The Points (7,8),(1,9)M(7,8),N(1,9), And (0,3)O(0,3) Form A Triangle. Find The Desired Slopes And Lengths,

The Points (7,8),(1,9)M(7,8),N(1,9), And (0,3)O(0,3) Form A Triangle. Find The Desired Slopes And Lengths,
Understanding the geometric relationships between points on a coordinate plane is fundamental in mathematics, especially when analyzing the properties of triangles. The given points—(7,8), (1,9), M(7,8), N(1,9), and (0,3), O(0,3)—are key to constructing and analyzing a triangle, and determining their slopes and lengths can reveal important geometric insights. This article aims to guide you through the process of identifying the triangle formed by these points and calculating the desired slopes and lengths with clarity and precision.

Understanding the Given Points and Their Significance

Analyzing the Coordinates

The provided points are as follows:
- (7,8)
- (1,9)
- M(7,8)
- N(1,9)
- (0,3)
- O(0,3)

Notice that points (7,8) and M are the same, as are (1,9) and N, and (0,3) and O. This suggests that M, N, and O are likely designated points or labels for the same coordinates. These points are positioned on the coordinate plane in a way that they could form sides of a triangle, especially when considering their connections.

Key observations:
- The points (7,8) and (1,9) are distinct and could serve as vertices.
- The points (0,3) may act as a third vertex or as points defining specific segments.

Understanding their placement helps us visualize potential triangles.

Constructing the Triangle from the Points

Identifying the Vertices

Based on the points, the natural vertices of the triangle are likely:
- Point A: (7,8)
- Point B: (1,9)
- Point C: (0,3)

Alternatively, if M, N, and O are labels for these points, they represent the same vertices.

Visual Representation:
To better understand the shape, plotting these points on a coordinate plane can be very helpful. This can be done manually or through graphing tools for accuracy.

Connecting the Points

The triangle is formed by connecting points:
- A (7,8) to B (1,9)
- B (1,9) to C (0,3)
- C (0,3) to A (7,8)

This creates a triangle with sides AB, BC, and CA.

Calculating the Lengths of the Triangle Sides

Distance Formula

The length between any two points (x_1, y_1) and (x_2, y_2) is given by the distance formula:

\[

d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

This formula will be used to determine each side's length.

Calculating Side Lengths

Let's compute each side:

1. Side AB (between (7,8) and (1,9)):
\[
AB = \sqrt{(1 - 7)^2 + (9 - 8)^2} = \sqrt{(-6)^2 + 1^2} = \sqrt{36 + 1} = \sqrt{37} \approx 6.08
\]

2. Side BC (between (1,9) and (0,3)):
\[
BC = \sqrt{(0 - 1)^2 + (3 - 9)^2} = \sqrt{(-1)^2 + (-6)^2} = \sqrt{1 + 36} = \sqrt{37} \approx 6.08
\]

3. Side CA (between (0,3) and (7,8)):
\[
CA = \sqrt{(7 - 0)^2 + (8 - 3)^2} = \sqrt{7^2 + 5^2} = \sqrt{49 + 25} = \sqrt{74} \approx 8.60
\]

Summary of side lengths:
- AB ≈ 6.08 units
- BC ≈ 6.08 units
- CA ≈ 8.60 units

This indicates that sides AB and BC are equal, suggesting the triangle might be isosceles.

Calculating the Slopes of the Triangle Sides

Slope Formula

The slope (m) between two points (x_1, y_1) and (x_2, y_2) is:

\[

$$
m = \frac{y_2 - y_1}{x_2 - x_1}
$$

Calculating slopes helps determine the angles of the sides and the nature of the triangle.

Calculating Each Slope

1. Slope of AB (between (7,8) and (1,9)):

$$
m_{AB} = \frac{9 - 8}{1 - 7} = \frac{1}{-6} = -\frac{1}{6}
$$

2. Slope of BC (between (1,9) and (0,3)):

$$
m_{BC} = \frac{3 - 9}{0 - 1} = \frac{-6}{-1} = 6
$$

3. Slope of CA (between (0,3) and (7,8)):

$$
m_{CA} = \frac{8 - 3}{7 - 0} = \frac{5}{7}
$$

Summary of slopes:
- AB: -1/6
- BC: 6
- CA: 5/7

The varying slopes indicate different inclinations of the sides, with some sides being steep and others more gentle.

Interpreting the Results and Geometric Properties

Type of Triangle

Given the side lengths, the triangle is likely scalene (all sides different) with two sides approximately equal (AB and BC). The approximate calculations suggest the triangle might be isosceles with sides AB and BC equal, or at least very close in length.

Angles and shape:
- The slopes suggest the arrangement of sides at different angles.
- The side BC has a steep positive slope, indicating a sharp inclination.
- Side AB has a gentle negative slope, indicating a slight downward incline.

Area and Perimeter

Beyond side lengths and slopes, calculating the area and perimeter provides further understanding of the triangle's size.

- Perimeter: sum of side lengths:

$$
6.08 + 6.08 + 8.60 \approx 20.76
$$

- Area: can be calculated using Heron's formula or the coordinate geometry formula. Using Heron's formula:

$$
s = \frac{a + b + c}{2} = \frac{6.08 + 6.08 + 8.60}{2} \approx 10.38
$$

$$
\text{Area} = \sqrt{s(s - a)(s - b)(s - c)} \approx \sqrt{10.38(10.38 - 6.08)(10.38 - 6.08)(10.38 - 8.60)}
$$

Calculating:

$$
= \sqrt{10.38 \times 4.30 \times 4.30 \times 1.78}
$$

The exact value can be computed with a calculator for precision.

Applications and Significance of Calculating Slopes and Lengths in Triangles

Understanding slopes and lengths is vital in multiple fields:

- Geometry and Trigonometry: For calculating angles, areas, and understanding shapes.
- Engineering and Architecture: For designing structures with precise measurements.

- Navigation and GIS: For mapping and spatial analysis.
- Computer Graphics: For rendering shapes and understanding spatial relationships.

Summary and Key Takeaways

- The points (7,8), (1,9), and (0,3) form a triangle with sides approximately 6.08, 6.08, and 8.60 units long.
- The slopes of the triangle's sides are -1/6, 6, and 5/7, indicating the sides' inclinations and the shape's overall geometry.
- The triangle appears to be isosceles, with two sides of equal length.
- Calculating the side lengths and slopes helps in understanding the triangle's shape, size, and properties.

This comprehensive analysis demonstrates how coordinate geometry allows us to derive meaningful insights about geometric figures based on their vertex points. Whether for academic purposes, engineering design, or spatial analysis, mastering these calculations is essential for precise and effective problem-solving.

Further Resources:
- Coordinate Geometry

Frequently Asked Questions

What are the coordinates of points M and N in the given triangle?

Point M is at (7,8) and point N is at (1,9).

How do you find the slope of the line segment between points M(7,8) and N(1,9)?

The slope is calculated as (9 - 8) / (1 - 7) = 1 / (-6) = -1/6.

What is the length of the segment connecting points M(7,8) and N(1,9)?

Using the distance formula: $\sqrt{[(1 - 7)^2 + (9 - 8)^2]} = \sqrt{[(-6)^2 + 1^2]} = \sqrt{[36 + 1]} = \sqrt{37} \approx 6.08$ units.

What are the coordinates of points O in the given figure?

Point O is at (0,3).

What is the slope of the segment connecting point O(0,3) to points M(7,8) and N(1,9)?

For O to M: (8 - 3) / (7 - 0) = 5/7. For O to N: (9 - 3) / (1 - 0) = 6/1 = 6.

Are points M, N, and O collinear based on the slopes calculated?

No, since the slopes O-M (5/7) and O-N (6) are different, the points are not collinear and form a triangle.

What are the lengths of segments OM, ON, and MN in the triangle?

OM: √[(7 - 0)^2 + (8 - 3)^2] = √[49 + 25] = √74 ≈ 8.60 units. ON: √[(1 - 0)^2 + (9 - 3)^2] = √[1 + 36] = √37 ≈ 6.08 units. MN: as previously calculated, approximately 6.08 units.

How can you verify that points M, N, and O form a triangle with the given points?

By calculating the slopes of the sides and confirming they are not all equal, and by ensuring the sum of the lengths confirms a triangle shape, verifying that the points are non-collinear.

What is the significance of calculating slopes and lengths in forming the triangle from these points?

Calculating slopes helps determine the orientation of the sides, while lengths help confirm the size and shape of the triangle; both are essential for understanding the geometric relationships among the points.

Additional Resources

The Points (7,8), (1,9), M(7,8), N(1,9), and O(0,3) form a triangle: Find the desired slopes and lengths is a comprehensive geometric problem that involves analyzing the relationships between various points on the coordinate plane. This problem combines elements of coordinate geometry, including calculating distances between points and determining the slopes of lines connecting these points. It also presents an opportunity to explore properties such as collinearity, parallelism, and the formation of triangles. This article aims to dissect this problem thoroughly, providing detailed explanations, step-by-step calculations, and insights into the underlying geometric principles.

Understanding the Given Points and Their Significance

Before diving into calculations, it's crucial to interpret the points provided and understand their roles in forming the triangle. The points given are:
- (7,8)
- (1,9)
- M(7,8)
- N(1,9)
- O(0,3)

At first glance, some points appear to be repeated or labeled for specific purposes. Notably, (7,8) and M(7,8) are the same point, as are (1,9) and N(1,9). Point O(0,3) seems to be an additional point that might serve as a vertex or an auxiliary point in the geometric configuration.

Key observations:
- The points (7,8) and M(7,8) are identical.
- The points (1,9) and N(1,9) are identical.
- The points (7,8) and (1,9) can be connected to form a segment or serve as vertices of a triangle.
- The point O(0,3) may be another vertex or an external point relevant to the problem.

Establishing the Triangle: Which Points Form It?

Given the points, the primary candidates for forming a triangle are:
- (7,8)
- (1,9)
- O(0,3)

Since M(7,8) and N(1,9) are the same as (7,8) and (1,9), respectively, they can be considered as labeled points for clarity. The core triangle is likely formed by the points:

A (7,8), B (1,9), and C (0,3).

This setup suggests that the triangle in question is ABC with vertices at these points.

Calculating the Lengths of the Sides

To fully analyze the triangle, we need to find the lengths of all sides: AB, BC, and AC. The distance formula in coordinate geometry is:

$$ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} $$

Side AB (between points A(7,8) and B(1,9)):
$$ AB = \sqrt{(1 - 7)^2 + (9 - 8)^2} = \sqrt{(-6)^2 + 1^2} = \sqrt{36 + 1} = \sqrt{37} \approx 6.08 $$

Side AC (between points A(7,8) and C(0,3)):
$$ AC = \sqrt{(0 - 7)^2 + (3 - 8)^2} = \sqrt{(-7)^2 + (-5)^2} = \sqrt{49 + 25} = \sqrt{74} \approx 8.60 $$

Side BC (between points B(1,9) and C(0,3)):
$$ BC = \sqrt{(0 - 1)^2 + (3 - 9)^2} = \sqrt{(-1)^2 + (-6)^2} = \sqrt{1 + 36} = \sqrt{37} \approx 6.08 $$

Summary of side lengths:
- AB ≈ 6.08 units
- AC ≈ 8.60 units
- BC ≈ 6.08 units

This indicates that sides AB and BC are equal, suggesting the triangle is isosceles with AB and BC as equal sides.

Determining the Slopes of the Sides

The slope of a line passing through two points (x_1, y_1) and (x_2, y_2) is:

$$ m = \frac{y_2 - y_1}{x_2 - x_1} $$

Calculating the slopes of each side:

Slope of AB:
$$ m_{AB} = \frac{9 - 8}{1 - 7} = \frac{1}{-6} = -\frac{1}{6} $$

Slope of AC:
$$ m_{AC} = \frac{3 - 8}{0 - 7} = \frac{-5}{-7} = \frac{5}{7} $$

Slope of BC:
$$ m_{BC} = \frac{3 - 9}{0 - 1} = \frac{-6}{-1} = 6 $$

Summary of slopes:
- AB: $-\frac{1}{6}$
- AC: $\frac{5}{7}$
- BC: 6

The slopes reveal the orientations of the sides and can be used to determine angles between sides or check for properties like perpendicularity.

Analyzing the Triangle’s Properties

With side lengths and slopes computed, we can analyze the triangle’s features:

Isosceles Nature:
- Sides AB and BC are equal in length (~6.08 units), indicating an isosceles triangle with AB and BC as the equal sides.

Angles:
- The angles at vertices A, B, and C can be calculated using the Law of Cosines or by examining slopes.

Perpendicularity:
- To check if any sides are perpendicular, see if the slopes multiply to -1.
- For example, the slopes of AB and AC: $-\frac{1}{6} \times \frac{5}{7} = -\frac{5}{42} \neq -1$, so not perpendicular.
- Sides with slopes m_1 and m_2 are perpendicular if $m_1 \times m_2 = -1$.

Angles at Vertices:
Using the slopes, the angles between sides can be found via the tangent of the angle between two lines:

$$
\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|
$$

Calculations:

- Angle at A (between AB and AC):
\[

$$
\tan \theta_A = \left| \frac{-\frac{1}{6} - \frac{5}{7}}{1 + (-\frac{1}{6})(\frac{5}{7})} \right| = \left| \frac{-\frac{7}{42} - \frac{30}{42}}{1 - \frac{5}{42}} \right| = \left| \frac{-\frac{37}{42}}{\frac{37}{42}} \right| = 1
$$

$$
\Rightarrow \theta_A = 45^\circ
$$

Similarly, other angles can be calculated, but the key point is that the triangle has known angles based on slopes.

Constructing the Triangle and Visualizing

Having determined side lengths and slopes, plotting the points and the triangle is straightforward:

- Plot points A(7,8), B(1,9), and C(0,3).
- Draw segments AB, BC, and AC.
- Observe the shape: an isosceles triangle with two sides approximately 6.08 units and base approximately 8.60 units.

Visualization aids in understanding the geometric relationships and verifying calculations.

Additional Considerations and Extensions

Finding Coordinates of M, N, and O:
- M and N are given as points identical to A and B, respectively.
- O(0,3) may serve as a point of interest; perhaps it’s the intersection of certain lines or a point forming a special triangle with the original points.

Possible Tasks in the Original Problem:
- Find the slopes of lines connecting these points.
- Calculate lengths of segments involving O or other points.
- Determine if the triangle is right-angled, equilateral, or other types based on lengths and slopes.

Features and Tools Used:

- Distance formula for lengths
- Slope formula for line orientation
- Law of Cosines for angles
- Visualization for geometric intuition

Pros and Cons of the Approach

Pros:
- Clear step-by-step methodology makes the problem manageable.
- Use of coordinate geometry allows precise calculations.
- Visualization helps confirm calculations and understand geometric relationships.
- Generalizable approach applicable to similar problems.

Cons:
- Assumes points are accurately plotted, which may lead to errors in manual sketching.
- Slope calculations can be undefined or tricky for vertical lines

The Points 7 8 1 9 M 7 8 N 1 9 And 0 3 O 0 3 Form A Triangle Find The Desired Slopes And Lengths

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