Twice A Number Added To A Smaller Number Is 5. The Difference Of 5 Times The Smaller Number And The Larger

Understanding the Problem Statement

Introduction to the Problem

The statement "Twice A Number Added To A Smaller Number Is 5. The Difference Of 5 Times The Smaller Number And The Larger" presents a mathematical scenario involving two numbers, one larger and one smaller. This problem requires translating a verbal statement into algebraic expressions and solving for the unknowns.

Breaking Down the Statement

The statement contains two parts:

1. "Twice A Number Added To A Smaller Number Is 5"
This indicates a relationship between two numbers, where one is doubled and then added to the other, resulting in 5.

2. "The Difference Of 5 Times The Smaller Number And The Larger"
This suggests an expression involving the difference between five times the smaller number and the larger number.

Understanding these parts is essential before formulating equations.

Assigning Variables

Choosing Symbols for the Unknowns

To translate the problem into algebraic form, assign variables:

- Let x = the smaller number
- Let y = the larger number

With these variables, we can write the relationships described in the problem.

Establishing the First Equation

From "Twice A Number Added To A Smaller Number Is 5," we get:

\[2y + x = 5 \]

Note: Since the statement emphasizes "twice a number" and "a smaller number," and the phrase "twice a number" is typical, we need to clarify whether "twice a number" refers to the smaller or larger number. The natural interpretation here is:

- "Twice a number" refers to twice the larger number (y),
- "A smaller number" refers to the smaller number (x).

So, the first equation becomes:

$$ 2y + x = 5 $$

Important Clarification:
The problem wording can sometimes be ambiguous. Based on common problem structures, it's more logical to interpret "Twice a number" as "twice the larger number," especially if the second part involves the smaller number. For our purposes, we'll proceed with:

- Twice the larger number: $2y$
- Smaller number: x

Thus, the first equation is:

$$ 2y + x = 5 $$

Formulating the Second Equation

Interpreting the Second Part of the Problem

"The Difference Of 5 Times The Smaller Number And The Larger" suggests an expression:

$$ \text{Difference} = 5x - y $$

But since the phrase is incomplete, it likely indicates a relationship involving this difference. Typically, in algebra problems, such phrases are completed with an equality to a certain value or another expression.

Given the structure of the problem and common algebraic problem patterns, a reasonable assumption is that:

> The difference of 5 times the smaller number and the larger number equals some constant, possibly zero or another number.

In this case, since the statement is incomplete, it is often interpreted as:

$$ 5x - y = \text{some value} $$

But the problem statement seems truncated. To proceed logically, let's assume the problem is:

- "The difference of 5 times the smaller number and the larger number is 0," meaning:

$$ 5x - y = 0 $$

This is a common pattern in algebra problems, where the difference equals zero or some specified value.

Summary of the Equations

Based on the above interpretation, we have:

1. $2y + x = 5$
2. $5x - y = 0$

Note: If the problem has a different intended second part, adjustments can be made later.

Solving the System of Equations

Step 1: Express y in terms of x

From the second equation:

$$ 5x - y = 0 \implies y = 5x $$

Step 2: Substitute into the first equation

Plug $y = 5x$ into the first equation:

$$ 2(5x) + x = 5 $$
$$ 10x + x = 5 $$
$$ 11x = 5 $$
$$ x = \frac{5}{11} $$

Step 3: Find y

Using $y = 5x$:

$$ y = 5 \times \frac{5}{11} = \frac{25}{11} $$

Summary of Solutions

- Smaller number $x = \frac{5}{11}$
- Larger number $y = \frac{25}{11}$

These are the values satisfying the assumed equations.

Verifying the Solution

Check the First Equation

$$ 2y + x = 2 \times \frac{25}{11} + \frac{5}{11} = \frac{50}{11} + \frac{5}{11} = \frac{55}{11} = 5 $$

Correct.

Check the Second Equation

$$ 5x - y = 5 \times \frac{5}{11} - \frac{25}{11} = \frac{25}{11} - \frac{25}{11} = 0 $$

Correct.

Alternative Interpretations and Adjustments

Possibility of Different Second Parts

If the original problem intended the second part to be different, such as:

- The difference of 5 times the smaller number and the larger number equals a particular value,
- Or the phrase was incomplete, and the actual statement was:

"Twice a number added to a smaller number is 5. The difference of 5 times the smaller number and the larger number is some value."

In such cases, the equations would adjust accordingly.

Example with Different Assumptions

Suppose the second part is "The difference of 5 times the smaller number and the larger is 3," then:

$$ 5x - y = 3 $$

The system becomes:

1. $2y + x = 5$
2. $5x - y = 3$

Solving similarly:

- From the second: $y = 5x - 3$

- Substitute into the first:

$$2(5x - 3) + x = 5$$
$$10x - 6 + x = 5$$
$$11x - 6 = 5$$
$$11x = 11$$
$$x = 1$$
$$y = 5(1) - 3 = 2$$

Check:

- $2(2) + 1 = 4 + 1 = 5$ (correct)
- $5(1) - 2 = 5 - 2 = 3$ (matches assumption)

Solution:

- Smaller number $x = 1$
- Larger number $y = 2$

Conclusion and Summary

Key Takeaways

- Carefully interpret the verbal problem to formulate accurate equations.
- Assign variables logically; typically, the smaller number is x, the larger is y.
- Use substitution to solve the system of equations.
- Verify solutions by substituting back into original equations.
- Recognize that ambiguous wording may lead to multiple interpretations; clarify assumptions if possible.

Final Remarks

Mathematical word problems require precision in interpretation. When faced with incomplete or ambiguous statements, it's essential to consider common problem patterns and make logical assumptions. The approach outlined here demonstrates how to translate a verbal description into algebra, solve systematically, and verify results, providing a comprehensive understanding of the problem involving two numbers related by specific algebraic expressions.

If you encounter similar problems, remember to:

- Break down the statement into parts.
- Assign clear variables.
- Formulate equations based on the description.
- Solve the system step-by-step.
- Verify your solutions thoroughly.

This methodology ensures accurate problem-solving and enhances your algebraic reasoning skills.

Frequently Asked Questions

What does the equation 'Twice a number added to a smaller number is 5' translate to mathematically?

It translates to $2x + y = 5$, where x is the larger number and y is the smaller number.

How can we express 'The difference of 5 times the smaller number and the larger' in an equation?

It can be written as $5y - x$, representing 5 times the smaller number minus the larger number.

Given these conditions, how can we set up a system of equations to find the two numbers?

We can set up the system: $2x + y = 5$ and $5y - x =$ difference value (if given), then solve for x and y.

If the difference of 5 times the smaller number and the larger is 2, what are the values of the numbers?

Set up equations: $2x + y = 5$ and $5y - x = 2$. Solving these simultaneously gives the values of x and y.

What methods can be used to solve for the larger and smaller numbers in these types of problems?

Methods include substitution or elimination to solve the system of equations derived from the problem's conditions.

Additional Resources

Mathematical Problem-Solving: An In-Depth Exploration of the Relationship Between Two Numbers

When delving into the realm of algebra, one of the most fundamental yet powerful skills is translating real-world scenarios into mathematical expressions and solving for unknowns. Today, we explore a classic problem involving two numbers, which combines the concepts of linear equations, substitution, and logical reasoning. The problem is as follows:

"Twice a number added to a smaller number is 5. The difference of 5 times the smaller number and the larger."

This problem, at first glance, may seem straightforward but invites a deep understanding of algebraic principles, problem modeling, and solution strategies. As an expert in mathematics education and problem analysis, I will guide you through each step of understanding, modeling, and solving this intriguing problem.

Understanding the Problem: Breaking Down the Statement

Before jumping into equations and calculations, it’s crucial to interpret the problem carefully. The statement can be paraphrased as:

- "Twice a number" suggests we have a certain number, say x, and we're considering 2x.
- "Added to a smaller number" indicates the presence of another number, say y, which is smaller than x.
- The total of these two quantities is 5:
$2x + y = 5$

- The second part of the statement, "The difference of 5 times the smaller number and the larger," appears incomplete. However, in context, it suggests a comparison or an expression involving these numbers:

"The difference of 5 times the smaller number and the larger" likely refers to:
$(5y) - x$

- Typically, in such algebra problems, the phrase "difference" indicates subtraction, and the mention of "the larger" and "the smaller" refers to the two numbers, with the larger being x and the smaller being y, or vice versa.

Key assumptions and interpretations:

- The two numbers are x (larger) and y (smaller).
- The initial equation:
$2x + y = 5$
- The second expression involves the difference:
$(5y) - x$

Clarification of the problem statement:

The full statement is somewhat ambiguous, but typical algebraic problems of this style are structured as:

1. Twice the larger number added to the smaller number equals 5:
$2x + y = 5$

2. The difference between 5 times the smaller number and the larger number is a certain value, possibly 0 or some specific number, or perhaps the problem intends for us to find the relationship between these expressions, such as:
$(5y) - x = ?$

Given the incomplete nature, a common interpretation is:

> "Twice a number (x) added to a smaller number (y) is 5, and the difference of 5 times the smaller number and the larger number is zero."

or perhaps:

> "Find the values of x and y such that these conditions hold."

For the purpose of this comprehensive analysis, we'll assume the most common form:

- Equation 1:
$2x + y = 5$

- Equation 2:
$(5y) - x = 0$ (meaning the difference of 5 times the smaller and the larger is zero, i.e., they are equal in some proportion)

Formulating the Mathematical Model

Once the problem is interpreted, the next step is to formalize it into algebraic equations. Based on the above assumptions, we have:

Equation 1: Relationship Between x and y
$2x + y = 5$

This equation states that twice the larger number plus the smaller number equals 5.

Equation 2: Difference of 5 times the smaller and the larger
$(5y) - x = 0$

This simplifies to:
$5y = x$

This indicates the larger number x is five times the smaller number y.

Solving the System of Equations

With two equations:

1. $2x + y = 5$
2. $x = 5y$

we can substitute equation 2 into equation 1 to find y, then back-calculate x.

Step 1: Substitution

Substitute $x = 5y$ into $2x + y = 5$:

$2(5y) + y = 5$

Simplify:

$10y + y = 5$

$11y = 5$

Step 2: Solving for y

Divide both sides by 11:

$y = 5/11$

Step 3: Find x

Using $x = 5y$:

$x = 5\ (5/11) = 25/11$

Interpreting the Solutions: Numerical and Conceptual Insights

Our solutions are:

- $x = 25/11 \approx 2.2727$
- $y = 5/11 \approx 0.4545$

These fractional solutions reveal that the two numbers are not necessarily integers, which is common in algebraic problems.

Understanding these values:

- The larger number x is approximately 2.27, and the smaller number y is approximately 0.45.
- The ratio x/y is exactly 5, confirming the relation $x = 5y$.

Validation:

Check if these satisfy the original equations:

- Equation 1:
$2x + y = 2(25/11) + 5/11 = (50/11) + (5/11) = 55/11 = 5$ ✔️

- Equation 2:
$x = 5y$
$25/11 = 5(5/11) = 25/11$ ✔️

Both equations are satisfied, confirming the correctness of the solutions.

Deeper Analysis and Broader Context

This problem exemplifies several key concepts in algebra:

- Linear Equations: The system involves two linear equations, which are straightforward to solve through substitution.
- Variable Relationships: Recognizing that x and y are proportional simplifies the problem dramatically.

- Fractional Solutions: Not all problems yield integer solutions; understanding how to interpret and verify fractional solutions is vital.
- Problem Modeling: Correctly translating verbal statements into algebraic equations requires careful analysis and assumptions, especially when the problem statement is ambiguous.

Potential variations of this problem include:

- Changing the coefficients or constants in the equations.
- Introducing inequalities instead of equalities.
- Considering the problem with constraints, such as x and y being positive integers.

Practical Applications and Educational Significance

Understanding such problems is essential not only for mastering algebra but also for developing critical thinking skills applicable across disciplines:

- In Engineering: Balancing equations and solving for unknowns.
- In Economics: Modeling relationships between variables like supply and demand.
- In Computer Science: Algorithm design often involves solving systems of equations.

From an educational perspective, this problem serves as an excellent case study for:

- Teaching variable substitution.
- Emphasizing the importance of clear problem statements.
- Developing problem-solving strategies for real-world scenarios.

Summary and Key Takeaways

- The problem involves translating a verbal description into algebraic equations, highlighting the importance of precise interpretation.
- Formulating the system of equations is crucial:
- $2x + y = 5$
- $x = 5y$
- Solving the system yields fractional solutions:
- $x = 25/11$
- $y = 5/11$
- Validating solutions ensures correctness and reinforces understanding.
- Recognizing the proportional relationship $x = 5y$ simplifies the problem significantly.
- The process exemplifies core algebra skills: substitution, solving linear systems, and interpreting results.

Final Thoughts

By dissecting and solving this classic problem, we observe how algebra serves as a powerful tool for modeling and solving real-world situations. Whether in academic settings, professional contexts, or everyday problem-solving, mastering these techniques enhances logical thinking and analytical skills. Remember, every problem is an opportunity to explore, analyze, and understand the elegant structure of mathematics.

Interested in more advanced problem-solving techniques or real-world applications? Stay tuned for our upcoming articles where we delve into systems of equations, inequalities, and their diverse applications across various fields.

Twice A Number Added To A Smaller Number Is 5 The Difference Of 5 Times The Smaller Number And The Larger

Twice A Number Added To A Smaller Number Is 5 The Difference Of 5 Times The Smaller Number And The Larger

Related Articles

- Three Forces Of 12, 15, And 20 Newtons Are In Equilibrium. If The 12 Newton Force Is Directed Horizontally
- The Following Data Is Provided For Garcon Company And Pepper Company For The Year Ended December 31.Garcon
- A Crate Is Lifted Vertically 1.5 M And Then Held At Rest. The Crate Has Weight 100 N (i.e., It Is Supported

Back to Home