Use The Graph F(x)=-x^2+8x+20 To Determine The Values Of M For Which M= -x^2+8x+m=0 Will Have Two Positive

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Understanding quadratic functions and their intersections is fundamental in algebra and calculus. In this article, we explore the quadratic function $F(x) = -x^2 + 8x + 20$ and analyze how it helps determine the values of m for which the equation $M = -x^2 + 8x + m = 0$ has two positive solutions. This problem combines graphing techniques, quadratic properties, and inequality analysis to arrive at a comprehensive understanding of the conditions necessary for two positive roots, providing insight into the application of quadratic functions in problem-solving.

Understanding the Given Functions and Their Significance

Before delving into the problem, it's essential to understand the functions involved:

- The Parabola $F(x) = -x^2 + 8x + 20$:
This quadratic function opens downward (since the coefficient of x^2 is negative) and has specific intercepts and vertex, which influence the behavior of solutions.

- The Equation $M = -x^2 + 8x + m = 0$:
For a fixed m, this quadratic equation's roots are of interest, especially when both roots are positive.

The goal is to find the range of m such that the quadratic in x:

$$
-x^2 + 8x + m = 0
$$

has two positive solutions.

Analyzing the Quadratic $M = -x^2 + 8x + m = 0$

To understand for which m this quadratic has two positive roots, we analyze its properties:

2.1 Standard Form and Discriminant

Rewrite in standard form:

$$
-x^2 + 8x + m = 0 \quad \Rightarrow \quad x^2 - 8x - m = 0
$$

The discriminant Δ determines the nature of roots:

$$
\Delta = (-8)^2 - 4 \times 1 \times (-m) = 64 + 4m
$$

For real roots, $\Delta \geq 0$:

$$
64 + 4m \geq 0 \quad \Rightarrow \quad m \geq -16
$$

2.2 Roots of the Quadratic

The roots x_1 and x_2 are:

$$
x_{1,2} = \frac{8 \pm \sqrt{\Delta}}{2} = \frac{8 \pm \sqrt{64 + 4m}}{2}
$$

Since the coefficient of x^2 is negative, the parabola opens downward, and the quadratic in x crosses the x-axis at these roots.

Conditions for Both Roots to Be Positive

To ensure both solutions are positive, the roots must satisfy:

$$
x_1 > 0 \quad \text{and} \quad x_2 > 0
$$

Given the roots:

$$
x_1 = \frac{8 - \sqrt{64 + 4m}}{2}
$$
$$
x_2 = \frac{8 + \sqrt{64 + 4m}}{2}
$$

- x_2 is always greater than or equal to 4, since $\sqrt{64 + 4m} \geq 0$.

- For both roots to be positive:

1. The smaller root x_1 must be positive:

$$
x_1 > 0 \quad \Rightarrow \quad \frac{8 - \sqrt{64 + 4m}}{2} > 0
$$

Multiply both sides by 2:

$$
8 - \sqrt{64 + 4m} > 0
$$

$$
\sqrt{64 + 4m} < 8
$$

2. Since the square root is non-negative, the condition is:

$$
\sqrt{64 + 4m} < 8
$$

Square both sides:

$$
64 + 4m < 64
$$
$$
4m < 0
$$
$$
m < 0
$$

- Additionally, the discriminant condition requires:

$$
64 + 4m \geq 0 \quad \Rightarrow \quad m \geq -16
$$

\]

- Combining these constraints:

$$
-16 \leq m < 0
$$

- Summary: For both roots to be real and positive, m must satisfy:

$$
-16 \leq m < 0
$$

Understanding the Role of $F(x) = -x^2 + 8x + 20$ in the Problem

The function $F(x) = -x^2 + 8x + 20$ provides a visual and analytical tool to understand the behavior of the quadratic equation $M = -x^2 + 8x + m=0$.

2.1 Graphical Interpretation

- The parabola $F(x)$ opens downward with a vertex at:

$$
x_{vertex} = -\frac{b}{2a} = -\frac{8}{2 \times (-1)} = 4
$$

- The maximum value of $F(x)$:

$$
F(4) = -4^2 + 8 \times 4 + 20 = -16 + 32 + 20 = 36
$$

- The graph of $F(x)$ reaches its maximum at $(4, 36)$.

2.2 Connection to the Equation $M = -x^2 + 8x + m$

- For any fixed m, the quadratic $-x^2 + 8x + m$ is essentially the parabola $F(x)$ shifted vertically downward by $(20 - m)$.

- The roots of the quadratic correspond to the points where the parabola intersects the x-axis.

2.3 Geometrical Insight

- The condition that $M = 0$ has two positive roots corresponds to the parabola $F(x)$ intersecting the x-axis at two points with positive x-values.

- Since $F(4) = 36$, for the quadratic $-x^2 + 8x + m$, the maximum point is at $(4, 36)$.

- To have two positive roots, the parabola must cross the x-axis at points greater than 0, which depends on the value of m.

Determining the Range of m for Which $M=0$ Has Two Positive Roots

Based on the previous analysis, the roots are:

$$
x_{1,2} = \frac{8 \pm \sqrt{64 + 4m}}{2}
$$

To have both roots positive:

$$
-16 \leq m < 0
$$

This can be summarized as:

- When $m = -16$:
$\sqrt{64 + 4(-16)} = \sqrt{64 - 64} = 0$, roots:

$$
x = \frac{8 \pm 0}{2} = 4
$$

Both roots coincide at $x=4$, so the quadratic touches the x-axis at one point (a double root), which is positive.

- When $-16 < m < 0$:
Roots are real and satisfy:

$$
x_1 = \frac{8 - \sqrt{64 + 4m}}{2} > 0
$$
$$
x_2 = \frac{8 + \sqrt{64 + 4m}}{2} > 0
$$

Both roots are positive, and the quadratic intersects the x-axis at two points, both with positive x-values.

- When $m \geq 0$:
$\sqrt{64 + 4m} \geq 8$, making $x_1 \leq 0$, so at least one root is non-positive, violating the conditions for two positive roots.

Summary of the Conditions for m

To conclude, the values of m for which the quadratic $M = -x^2 + 8x + m = 0$ has two positive roots are:

$$
\boxed{
-16 \leq m < 0
}
$$

This interval ensures:

- The discriminant is non-negative ($\Delta \geq 0$), guaranteeing real roots.
- Both roots are positive

Frequently Asked Questions

What is the given quadratic function for which we need to find values of m?

The given quadratic function is $F(x) = -x^2 + 8x + 20$.

How is the quadratic equation M = -x^2 + 8x + m related to the function F(x)?

The quadratic equation $M = -x^2 + 8x + m$ is similar to $F(x)$, with the only difference being the constant term m, which shifts the graph vertically.

What condition must be satisfied for the quadratic equation to have two positive roots?

The quadratic must have a positive discriminant ($D > 0$) for two real roots, and both roots must be positive, which depends on the quadratic's roots being greater than zero.

How do we find the roots of the quadratic equation M = $-x^2 + 8x + m$?

The roots are found using the quadratic formula: $x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$, where $a = -1$, $b = 8$, and $c = m$.

What is the discriminant of the quadratic equation in terms of m?

The discriminant $D = b^2 - 4ac = 64 - 4(-1)(m) = 64 + 4m$.

What inequality must m satisfy for the quadratic to have two real roots?

Since $D > 0$, $64 + 4m > 0$, which simplifies to $m > -16$.

How do we ensure both roots are positive?

Using the roots formula, both roots are positive if the quadratic's vertex is to the left of the y-axis and the roots themselves are positive, which involves analyzing the roots and the sign of the quadratic at specific points.

What is the final range of m for which the quadratic equation has two positive roots?

The quadratic will have two positive roots when $m > -16$ and the roots, calculated from the quadratic formula, are both positive, which requires m to satisfy $m > -16$ and the roots to be greater than zero—specifically, m must be greater than 4 to ensure both roots are positive.

Additional Resources

Understanding how to determine the values of m for which the quadratic equation M = $-x^2 + 8x + m = 0$ has two positive solutions is a fundamental concept in algebra that combines graphing, quadratic functions, and discriminant analysis. In this detailed guide, we'll explore how the Use The Graph $F(x) = -x^2 + 8x + 20$ to determine the values of m for which M = $-x^2 + 8x + m = 0$ will have two positive solutions. By understanding the behavior of the quadratic function and the conditions for solutions, you'll gain insights into how the parameters influence the roots and how to identify the precise range of m that satisfy the given criteria.

Introduction to the Problem

Imagine you have a quadratic function $F(x) = -x^2 + 8x + 20$, which opens downward because the coefficient of x^2 is negative. Now, consider a similar quadratic function:

$$
M = -x^2 + 8x + m
$$

where m is a parameter that shifts the graph vertically. The question posed is: For which values of m does the quadratic equation $-x^2 + 8x + m = 0$ have two positive solutions?

This problem combines several key concepts:

- The shape and position of the quadratic graph $y = -x^2 + 8x + m$
- The nature of the roots of the quadratic (real and positive)
- The relationship between the parameter m and the roots' properties

By analyzing the quadratic function and its roots, we can find the precise values of m that satisfy the condition of having two positive solutions.

Understanding the Basic Quadratic Function

The General Form and Key Features

The quadratic function:

$$
F(x) = -x^2 + 8x + 20
$$

has:

- A downward opening parabola (since the coefficient of x^2 is negative).
- A vertex that determines the maximum point of the parabola.
- Roots (or zeros) where the graph intersects the x-axis.

Vertex of the Parabola

The vertex (x_v, y_v) of a parabola $y = ax^2 + bx + c$ is given by:

$$
x_v = -\frac{b}{2a}
$$
$$
y_v = c - \frac{b^2}{4a}
$$

For $F(x) = -x^2 + 8x + 20$:

$$
a = -1, \quad b = 8, \quad c = 20
$$

Calculating:

$$
x_v = -\frac{8}{2 \times -1} = -\frac{8}{-2} = 4
$$

$$
y_v = 20 - \frac{8^2}{4 \times -1} = 20 - \frac{64}{-4} = 20 + 16 = 36
$$

Interpretation: The vertex is at $(4, 36)$, which is the maximum point of the parabola. This indicates the maximum value of the original function $F(x)$.

Connecting $F(x)$ and the Parameter m

From $F(x)$ to M

The function:

$$
M = -x^2 + 8x + m
$$

is similar to $F(x)$, just with a different constant term m. It can be viewed as shifting the graph of $F(x)$ vertically up or down depending on m.

- When $m = 20$, the graphs of $F(x)$ and M coincide.
- For other values of m, the graph of M is shifted vertically relative to $F(x)$.

Graphical Interpretation

- The solutions to $M = 0$ correspond to the x-intercepts of the parabola $y = -x^2 + 8x + m$.
- To have two solutions, the parabola must intersect the x-axis at two points.
- To have two positive solutions, both these x-intercepts must be positive numbers.

Conditions for the Roots of the Quadratic

The Quadratic Equation

$$
-x^2 + 8x + m = 0
$$

can be rewritten as:

\[

$$
x^2 - 8x - m = 0
$$

by multiplying through by -1 for simplicity.

Discriminant Analysis

The discriminant Δ:

$$
\Delta = b^2 - 4ac
$$

determines the nature of roots:

- $\Delta > 0$: two real roots
- $\Delta = 0$: one real root (double root)
- $\Delta < 0$: no real roots

For our quadratic:

$$
a = 1, \quad b = -8, \quad c = -m
$$

Calculating:

$$
\Delta = (-8)^2 - 4 \times 1 \times (-m) = 64 + 4m
$$

For two real roots:

$$
64 + 4m > 0 \implies 4m > -64 \implies m > -16
$$

Roots of the Quadratic

Using the quadratic formula:

$$
x = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{8 \pm \sqrt{64 + 4m}}{2}
$$

which simplifies to:

$$
x = 4 \pm \frac{\sqrt{64 + 4m}}{2}
$$

Ensuring Both Roots Are Positive

Since we want both solutions to be positive, the following conditions must hold:

1. Both roots are real: $m > -16$
2. Both roots are greater than zero:

- First root:

$$
x_1 = 4 - \frac{\sqrt{64 + 4m}}{2}
$$

- Second root:

$$
x_2 = 4 + \frac{\sqrt{64 + 4m}}{2}
$$

For both roots to be positive:

$$
x_1 > 0 \quad \text{and} \quad x_2 > 0
$$

The second root x_2 is always greater than 4 (since the square root term is non-negative), so it is always positive if the roots are real.

Focus on the first root:

$$
x_1 = 4 - \frac{\sqrt{64 + 4m}}{2} > 0
$$

Solve for m:

$$
4 > \frac{\sqrt{64 + 4m}}{2}
$$

Multiply both sides by 2:

$$
8 > \sqrt{64 + 4m}
$$

Square both sides:

$$

$$
64 > 64 + 4m
$$

Subtract 64:

$$
0 > 4m \implies m < 0
$$

Summary of Conditions for m

Putting it all together:

- Discriminant condition for two real roots:

$$
m > -16
$$

- Positivity of both roots:

$$
m < 0
$$

- Ensures both roots are positive:

$$
-16 < m < 0
$$

This interval guarantees that:

- The quadratic $-x^2 + 8x + m$ has two real roots.
- Both roots are positive numbers.

Visualizing the Graph and Roots

Graphical Representation

- When m is within $(-16, 0)$, the parabola $y = -x^2 + 8x + m$ intersects the x-axis at two points, both to the right of the y-axis (positive x-values).
- The maximum point (vertex at $(4, 36 + m - 20)$) remains above the x-axis, ensuring two positive roots.

Significance of the Parameter m

- As m approaches -16 from above, the roots coalesce at the origin, approaching zero.
- As m approaches zero from below, the roots are close to $(0, \text{positive})$.

Practical Applications and Additional Insights

Real-World Contexts

This analysis applies in various scenarios such as:

- Projectile motion: Determining initial velocities or heights that yield two positive time solutions.
- Optimization problems: Finding parameters that produce two feasible positive solutions.
- Engineering designs: Ensuring systems have two positive states under certain parameter constraints.

Extending the Analysis

- What if the parabola opens upward? The conditions for roots would change, and the analysis would need to be adjusted.
- What about roots that are positive but not necessarily both? Similar methods can be used to isolate conditions for exactly one or no positive roots.

Conclusion

In conclusion, by analyzing the quadratic function \(M = -x^

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