The Graph Of F(x) = X2 Is Translated To Form G(x) = (x 5)2 + 1. On A Coordinate Plane, A Parabola, Labeled

The Graph Of F(x) = X^2 Is Translated To Form G(x) = (x - 5)^2 + 1. On A Coordinate Plane, A Parabola, Labeled

Understanding how functions transform on the coordinate plane is fundamental in algebra and calculus. In particular, analyzing the translation of the basic quadratic function f(x) = x^2 into a new function g(x) = (x - 5)^2 + 1 offers insight into how shifts affect the graph's position without changing its shape. This article explores the translation process, the resulting graph's features, and how to accurately plot and label the parabola on a coordinate plane for clear visualization and understanding.

Understanding the Basic Quadratic Function f(x) = x^2

The Standard Parabola

The function f(x) = x^2 is one of the most fundamental quadratic functions, producing a symmetric parabola opening upward on the coordinate plane. Its key features include:

- Vertex at the origin (0, 0)
- Axis of symmetry along the y-axis (x = 0)
- Y-intercept at (0, 0)
- Symmetric about the y-axis

Graph Characteristics

The graph of y = x^2 is a smooth, U-shaped curve. Its width and steepness are determined by the coefficient of x^2, which in this case is 1, giving a standard parabola.

Transforming f(x) = x^2 to g(x) = (x - 5)^2 + 1

Translation of the Graph

The function $g(x) = (x - 5)^2 + 1$ is a translated version of $f(x)$. The transformations involve shifting the original parabola horizontally and vertically.

- **Horizontal Shift:** The term $(x - 5)^2$ indicates a shift to the right by 5 units. In general, replacing x with $(x - h)$ shifts the graph right by h units if $h > 0$.
- **Vertical Shift:** The +1 outside the squared term shifts the graph upward by 1 unit. Adding a constant c to the function moves the graph vertically by c units.

Summary of the Transformation

The combined effect of these transformations results in a parabola that is shifted:

- 5 units to the right
- 1 unit upward

compared to the original $f(x) = x^2$.

Plotting the Transformed Parabola $g(x) = (x - 5)^2 + 1$

Key Points to Plot

Identifying specific points on the original parabola helps in plotting the new graph accurately.

- Vertex: The vertex of the transformed parabola is at $(h, k) = (5, 1)$.
- Additional points: To plot accurately, select x-values around the vertex and compute corresponding y-values.

Calculating Specific Points

Choose x-values around 5, such as 3, 4, 5, 6, 7, and substitute into $g(x)$:

- At $x = 3$: $g(3) = (3 - 5)^2 + 1 = (-2)^2 + 1 = 4 + 1 = 5$ → Point (3, 5)

- At x = 4: g(4) = (4 - 5)^2 + 1 = (-1)^2 + 1 = 1 + 1 = 2 → Point (4, 2)
- At x = 5: g(5) = (5 - 5)^2 + 1 = 0 + 1 = 1 → Vertex (5, 1)
- At x = 6: g(6) = (6 - 5)^2 + 1 = 1^2 + 1 = 1 + 1 = 2 → Point (6, 2)
- At x = 7: g(7) = (7 - 5)^2 + 1 = 2^2 + 1 = 4 + 1 = 5 → Point (7, 5)

Plotting the Parabola

Using the points calculated, plot the key points on the coordinate plane:

- (3, 5)
- (4, 2)
- (5, 1) (vertex)
- (6, 2)
- (7, 5)

Connect these points with a smooth, symmetrical curve that opens upward, ensuring the parabola's shape remains consistent.

Labeling the Graph for Clarity

Labeling Key Features

Clear labels enhance understanding when presenting the graph:

- **Vertex:** Label the point (5, 1) as "Vertex."
- **Axis of Symmetry:** Draw and label the line x = 5 as the axis of symmetry, highlighting the parabola's symmetry about this line.
- **Points:** Mark and label points (3, 5), (4, 2), (5, 1), (6, 2), and (7, 5).
- **Y-intercept:** Confirm the y-intercept at (0, 1) by calculating g(0) = (0 - 5)^2 + 1 = 25 + 1 = 26; thus, the y-intercept is at (0, 26).
- **X-intercepts:** Find x-intercepts by solving (x - 5)^2 + 1 = 0, which has no real solutions, indicating the parabola does not cross the x-axis.

Adding Descriptive Labels and Title

To make the graph educational and easy to interpret, include:

- A descriptive title such as "Graph of Translated Parabola $g(x) = (x - 5)^2 + 1$"
- Labels for axes (e.g., "x-axis" and "y-axis")
- Markers or dots at key points with labels for clarity

Understanding the Significance of Translations in Graphs

Real-World Applications

Translations of quadratic functions are used in various fields including physics, engineering, and economics to model real-world phenomena:

- Projectile motion, where the parabola represents the path of an object
- Optimizing profit or minimizing costs in economics, modeled via quadratic functions

Mathematical Insights

Understanding how to manipulate graphs through translations allows students and professionals to:

- Predict the movement of graphs in response to changes in equations
- Design functions to fit data by shifting and scaling
- Develop a deeper understanding of the relationships between algebraic expressions and their graphical representations

Summary

The transformation from $f(x) = x^2$ to $g(x) = (x - 5)^2 + 1$ exemplifies how horizontal and vertical shifts affect a parabola's position on the coordinate plane. Plotting and labeling the graph with key points such as the vertex, axis of symmetry, and intercepts enhance comprehension and visual clarity. Recognizing these transformations is essential for mastering quadratic functions and their applications across various disciplines.

By mastering the process of translating graphs, students can better understand the dynamic nature of functions and develop skills to analyze more complex transformations. Whether for academic purposes or practical applications, visualizing how functions shift helps demystify the algebraic expressions and their geometric counterparts on the coordinate plane.

Frequently Asked Questions

What is the original function in the given translation?

The original function is $f(x) = x^2$.

How has the graph of $f(x) = x^2$ been translated to form g(x)?

The graph has been shifted 5 units to the left and 1 unit upward.

What is the equation of the translated parabola g(x)?

$g(x) = (x + 5)^2 + 1$.

What do the transformations x + 5 and +1 represent in the function g(x)?

x + 5 indicates a horizontal shift 5 units to the left, and +1 indicates a vertical shift 1 unit up.

How does the vertex of the parabola change after the translation?

The vertex moves from (0, 0) to (-5, 1).

What is the significance of the labels on the coordinate plane for this parabola?

The labels help identify the vertex and key points of the parabola after translation, illustrating the shifts.

How does the graph of g(x) compare to the original

parabola in terms of shape?

The shape and width of the parabola remain the same; only its position has changed.

Can the translation be described as a function composition? If so, how?

Yes, g(x) can be viewed as f(x + 5) + 1, representing a horizontal shift of 5 units left and a vertical shift of 1 unit up.

Why is understanding translations important in graphing functions?

Understanding translations helps in visualizing how changes in the function's equation affect the graph's position without altering its fundamental shape.

Additional Resources

The Graph Of F(x) = X^2 Is Translated To Form G(x) = $(x - 5)^2 + 1$. On A Coordinate Plane, A Parabola, Labeled

Understanding how functions transform on a coordinate plane is fundamental in algebra and analytic geometry. In particular, the parabola represented by the basic quadratic function $f(x) = x^2$ offers a clear example of how shifts and translations alter the graph's position without changing its shape. When the function is modified to $g(x) = (x - 5)^2 + 1$, it signifies a translation—shifting the parabola horizontally and vertically. This article explores this transformation in detail, examining how the graph of $f(x) = x^2$ is translated to form $g(x)$, what that looks like on the coordinate plane, and how to interpret these changes effectively.

Understanding the Basic Parabola: The Graph of $f(x) = x^2$

Before delving into the translation, it's essential to grasp the characteristics of the original parabola.

The Shape and Features of $f(x) = x^2$

- Vertex: The lowest point of the parabola, located at the origin $(0,0)$.
- Axis of Symmetry: The vertical line $x=0$ that divides the parabola into two mirror-image halves.

- Direction: Opens upward, indicating the parabola extends infinitely in the positive y-direction.
- Intercepts:
- Vertex (also the y-intercept): $(0, 0)$
- X-intercepts: The points where $x^2=0$, which is only at $x=0$.
- Y-intercept: When $x=0$, $f(0)=0$.

- Graphical Representation: The parabola is symmetric with respect to the y-axis, with a smooth, U-shaped curve.

Plotting $f(x) = x^2$

To visualize this graph:

1. Plot key points such as $(-2, 4)$, $(-1, 1)$, $(0, 0)$, $(1, 1)$, $(2, 4)$.
2. Connect these points with a smooth curve opening upward.
3. Recognize the symmetry about the y-axis.

This basic understanding provides a foundation for understanding how shifts affect the graph.

Transformation of the Function: From $f(x) = x^2$ to $g(x) = (x - 5)^2 + 1$

Transformations modify the position of the graph without altering its fundamental shape. The function $g(x) = (x - 5)^2 + 1$ includes specific modifications:

- A horizontal shift (or translation) represented by $(x - 5)$.
- A vertical shift represented by $+1$.

Deciphering the Horizontal Shift: The Role of $(x - 5)$

In the original function, the input x is simply squared. In the new function, the input is $(x - 5)$, which shifts the graph horizontally.

- When you replace x with $(x - h)$, the graph shifts h units to the right if $h > 0$, and h units to the left if $h < 0$.

For our case:

- $g(x) = (x - 5)^2 + 1$ indicates a shift 5 units to the right because of $(x - 5)$.

Vertical Shift: The Effect of $+1$

Adding a constant outside the squared term shifts the entire graph vertically:

- When adding $+k$, the graph moves k units upward.
- When subtracting k, it moves k units downward.

In this case:

- The $+1$ shifts the parabola 1 unit upward.

Summary of Transformations

Transformation	Effect on the Graph	Description
Horizontal shift	5 units to the right	Due to $(x - 5)$
Vertical shift	1 unit upward	Due to $+1$

These combined shifts translate the original parabola to a new position on the coordinate plane, maintaining its shape but relocating it.

Visualizing the Translated Parabola on the Coordinate Plane

To better understand the translation, plotting specific points helps visualize the change. Let's analyze how key points on $f(x) = x^2$ translate to $g(x) = (x - 5)^2 + 1$.

Mapping Key Points

- Vertex:

- For $f(x) = x^2$, vertex at $(0, 0)$.
- For $g(x)$:

$$
g(x) = (x - 5)^2 + 1
$$

To find the vertex, set the squared term to zero:

\[

$$
x - 5 = 0 \Rightarrow x=5
$$

Then:

$$
g(5) = (5 - 5)^2 + 1 = 0 + 1 = 1
$$

Vertex of $g(x)$: $\boxed{(5, 1)}$

- X-intercepts:

- Solve $g(x) = 0$:

$$
(x - 5)^2 + 1 = 0
$$

Since $(x - 5)^2 \geq 0$, the sum ≥ 1, so the equation has no real solutions. The parabola does not cross the x-axis.

- Y-intercept:

- Set $x=0$:

$$
g(0) = (0 - 5)^2 + 1 = 25 + 1 = 26
$$

Y-intercept: $\boxed{(0, 26)}$

- Additional points:

- For $x=4$:

$$
g(4) = (4 - 5)^2 + 1 = 1 + 1 = 2
$$

- For $x=6$:

$$
g(6) = (6 - 5)^2 + 1 = 1 + 1 = 2
$$

- For $x=3$:

$$
g(3) = (3 - 5)^2 + 1 = 4 + 1 = 5
$$

- For $x=7$:

$$
g(7) = (7 - 5)^2 + 1 = 4 + 1 = 5
$$

Plotting the Translated Parabola

Using these points:

- Plot the vertex at $(5, 1)$.
- Mark the y-intercept at $(0, 26)$, which is far above the vertex, indicating the parabola opens upward.
- Plot points at $(4, 2)$, $(6, 2)$, $(3, 5)$, and $(7, 5)$.

Connecting these points with a smooth curve demonstrates the shift of the original parabola to the right and upward.

Interpreting the Transformation Geometrically

The translation of the parabola from $f(x) = x^2$ to $g(x) = (x - 5)^2 + 1$ can be visualized as:

- Moving the entire parabola rightward 5 units: Every point (x, y) on $f(x)$ is mapped to $(x + 5, y)$, resulting in the vertex shifting from $(0, 0)$ to $(5, 0)$, then adjusted upward by 1 unit.
- Moving upward 1 unit: The vertex at $(0, 0)$ shifts to $(5, 1)$, and the entire parabola moves accordingly.

This translation preserves the parabola’s shape, width, and opening direction, emphasizing the property that transformations involving addition or subtraction inside the function's argument correspond to horizontal shifts, while additions outside affect vertical shifts.

Practical Applications and Significance

Understanding translations of quadratic functions is not just an academic exercise; it has real-world applications:

- Physics: Modeling projectile motion where the path of an object is a parabola, and shifting the parabola corresponds to changes in initial position or height.
- Engineering: Designing structures or trajectories that require precise adjustments in position.

- Data Modeling: Adjusting quadratic models to fit data that is shifted vertically or horizontally.

In educational contexts, mastering these transformations enhances comprehension of function behavior and graph interpretation, which are critical skills in advanced mathematics and related disciplines.

Conclusion: The Power of Function Transformations

The transition from

The Graph Of F X X2 Is Translated To Form G X X 5 2 1 On A Coordinate Plane A Parabola Labeled

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