

The Slope Of A Line Is $-1/5$, And The Y-intercept Is 5. What Is The Equation Of The Line Written In General

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Understanding the fundamental aspects of linear equations is essential for mastering algebra and coordinate geometry. When given the slope and y-intercept of a line, it becomes straightforward to determine its equation in various forms. In this comprehensive guide, we will explore how to find the equation of a line given its slope and y-intercept, focusing specifically on the case where the slope is $-1/5$ and the y-intercept is 5. We will delve into the slope-intercept form, the general form, and related concepts to deepen your understanding of linear equations.

Introduction to Linear Equations

Before diving into the specific problem, it's vital to understand what a linear equation is and the significance of its key components.

What Is a Linear Equation?

A linear equation describes a straight line on the coordinate plane. It can be written in various forms, but the most common are:

- Slope-intercept form: $y = mx + b$
- Standard form (or general form): $Ax + By + C = 0$

Where:

- m is the slope of the line,
- b is the y-intercept,
- A , B , and C are coefficients in the standard form.

The Importance of Slope and Y-Intercept

- Slope (m): Indicates the steepness of the line, i.e., how much y changes for a unit increase in x .
- Y-intercept (b): The point where the line crosses the y-axis ($x=0$).

Knowing these two components allows us to write the equation of the line quickly in slope-intercept form.

Given Data: Slope and Y-Intercept

In this problem, the given information is:

- Slope (m) = $-1/5$
- Y-intercept (b) = 5

This information guides us directly to the slope-intercept form of the line's equation.

Understanding the Slope $-1/5$

A slope of $-1/5$ indicates that for every increase of 5 units in x, y decreases by 1 unit. The negative sign suggests the line slants downward from left to right.

Understanding the Y-Intercept 5

The y-intercept at 5 means the line crosses the y-axis at the point (0, 5).

Writing the Equation in Slope-Intercept Form

Given the slope and y-intercept, the slope-intercept form is:

$$\begin{aligned} & \backslash [\\ y &= mx + b \\ & \backslash] \end{aligned}$$

Substituting the specific values:

$$\begin{aligned} & \backslash [\\ y &= -\frac{1}{5}x + 5 \\ & \backslash] \end{aligned}$$

This is the simplest form of the line's equation, representing it explicitly in terms of x and y.

Converting the Equation to the General (Standard) Form

While the slope-intercept form is useful for graphing and understanding the slope and intercept, the general form of a line is often preferred for algebraic manipulations and solving systems of equations.

Standard (General) Form of a Line

The standard form is:

$$Ax + By + C = 0$$

Where A, B, and C are integers, and $A \geq 0$ for convention.

Steps to Convert from Slope-Intercept to Standard Form

To convert:

1. Start with the slope-intercept form:

$$y = -\frac{1}{5}x + 5$$

2. Eliminate the fraction by multiplying through by the denominator (5):

$$5y = -x + 25$$

3. Rearrange to bring all terms to one side:

$$x + 5y = 25$$

4. Write in standard form:

$$x + 5y - 25 = 0$$

This is the equation of the line in general form.

Final Equation of the Line in General Form

Based on the steps above, the equation of the line with a slope of $-1/5$ and y-intercept of 5, written in the general form, is:

$$x + 5y - 25 = 0$$

This form is often useful for algebraic operations, such as solving systems of equations, finding intersections, or applying certain geometric concepts.

Additional Considerations and Tips

To deepen your understanding and ensure accuracy, consider these additional points:

Verifying the Equation

- Check the y-intercept: Set $x=0$:

$$\begin{aligned} & \backslash [\\ 0 + 5(0) - 25 &= 0 + 25 - 25 = 0 \\ & \backslash] \end{aligned}$$

The equation holds, confirming the y-intercept at $(0, 5)$.

- Check the slope: Rewrite in slope-intercept form:

$$\begin{aligned} & \backslash [\\ x + 5y = 25 & \rightarrow 5y = -x + 25 \rightarrow y = -\frac{1}{5}x + 5 \\ & \backslash] \end{aligned}$$

The slope is $-1/5$, matching the given data.

Graphing the Line

Plotting the line can solidify understanding:

- Plot the y-intercept at $(0, 5)$.
- Use the slope to find another point: from $(0, 5)$, move 5 units right and 1 unit down to $(5, 4)$.
- Draw the line through these points.

Applications of the Equation

Understanding the equation allows for:

- Graphing the line accurately.
- Finding x-intercepts by setting $y=0$.
- Solving for intersections with other lines.
- Applying in real-world problems involving rates of change and linear relationships.

Conclusion

Determining the equation of a line from its slope and y-intercept is a fundamental skill in algebra. Given the slope of $-1/5$ and y-intercept of 5, the most straightforward form is the slope-intercept form:

```
\[
\boxed{y = -\frac{1}{5}x + 5}
\]
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Transforming this into the general form yields:

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\[
\boxed{x + 5y - 25 = 0}
\]
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Mastering these conversions and understanding the significance of each form enhances your ability to analyze and interpret linear relationships confidently. Whether you're graphing, solving systems, or applying these concepts to real-world scenarios, knowing how to write and manipulate the equation of a line is an essential algebraic skill.

Meta Description: Learn how to find the equation of a line given its slope of $-1/5$ and y-intercept of 5. Explore conversions between slope-intercept and general forms with detailed explanations and examples.

Frequently Asked Questions

What is the slope of the line given in the problem?

The slope of the line is $-1/5$.

What is the y-intercept of the line?

The y-intercept of the line is 5.

How do you write the equation of a line in slope-intercept form?

The slope-intercept form is $y = mx + b$, where m is the slope and b is the y-intercept.

What is the equation of the line in slope-intercept form based on the given information?

The equation is $y = -\frac{1}{5}x + 5$.

How do you convert the slope-intercept form to the general form of a line?

Rewrite $y = -\frac{1}{5}x + 5$ as standard form by moving all terms to one side: $\frac{1}{5}x + y - 5 = 0$.

What is the general form of the line's equation with

the given slope and y-intercept?

$$\frac{1}{5}x + y - 5 = 0.$$

Is the line increasing or decreasing, given the slope $-1/5$?

The line is decreasing because the slope is negative $(-1/5)$.

Can the equation $\frac{1}{5}x + y - 5 = 0$ be rewritten in standard form?

Yes, it is already in standard form: $Ax + By + C = 0$, with $A = \frac{1}{5}$, $B = 1$, $C = -5$.

What is the significance of the y-intercept in the line's equation?

The y-intercept indicates where the line crosses the y-axis, which is at the point $(0, 5)$.

Additional Resources

Understanding the Equation of a Line with a Given Slope and Y-Intercept

When exploring the fundamentals of linear equations, understanding how to formulate the equation of a line from given parameters—such as slope and y-intercept—is essential. This detailed review delves into the specifics of the line characterized by a slope of $-\frac{1}{5}$ and a y-intercept of 5, focusing on deriving its equation in the most common form: the general form.

Introduction to the Equation of a Line

A line in a two-dimensional Cartesian coordinate system can be described mathematically through various forms of equations, including:

- Slope-Intercept Form: $y = mx + b$
- Standard (General) Form: $Ax + By + C = 0$
- Point-Slope Form: $y - y_1 = m(x - x_1)$

Each form offers unique advantages for different applications. The slope-intercept form is particularly straightforward when the slope and y-intercept are known, making it an ideal starting point for converting to the general form.

Key Concepts and Definitions

Before proceeding, let's clarify some crucial terms:

- Slope (m): Represents the rate of change of y with respect to x . It indicates how steep the line is.
- Y-intercept (b): The point where the line crosses the y-axis, i.e., when $x=0$.
- General (Standard) Form: An algebraic representation where the equation is written as $Ax + By + C = 0$, with A , B , and C being real numbers, and A typically non-negative.

Given Parameters and Their Significance

The problem specifies:

- Slope (m): $-\frac{1}{5}$
- Y-intercept (b): 5

Understanding these parameters is crucial:

1. The slope of $-\frac{1}{5}$ indicates the line decreases by 1 unit in y for every 5 units increase in x . The negative sign indicates a downward slope from left to right.
2. The y-intercept of 5 means the line crosses the y-axis at the point $(0, 5)$.

Formulating the Line Equation in Slope-Intercept Form

Using the slope-intercept form:

$$y = mx + b$$

Substituting the known values:

$$y = -\frac{1}{5}x + 5$$

This equation succinctly captures the line's behavior: for any x , the corresponding y is obtained by multiplying x by $-\frac{1}{5}$ and then adding 5.

Converting to the General (Standard) Form

The main objective here is to express the line's equation in the form:

$$Ax + By + C = 0$$

This process involves algebraic manipulation of the slope-intercept form.

Step-by-Step Conversion Process

1. Start with the slope-intercept form:

$$y = -\frac{1}{5}x + 5$$

2. Eliminate the fraction for clarity and ease of conversion. Multiply both sides of the equation by 5 (the denominator):

$$5y = -x + 25$$

3. Rearranged to standard form:

$$x + 5y = 25$$

4. Bring all terms to one side to set the equation equal to zero:

$$x + 5y - 25 = 0$$

This is the general form of the line's equation:

$$\boxed{x + 5y - 25 = 0}$$

Verification of the Equation

It's essential to verify that the derived general form accurately reflects the original parameters.

- Y-intercept check: When $(x=0)$:

$$\begin{aligned} & \backslash[\\ & 0 + 5y - 25 = 0 \rightarrow 5y = 25 \rightarrow y=5 \\ & \backslash] \end{aligned}$$

This matches the given y-intercept.

- Slope check: Rewrite the general form to slope-intercept form:

$$\begin{aligned} & \backslash[\\ & x + 5y = 25 \rightarrow 5y = -x + 25 \rightarrow y = -\frac{1}{5}x + 5 \\ & \backslash] \end{aligned}$$

which matches the original slope-intercept form, confirming correctness.

Additional Insights into the Line Equation

Understanding the form and characteristics of the line provides deeper insights:

- Graphical Representation: The line crosses the y-axis at 5 and declines with a slope of $(-\frac{1}{5})$, meaning for every 5 units moved right along the x-axis, the line drops 1 unit in the y-direction.

- Intercepts:

- Y-intercept: $((0, 5))$

- X-intercept: To find the x-intercept, set $(y=0)$:

$$\begin{aligned} & \backslash[\\ & x + 5(0) - 25=0 \rightarrow x - 25=0 \rightarrow x=25 \\ & \backslash] \end{aligned}$$

Thus, the x-intercept is at $((25, 0))$.

- Line Characteristics:

- The line is decreasing.

- The slope's magnitude indicates a gentle decline.

- The equation's coefficients are integers, simplifying interpretation.

Alternate Forms and Their Uses

While the general form is often used for algebraic manipulations and understanding the line's properties, different forms serve different purposes:

- Slope-Intercept Form: Useful for quick graphing and understanding y-values for given x-values.

- Point-Slope Form: Useful when you know a point on the line and the slope, especially for writing equations quickly.

- Standard (General) Form: Useful for solving systems of equations, applying certain algebraic techniques, and when integrating with other algebraic structures.

Practical Applications and Real-World Context

Knowing how to derive the general form from given slope and y-intercept isn't just a theoretical exercise; it has real-world applications:

- Engineering: Designing roads or bridges where certain slopes and intercepts are necessary.
- Economics: Modeling cost or revenue functions where intercepts and slopes are known.
- Science: Describing relationships between variables, such as speed and distance over time.
- Data Analysis: Fitting a line to data points and expressing the model in various forms.

Summary and Key Takeaways

- The line with slope $-\frac{1}{5}$ and y-intercept 5 has the slope-intercept form:

$$y = -\frac{1}{5}x + 5$$

- Converting to the general form involves clearing fractions and rearranging:

$$x + 5y - 25 = 0$$

- The process demonstrates algebraic manipulation techniques, emphasizing the importance of understanding different representations of linear equations.
- The derived equation accurately reflects the line's properties, verified through intercept calculations.
- Recognizing the importance of various forms aids in solving, graphing, and applying linear equations to practical problems.

Conclusion

Mastering the conversion of a line's equation from slope-intercept form to the general form is a fundamental skill in algebra, vital for both academic

understanding and practical applications. Given the specific parameters—slope of $-\frac{1}{5}$ and y-intercept of 5—the derived equation $(x + 5y - 25 = 0)$ succinctly encapsulates the line's essence. This comprehensive exploration underscores not only the procedural steps but also the deeper understanding necessary to interpret and utilize linear equations effectively in diverse contexts.

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