 0$ → graph is concave up.
- $F''(x) < 0$ when $x < 0$ → graph is concave down.
- The point $x = 0$ is an inflection point where concavity changes.

Additional Tips for Analyzing Concavity

- Always simplify the function first if possible.
- Find the first and second derivatives carefully.
- Determine the critical points of the second derivative (where $F''(x) = 0$ or undefined).
- Test points in each interval to determine the sign of $F''(x)$.

- Remember that the interval where the second derivative is positive corresponds to concave up.

Final Remarks

Understanding the concavity of a function like F(x) = 4x^7 provides deeper insights into its geometric behavior and helps identify points of inflection and shape. In this case, the function is concave up on (0, ∞) and concave down on (-∞, 0). Recognizing these patterns is fundamental in calculus, especially when analyzing the curvature and optimizing functions.

In conclusion, the graph of F(x) = 4x^7 is concave up on the interval (0, ∞). The given multiple-choice option (-∞, 0) corresponds to the region where the graph is concave down, highlighting the importance of careful analysis when interpreting concavity and selecting the correct intervals.

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