The Position Of A 50 G Oscillating Mass Is Given By X(t)=(2.0cm)cos(10t/4), Where T Is In S. If Necessary,

The Position Of A 50 G Oscillating Mass Is Given By X(t)=(2.0cm)cos(10t/4), Where T Is In S. If Necessary,

Understanding the motion of oscillating masses is fundamental in physics, especially in the study of simple harmonic motion (SHM). This article explores a specific case: a 50-gram mass undergoing oscillation described by the function $x(t) = (2.0\,cm) \cos \left(\frac{10t}{4} \right)$, where t is measured in seconds. By analyzing this equation, we gain insights into the characteristics of the oscillation, such as amplitude, period, frequency, and phase. This detailed breakdown is essential for students, educators, and enthusiasts aiming to deepen their understanding of harmonic motion and related physical phenomena.

Understanding the Context of the Oscillation Equation

Before delving into specific calculations, it's important to understand the significance of the given displacement function:

$$
x(t) = (2.0\,cm) \cos \left(\frac{10t}{4} \right)
$$

This mathematical expression models the position $x(t)$ of a mass at any given time t. The key components include:

- Amplitude (A): The maximum displacement from the mean position, here 2.0 cm.
- Angular frequency (ω): The rate of oscillation, derived from the argument of the cosine function.
- Time (t): The independent variable, measured in seconds.
- Phase: The initial angle, which in this case is zero, indicating the mass starts at maximum displacement when $t=0$.

Deciphering the Mathematical Representation

Identifying the Amplitude

The amplitude A indicates how far the mass moves from its equilibrium position during oscillation.

- Amplitude (A): 2.0 cm

This means the mass oscillates between +2.0 cm and -2.0 cm relative to the equilibrium point.

Extracting the Angular Frequency (ω)

The argument of the cosine function is:

$$
\frac{10t}{4} = \left(\frac{10}{4} \right) t = 2.5 t
$$

This implies:

$$
\omega = 2.5\, \text{rad/sec}
$$

where:

$$
\omega = \text{angular frequency}
$$

Understanding the Physical Significance of ω

The angular frequency relates to how quickly the mass oscillates. It connects directly to the period T and frequency f:

$$
T = \frac{2\pi}{\omega}
$$
$$
f = \frac{1}{T}
$$

Calculating the Oscillation Period and Frequency

Period (T)

Using the relation:

$$
T = \frac{2\pi}{\omega}
$$

Substituting $\omega = 2.5\, \text{rad/sec}$:

$$
T = \frac{2\pi}{2.5} = \frac{6.2832}{2.5} \approx 2.513\, \text{seconds}
$$

Therefore, the period of oscillation is approximately 2.51 seconds.

Frequency (f)

Given:

$$
f = \frac{1}{T}
$$

$$
f = \frac{1}{2.513} \approx 0.398\, \text{Hz}
$$

The mass completes approximately 0.398 oscillations per second.

Physical Parameters of the Oscillating System

Mass of the Object

Given as:

$$
m = 50\, \text{g} = 0.050\, \text{kg}
$$

Understanding the mass is crucial when analyzing the system's dynamics, especially in relation to the restoring force and energy considerations.

Restoring Force and Spring Constant (if applicable)

While the original equation doesn't specify a spring constant, in simple harmonic motion models, the restoring force F is proportional to displacement:

$$
F = -k x
$$

where k is the spring constant.

If we assume the oscillation is due to a mass attached to a spring, the relation between ω and k is:

$$
\omega = \sqrt{\frac{k}{m}}
$$

Rearranged to find k:

$$
k = m \omega^2
$$

Calculating:

$$
k = 0.050 \times (2.5)^2 = 0.050 \times 6.25 = 0.3125\, \text{N/m}
$$

This provides an estimate of the spring constant if the system is spring-based.

Phase and Initial Conditions

Given the cosine function:

$$
X(t) = 2.0\,cm \times \cos(2.5 t)
$$

- Initial position ($t=0$):

$$
X(0) = 2.0\,cm \times \cos(0) = 2.0\,cm
$$

- Initial velocity:

The velocity as a function of time is the derivative of $X(t)$:

$$
v(t) = -A \omega \sin(\omega t)
$$

At $t = 0$:

$$
v(0) = -2.0\,cm \times 2.5 \times \sin(0) = 0
$$

Thus, the mass starts at maximum displacement with zero initial velocity, consistent with the cosine form.

Energy Considerations in the Oscillating System

The total mechanical energy E in simple harmonic motion remains constant (assuming no damping):

$$
E = \frac{1}{2} k A^2
$$

Using the earlier estimate of k:

$$
E = \frac{1}{2} \times 0.3125\, \text{N/m} \times (0.02\, \text{m})^2

$$
E = 0.15625 \times 0.0004 = 6.25 \times 10^{-5}\, \text{J}
$$

This energy oscillates between kinetic and potential forms during the motion.

Applications and Real-World Examples

Understanding the behavior of such oscillating systems has numerous practical applications:

- Mechanical Clocks: Pendulums and spring-driven clocks rely on SHM principles.
- Seismology: Earthquake waves involve oscillatory motion captured by seismographs.
- Engineering Design: Vibration analysis of structures to prevent resonance failures.
- Medical Devices: Oscillatory motion in devices like MRI machines.

Summary of Key Findings

- The displacement function indicates a simple harmonic oscillator with an amplitude of 2.0 cm.
- The angular frequency ω is 2.5 rad/sec.
- The period T of oscillation is approximately 2.51 seconds.
- The frequency f is about 0.398 Hz.
- The system's energy and restoring force can be estimated based on the given parameters.
- The initial conditions suggest the mass starts at maximum displacement with zero initial velocity.

Conclusion

Analyzing the oscillating mass through its displacement function provides a comprehensive understanding of its motion characteristics. Recognizing the relationships between amplitude, period, frequency, and energy allows physicists and engineers to design and interpret systems involving harmonic oscillations effectively. This particular case, involving a 50 g mass oscillating with a 2.0 cm amplitude and a period of approximately 2.51 seconds, exemplifies fundamental principles of simple harmonic motion, serving as an educational model for broader physical phenomena.

Additional Resources for Further Study

- Textbooks:
- "Physics for Scientists and Engineers" by Serway and Jewett
- "Introduction to Classical Mechanics" by David Morin
- Online Tutorials:

- Khan Academy's Harmonic Motion Series
- HyperPhysics's Oscillations Section
- Simulation Tools:
- PhET Interactive Simulations on Oscillations
- GeoGebra for modeling SHM

By mastering the analysis of such oscillatory systems, students and professionals can better understand the dynamics that govern a wide array of physical processes and engineering applications.

Frequently Asked Questions

What is the amplitude of the oscillating mass in the given equation?

The amplitude is 2.0 cm, which is the maximum displacement of the mass from its equilibrium position.

How do you determine the angular frequency of the oscillation from the given equation?

The angular frequency ω is given by the coefficient of t inside the cosine function: ω = 10/4 = 2.5 rad/sec.

What is the period of oscillation for the mass, based on the given equation?

The period T is calculated using T = 2π/ω. With ω = 2.5 rad/sec, T = 2π/2.5 ≈ 2.51 seconds.

What is the phase of the oscillation at t=0?

At t=0, the phase is cos(0) = 1, so the mass starts at the maximum displacement of 2.0 cm.

How does the given position function relate to simple harmonic motion?

The function x(t) = (2.0 cm) cos(10t/4) describes simple harmonic motion, characterized by sinusoidal displacement, constant amplitude, and a specific period and frequency.

If the mass is at its maximum displacement, what is the velocity at that instant?

The velocity is zero at maximum displacement because the mass momentarily stops before reversing direction in simple harmonic motion.

Additional Resources

The Position Of A 50 G Oscillating Mass Is Given By x(t) = (2.0 cm) cos(10t/4), Where T Is In Seconds. If Necessary,

Understanding the dynamics of oscillating systems is fundamental to physics, engineering, and various applied sciences. The given expression for the position of a mass provides a window into the harmonic motion that underpins many physical phenomena — from simple pendulums to complex mechanical vibrations. This article delves into a comprehensive analysis of the oscillating mass described by the function \(x(t) = (2.0\, \text{cm}) \cos \left(\frac{10t}{4}\right) \), exploring its physical interpretation, mathematical foundations, and implications for real-world systems.

Introduction to Simple Harmonic Motion (SHM)

Simple Harmonic Motion (SHM) refers to a type of periodic motion where the restoring force acting on an object is directly proportional to its displacement and directed towards a fixed equilibrium point. This form of motion is characterized by sinusoidal functions—sine and cosine—and appears in diverse contexts such as pendulums, springs, and molecular vibrations.

Key features of SHM include:
- Amplitude (A): The maximum displacement from the equilibrium position.
- Period (T): The time taken to complete one full oscillation.
- Frequency (f): Number of oscillations per second.
- Angular frequency (ω): Rate at which the oscillation occurs in radians per second.
- Phase: The initial angle or offset at $t=0$.

Understanding these parameters is essential to analyze the given oscillating mass.

Deciphering the Given Function

Mathematical Formulation

The position function is:

$$
X(t) = (2.0\, \text{cm}) \cos \left(\frac{10t}{4} \right)
$$

- Amplitude (A): 2.0 cm.
- Angular argument: $\frac{10t}{4}$.

To interpret this, we first simplify the argument:

$$
\frac{10t}{4} = 2.5 t
$$

This reveals that the oscillatory motion can be expressed as:

$$
X(t) = 2.0\, \text{cm} \times \cos(2.5 t)
$$

Physical Significance

- The amplitude indicates that the mass oscillates ±2.0 cm around the equilibrium position.
- The angular frequency $\omega = 2.5 \, \text{rad/sec}$ governs how rapidly the mass oscillates.

Clarification on Variables

- The notation t is time in seconds.
- The problem mentions "T" in seconds, but it appears the variable used in the function is t. It's reasonable to assume the standard notation where t denotes time.

- The mass of 50 g (or 0.05 kg) is specified, but in harmonic motion analysis, mass's role is primarily in the calculation of acceleration, force, and energy, rather than position directly, unless considering the underlying physical system (like a mass-spring system).

Analyzing the Oscillating System

1. Determining the Period and Frequency

Given the angular frequency:

$$
\omega = 2.5\, \text{rad/sec}
$$

The period T of oscillation is related to angular frequency by:

$$
T = \frac{2\pi}{\omega}
$$

Calculating:

$$
T = \frac{2\pi}{2.5} = \frac{6.2832}{2.5} \approx 2.513\, \text{seconds}
$$

This means the mass completes one full oscillation approximately every 2.51 seconds.

The frequency f, the number of oscillations per second, is:

$$
f = \frac{1}{T} \approx \frac{1}{2.513} \approx 0.398\, \text{Hz}
$$

2. Phase and Initial Conditions

Since the function involves a cosine term with no phase shift (i.e., no additional constant inside the cosine), the initial position at $t=0$ is:

$$
X(0) = 2.0\, \text{cm} \times \cos(0) = 2.0\, \text{cm}
$$

Indicating that at $t=0$, the mass starts at maximum displacement.

Physical Interpretation and System Dynamics

1. Restoring Force and System Type

The harmonic motion suggests that the mass is attached to a restoring mechanism, such as a spring or a pendulum, which exerts a force proportional to displacement:

$$
F = -kx
$$

Where:
- k is the spring constant (or effective stiffness),
- x is the displacement.

For a mass-spring system, the angular frequency relates to k and the mass m as:

$$
\omega = \sqrt{\frac{k}{m}}
$$

Given that $m = 50\, \text{g} = 0.05\, \text{kg}$, and $\omega = 2.5\, \text{rad/sec}$, we can deduce:

$$
k = m \omega^2 = 0.05 \times (2.5)^2 = 0.05 \times 6.25 = 0.3125\, \text{N/m}
$$

This is the effective spring constant if the system is a simple harmonic oscillator.

2. Energy Considerations

The total mechanical energy in SHM, assuming no damping, remains constant and is the sum of kinetic and potential energy:

$$
E = \frac{1}{2} k A^2
$$

Where A is in meters:

$$
A = 2.0\, \text{cm} = 0.02\, \text{m}
$$

Calculating:

$$
E = \frac{1}{2} \times 0.3125 \times (0.02)^2 = 0.15625 \times 0.0004 = 6.25 \times 10^{-5}\, \text{J}
$$

This energy quantifies the maximum energy in the system, which oscillates between kinetic and potential forms as the mass moves.

Application of the Mathematical Model in Real-World Contexts

1. Engineering and Design

Understanding the motion characteristics enables engineers to design systems with desired oscillation periods and amplitudes. For example, in designing suspension systems, vibration isolators, or timing devices, precise knowledge of oscillation frequencies is crucial.

2. Seismology and Structural Analysis

Seismic waves induce oscillations similar to SHM. Analyzing such oscillations allows for assessing structural integrity and designing earthquake-resistant buildings.

3. Biological and Medical Physics

Oscillatory models describe phenomena like heartbeat rhythms, neuronal activity, or molecular vibrations.

Implications of the Mass and System Parameters

1. Role of Mass in Oscillations

While the position function primarily depends on ω and A, the mass influences:
- The system's natural frequency (ω),
- The response to external forces,
- Damping characteristics if present.

In this specific case, the mass of 50 g influences the stiffness k if modeled as a spring-mass system, as shown earlier.

2. Effects of Changing System Parameters

- Increasing the mass m (keeping k constant) would decrease ω, lengthening the period.
- Increasing the stiffness k would increase ω, shortening the period.
- Altering amplitude A affects the maximum energy stored within the system but does not influence ω.

Advanced Topics and Calculations

1. Velocity and Acceleration as Functions of Time

- Velocity: $v(t) = \frac{dx}{dt}$

$$
v(t) = -A \omega \sin(\omega t)
$$

At $t=0$:

$$
v(0) = 0
$$

- Acceleration: $a(t) = \frac{d^2 x}{dt^2}$

$$
a(t) = -A \omega^2 \cos(\omega t)
$$

At $t=0$:

$$
a(0) = -A \omega^2 = -0.02 \times (2.5)^2 = -0.02 \times 6.25 = -0.125\, \text{m/sec}^2
$$

The negative sign indicates acceleration directed toward the equilibrium position, characteristic of restoring force.

2. Phase Space Analysis

Plotting velocity against displacement yields a phase space trajectory, typically a closed ellipse for SHM, illustrating the energy exchange between kinetic and potential forms.

Summary and Concluding Remarks

The oscillating mass described by \(x(t) = 2.0\, \text{cm} \cos \left(\frac{10t}{4} \right

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