Three Subtracted From The Product Of Six And A Number Is Greater Than 17Use The Variable B For The Unknown

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Understanding how to translate word problems into algebraic expressions is a fundamental skill in mathematics. A common type of problem involves forming inequalities based on word descriptions, especially when dealing with unknown quantities. In this article, we will explore the problem statement: "Three subtracted from the product of six and a number is greater than 17." We will analyze how to interpret this problem, define the variable, set up the inequality, and solve it step by step. This comprehensive guide aims to help students and learners develop confidence in converting real-world scenarios into algebraic expressions and solving inequalities effectively.

Breaking Down the Word Problem

Before jumping into algebraic expressions, it's crucial to comprehend the problem statement thoroughly. The phrase:

- "Three subtracted from the product of six and a number" indicates a sequence of operations:

1. Take a number (the unknown quantity).
2. Multiply it by six.
3. Subtract three from this product.

- "is greater than 17" establishes an inequality: the result of the previous operation exceeds 17.

In essence, the problem describes a relationship involving a variable (the unknown number), an arithmetic operation (multiplication), subtraction, and an inequality.

Defining the Variable

In algebra, we often use symbols or letters to represent unknown quantities. Here, the problem specifies:

> "Use the variable B for the unknown."

Thus, we assign:

Let B represent the unknown number.

This choice is arbitrary but standard practice in algebra. Using B makes it easier to form the algebraic expression and solve the inequality systematically.

Formulating the Algebraic Expression

With the variable defined, translating the words into an algebraic expression is the next step.

- The phrase "product of six and a number" translates to:

$$ 6 \times B \quad \text{or} \quad 6B $$

- The phrase "Three subtracted from" indicates subtraction after the multiplication:

$$ 6B - 3 $$

- The phrase "is greater than 17" indicates an inequality:

$$ 6B - 3 > 17 $$

Therefore, the algebraic inequality representing the problem is:

$$
6B - 3 > 17
$$

Solving the Inequality Step by Step

Now that the inequality has been established, the goal is to find the range of values for B that satisfy the inequality.

Step 1: Isolate the term with B

Add 3 to both sides of the inequality to move constants to the right:

$$
6B - 3 + 3 > 17 + 3
$$

$$
6B > 20
$$

Step 2: Solve for B

Divide both sides by 6:

$$
B > \frac{20}{6}
$$

Simplify the fraction:

$$
B > \frac{10}{3}
$$

or as a decimal:

$$
B > 3.\overline{3}
$$

Final Solution:

The value of B must be greater than $\frac{10}{3}$, or approximately 3.33.

Interpreting the Solution

- The inequality $B > \frac{10}{3}$ indicates that any number greater than approximately 3.33 satisfies the original problem.
- Since the problem involves a real-world context, the solution can be expressed in various ways:
- In fractional form: $B > \frac{10}{3}$
- In decimal form: $B > 3.33$
- In interval notation: $B \in \left(\frac{10}{3}, \infty \right)$

Practical implications:

- If the number B represents, for example, a quantity of items, then more than 3.33 items (which practically means at least 4 items, since items are whole units) satisfy the inequality.
- For real numbers, the inequality includes all numbers greater than $\frac{10}{3}$.

Additional Examples and Practice

To reinforce understanding, consider similar problems:

1. Problem: "Four subtracted from the product of five and a number is less than 25. Use B as the variable."
- Solution:

$$
5B - 4 < 25
$$

$$
5B < 29
$$

\]

\[

B < \frac{29}{5} = 5.8

\]

- Range: \(B < 5.8 \)

2. Problem: "Eight more than twice a number is equal to 20. Use \(B \) as the variable."

- Solution:

\[

2B + 8 = 20

\]

\[

2B = 12

\]

\[

B = 6

\]

- Solution: \(B = 6 \)

3. Problem: "The sum of three times a number and 7 is less than or equal to 22."

- Solution:

\[

3B + 7 \leq 22

\]

\[

3B \leq 15

\]

\[

B \leq 5

\]

- Range: \(B \leq 5 \)

Graphing the Solution on a Number Line

Visualizing inequalities helps in understanding the range of solutions.

- For $B > \frac{10}{3}$, the solution set is all numbers greater than approximately 3.33.
- On a number line:
- Draw an open circle at $\frac{10}{3}$.
- Shade the region to the right of $\frac{10}{3}$.

This visual approach makes it easier to interpret the solution, especially when solving inequalities in real-world contexts.

Common Mistakes to Avoid

While solving inequalities, students often make errors that can be easily avoided:

- Incorrectly handling negative coefficients: When multiplying or dividing both sides by a negative number, remember to reverse the inequality sign.
- Misinterpreting the word problem: Ensure that the operations described are correctly translated into algebraic expressions.
- Neglecting to simplify fractions: Always simplify fractions for clarity.
- Ignoring the domain of the variable: For example, if the context restricts B to integers, interpret the solution accordingly.

Summary and Key Takeaways

- The phrase "Three subtracted from the product of six and a number is greater than 17" can be translated into the inequality:

$$
6B - 3 > 17
$$

- Solving the inequality involves:
1. Adding 3 to both sides.
2. Dividing both sides by 6.
- The solution is:

$$
B > \frac{10}{3}
$$

which means any number greater than approximately 3.33 satisfies the original problem.

- Understanding how to convert word problems into algebraic expressions is essential in mathematics. Practice with different problems helps build confidence and proficiency.

Conclusion

Mastering the translation of words into algebraic expressions and solving inequalities is a vital skill in algebra. The problem "Three subtracted from the product of six and a number is greater than 17," when approached systematically—by defining the variable, translating words into an expression, and solving step-by-step—becomes manageable and clear. Whether for academic purposes or real-life applications, these skills empower learners to analyze and solve a wide range of problems involving unknown quantities and inequalities.

Remember, practice makes perfect. Keep working through various problems to strengthen your understanding and become proficient in algebraic reasoning.

Frequently Asked Questions

What is the algebraic expression for 'Three subtracted from the product of six and a number'?

The expression is 6b - 3, where b is the unknown number.

How do you set up the inequality for 'Three subtracted from the product of six and a number is greater than 17'?

The inequality is 6b - 3 > 17.

What steps are involved in solving the inequality 6b - 3 > 17?

First, add 3 to both sides to get 6b > 20, then divide both sides by 6 to find b > 10/3.

What is the solution to the inequality 6b - 3 > 17?

The solution is b > 10/3, which is approximately b > 3.33.

How can I interpret the solution b > 10/3 in real-world terms?

It means the unknown number b must be greater than approximately 3.33 for the inequality to hold true.

Why is it important to check the solution in the original inequality?

Checking ensures that the solution satisfies the original inequality, confirming its correctness and validity.

Additional Resources

Three Subtracted From The Product Of Six And A Number Is Greater Than 17: An In-Depth Investigation

Mathematical expressions and inequalities often serve as the foundation upon which countless real-world problems are built. Among these, the inequality “Three subtracted from the product of six and a number is greater than 17” exemplifies the translation of verbal statements into algebraic expressions. In this article, we undertake a comprehensive examination of this inequality, which we will denote using the variable B for the unknown quantity. Our goal is to explore its formulation, interpret its meaning, analyze solutions, and understand its implications both within and beyond pure mathematics.

Understanding the Inequality: From Words to Algebra

At the heart of our investigation lies the core step of translating linguistic descriptions into algebraic expressions—a fundamental skill in mathematics. The statement:

"Three subtracted from the product of six and a number is greater than 17"

can be broken down systematically.

Step 1: Identify the unknown

- Let B represent the unknown number.

Step 2: Translate the phrase into an algebraic expression

- "The product of six and a number" translates to 6 × B.
- "Three subtracted from" indicates that 3 is to be subtracted from this product, giving 6B - 3.
- "Is greater than 17" corresponds to the inequality > 17.

Thus, the algebraic inequality becomes:

6B - 3 > 17

Analyzing the Inequality: Mathematical Interpretation

Having converted the verbal statement into an algebraic form, the next step involves analyzing the inequality to find the set of values for B that satisfy it.

Step 1: Isolate the variable

- Add 3 to both sides:

$6B - 3 + 3 > 17 + 3$

which simplifies to:

$6B > 20$

- Divide both sides by 6 (note: since $6 > 0$, the inequality sign remains the same):

$B > 20/6$

- Simplify the fraction:

$B > 10/3$

Step 2: Interpret the solution

- The solution set consists of all real numbers B greater than 10/3, which is approximately 3.333...

In interval notation, this can be expressed as:

$(10/3, \square)$

Deeper Insights: The Nature of the Inequality and Its Graphical Representation

Understanding the solution set is more than just solving the inequality; it involves visualizing its significance and implications.

Graphical Perspective

- Plotting the inequality 6B - 3 > 17 on a number line:
- Mark the critical point at B = 10/3.
- The solution includes all points to the right of this value.
- The graph illustrates a half-line extending infinitely to the right, indicating all B values greater than 10/3 satisfy the inequality.

Implications of the Solution

- The inequality models situations where a certain proportional relationship exceeds a threshold.
- For example, in real-world scenarios:
- If B represented a rate or quantity, this inequality could represent minimum thresholds.
- Understanding the domain of B helps in decision-making processes where constraints are modeled algebraically.

Extensions and Variations: Exploring Related Inequalities

The initial inequality is a specific case, but exploring related forms broadens our understanding.

1. Equality Case

- What if the inequality were an equality?

6B - 3 = 17

- Solving:

6B = 20

B = 10/3

- This single value demarcates the boundary between solutions satisfying the inequality and those that do not.

2. Inequality with Different Constants

- Adjusting the constants yields different solution sets:

Inequality	Solution for B	Description
6B - 3 > 17	B > 10/3	Values greater than 10/3
6B - 3 < 17	B < 10/3	Values less than 10/3
6B - 3 □ 17	B □ 10/3	Values greater than or equal to 10/3
6B - 3 □ 17	B □ 10/3	Values less than or equal to 10/3

3. Quadratic or Higher-Order Variations

- Introducing quadratic terms or more complex expressions can model more nuanced situations, such as:

6B^2 - 3 > 17

- Solving:

6B^2 > 20

B^2 > 20/6 = 10/3

- Solutions:

B < -√(10/3) or B > √(10/3)

This demonstrates how simple linear inequalities serve as gateways to more complex problem-solving.

Real-World Applications and Significance

Mathematical inequalities like the one under investigation are not mere academic exercises—they have practical relevance across various fields.

1. Economics and Business

- Determining minimum acceptable profit margins or thresholds for investment returns often involves inequalities.
- Example: Ensuring revenue (modeled as a function of quantity B) exceeds a certain target.

2. Engineering and Physics

- Designing systems where certain parameters must stay above or below thresholds.
- Example: The stress on a component (B) must be greater than a minimum to ensure safety but within limits.

3. Data Science and Analytics

- Establishing conditions for model parameters to satisfy performance criteria.
- Inequalities help define feasible solution spaces.

4. Education and Pedagogy

- Teaching students how to interpret and manipulate inequalities enhances problem-solving skills.

Potential Pitfalls and Common Misconceptions

While the inequality 6B - 3 > 17 appears straightforward, misconceptions can occur:

- Misreading the inequality: Confusing the direction of the inequality sign during operations.
- Neglecting the division step: Failing to divide both sides by 6 correctly, especially when dealing with negative coefficients in other contexts.
- Assuming the solution set includes equality: Remembering that > means strictly greater, not greater or equal.

Being attentive to these aspects ensures accurate solutions and interpretations.

Conclusion: The Power of Algebraic Inequalities

The investigation of the inequality "Three subtracted from the product of six and a number is greater than 17" illuminates the process of translating verbal statements into algebraic forms, solving inequalities systematically, and interpreting solutions within real-world contexts. This example underscores the elegance and utility of algebra as a language for modeling, analyzing, and solving problems across diverse disciplines.

By mastering such inequalities, learners and professionals alike can develop sharper analytical skills, foster critical thinking, and apply mathematical reasoning to a broad spectrum of challenges. Whether in academia, industry, or daily life, understanding inequalities like 6B - 3 > 17 empowers informed decision-making and problem-solving—core tenets of mathematical literacy.

In essence, this exploration demonstrates that even seemingly simple inequalities encapsulate complex reasoning and meaningful applications, reinforcing the importance of precise interpretation and solution strategies in mathematics.

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