We Want To Test The Null Hypothesis That Population Mean = 10. Using The Following Observations, Calculate

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In statistical analysis, hypothesis testing is a fundamental technique used to make inferences or draw conclusions about a population based on sample data. One common scenario involves testing whether the population mean equals a specific value, often referred to as the null hypothesis. In this article, we will explore the process of testing the null hypothesis that the population mean equals 10, using a set of observations. We will walk through the necessary calculations, interpret the results, and understand the significance of each step in the context of statistical inference.

Understanding the Null Hypothesis and Its Importance

What Is a Null Hypothesis?

The null hypothesis, denoted as H_0, is a statement that there is no effect or no difference in the population parameter being tested. In our case, it asserts that the population mean (μ) is equal to 10:

$$
H_0: \mu = 10
$$

This serves as the baseline assumption that we test against the alternative hypothesis.

Why Test the Null Hypothesis?

Testing the null hypothesis allows us to determine whether the observed data provides sufficient evidence to reject this baseline assumption. If the data significantly deviates from what we would expect under H_0, we might conclude that the true population mean differs from 10.

Gathering the Observations and Preparing for Calculation

Suppose we have collected a sample of observations from a population and wish to assess whether the mean of this population is 10. The observations are as follows:

- 12, 9, 11, 10, 8, 13, 7, 10, 9, 11

These observations serve as the basis for our hypothesis test.

Step 1: Calculate the Sample Mean ($\bar{x}$)

The sample mean provides an estimate of the population mean based on the data:

$$
\bar{x} = \frac{\sum_{i=1}^n x_i}{n}
$$

Where:
- x_i are individual observations
- n is the sample size

Calculating:

$$
\bar{x} = \frac{12 + 9 + 11 + 10 + 8 + 13 + 7 + 10 + 9 + 11}{10} = \frac{100}{10} = 10
$$

The sample mean is exactly 10, which aligns with the null hypothesis value.

Step 2: Calculate the Sample Standard Deviation (s)

The sample standard deviation measures how spread out the observations are:

$$
s = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n - 1}}
$$

Calculations:

Observation (x_i)	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
12	2	4
9	-1	1
11	1	1
10	0	0

8	-2	4
13	3	9
7	-3	9
10	0	0
9	-1	1
11	1	1

Sum of squared deviations:

$$
4 + 1 + 1 + 0 + 4 + 9 + 9 + 0 + 1 + 1 = 30
$$

Standard deviation:

$$
s = \sqrt{\frac{30}{10 - 1}} = \sqrt{\frac{30}{9}} \approx \sqrt{3.33} \approx 1.82
$$

Conducting the Hypothesis Test

To determine whether the sample supports the null hypothesis, we perform a t-test for the population mean.

Step 1: Formulate the Test Statistic

The t-statistic for testing the population mean is calculated as:

$$
t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}
$$

Where:
- $\bar{x}$ = sample mean
- μ_0 = hypothesized population mean (10)
- s = sample standard deviation
- n = sample size

Plugging in the values:

$$
t = \frac{10 - 10}{1.82 / \sqrt{10}} = \frac{0}{1.82 / 3.16} = 0
$$

The calculated t-value is 0.

Step 2: Determine the Degrees of Freedom and Critical Value

- Degrees of freedom (df): $n - 1 = 9$
- Significance level (α): commonly 0.05 (5%)

Using a t-distribution table or calculator, the critical t-value for a two-tailed test at $\alpha = 0.05$ and df = 9 is approximately:

$$
t_{critical} \approx \pm 2.262
$$

Interpreting the Results

Since our calculated t-statistic is 0, which lies well within the range -2.262 to 2.262, we do not reject the null hypothesis.

Conclusion: The sample data provides no evidence to suggest that the population mean differs from 10 at the 5% significance level.

Additional Considerations in Hypothesis Testing

Understanding P-Values

The p-value indicates the probability of observing a test statistic as extreme as the one calculated, assuming the null hypothesis is true. In our case, since $t=0$, the p-value is 1, meaning there is a very high probability of seeing such data if the null hypothesis is true.

Implications of Sample Size

A larger sample size enhances the power of the test, making it more capable of detecting small differences from the hypothesized mean. Conversely, small samples may not provide sufficient evidence to reject the null hypothesis, even if the true population mean differs.

Assumptions of the T-Test

- The data are approximately normally distributed, especially important with small samples.
- The observations are independent.
- The sample is randomly selected from the population.

Summary and Practical Applications

In this case, the sample mean matched the hypothesized population mean, leading to a t-statistic of zero and no grounds to reject $H_0: \mu=10$. This process exemplifies how hypothesis testing can be applied in various fields such as quality control, clinical trials, and social sciences to make informed decisions based on sample data.

Key steps in hypothesis testing include:

1. Defining the null and alternative hypotheses.
2. Calculating the sample mean and standard deviation.
3. Computing the test statistic.
4. Determining the critical value or p-value.
5. Making a decision to reject or not reject the null hypothesis based on the comparison.

Conclusion

Hypothesis testing remains a vital tool in statistical analysis, enabling researchers and analysts to assess claims about populations. By following systematic procedures—calculating sample statistics, understanding distribution characteristics, and interpreting results—it is possible to draw meaningful conclusions. In our scenario, the data supported the null hypothesis that the population mean equals 10, demonstrating the importance of thorough calculations and understanding underlying assumptions in statistical inference.

Remember: Always consider the context of your data, the assumptions underlying the tests, and the implications of your findings when making decisions based on hypothesis testing.

Frequently Asked Questions

What is the null hypothesis in testing whether the population mean equals 10?

The null hypothesis is that the population mean equals 10, expressed as H_0: $\mu = 10$.

How do you calculate the test statistic for testing the population mean against a hypothesized value?

The test statistic is calculated as $t = (\bar{x} - \mu_0) / (s / \sqrt{n})$, where $\bar{x}$ is the sample mean, μ_0 is the hypothesized mean (10), s is the sample standard deviation, and n is the sample size.

What information do I need from the observations to perform the hypothesis test?

You need the individual observations to compute the sample mean, sample standard deviation, and the sample size.

How do I interpret the p-value obtained from the t-test in this context?

The p-value indicates the probability of observing a sample mean as extreme as or more extreme than the one calculated, assuming the null hypothesis is true. A small p-value suggests evidence against H_0.

What are the assumptions underlying the t-test for the population mean?

The assumptions include that the data are approximately normally distributed (especially for small samples), the observations are independent, and the data are measured on an interval or ratio scale.

If the calculated p-value is less than the significance level (e.g., 0.05), what conclusion can be drawn?

You can reject the null hypothesis and conclude that there is statistically significant evidence that the population mean differs from 10.

How do I perform the calculation if I only have the observations and need to test the null hypothesis that the mean is 10?

First, compute the sample mean and standard deviation from the observations, then calculate the t-statistic using the formula $t = (\bar{x} - 10) / (s / \sqrt{n})$. Finally, compare this t-value to the critical t-value or compute the p-value to make your decision.

Additional Resources

Testing the Null Hypothesis That Population Mean = 10: An In-Depth Examination of Methodology and Calculation

In the realm of statistical inference, hypothesis testing serves as a fundamental tool for making informed decisions about population parameters based on sample data. Among these, the test of a population mean against a specified value—often the null hypothesis that the mean equals a particular figure—is one of the most commonly employed techniques. This article delves into the process of testing the null hypothesis that the population mean equals 10, illustrating the methodology with concrete calculations derived from sample observations. Through a systematic exploration, we will unpack the underlying principles, assumptions, and interpretative nuances that underpin this statistical procedure.

Understanding the Null Hypothesis and Its Significance

What is a Null Hypothesis?

The null hypothesis, denoted as H_0, is a statement of no effect or no difference that serves as the default assumption in hypothesis testing. In this context, it asserts that the population mean (μ) is equal to a specified value—here, 10. Formally:

$$
H_0: \mu = 10
$$

Testing this hypothesis involves determining whether the observed sample data provide sufficient evidence to reject H_0 in favor of an alternative hypothesis.

The Alternative Hypothesis

The alternative hypothesis (H_1 or H_A) represents a statement contrary to the null. Depending on the research question, it can be:

- Two-tailed: $H_A: \mu \neq 10$
- One-tailed: $H_A: \mu > 10$ or $H_A: \mu < 10$

In our analysis, the specific form of the alternative will influence the critical values and interpretation.

Why Test the Null Hypothesis?

Hypothesis testing provides a structured framework to assess evidence from sample data. It helps determine whether observed deviations from the hypothesized mean are statistically significant or likely due to random variation. This process is essential in scientific research, quality control, and decision-making in various fields.

Data Collection and Sample Observations

The Sample Data

Suppose we have collected a sample comprising the following observations:

Observation Number	Data Point
1	x_1
2	x_2
...	...
n	x_n

(Note: In your actual scenario, you would replace these placeholders with the specific data points you have.)

Importance of Sample Data

The sample serves as the basis for inference about the population. Key features such as the sample mean ($\bar{x}$) and sample standard deviation (s) are calculated from the observations. These statistics provide the necessary inputs to perform the hypothesis test.

Statistical Foundations of the Test

Assumptions

Before conducting the test, certain assumptions are typically made:

1. Random Sampling: The observations are randomly sampled from the population.
2. Normality: The distribution of the population is approximately normal, especially important for small sample sizes.
3. Independence: Observations are independent of each other.

Selecting the Appropriate Test

Depending on the sample size and variance knowledge, different tests may be appropriate:

- z-test: Used when the population variance is known or the sample size is large (usually $n \geq 30$)
- t-test: Used when the population variance is unknown and the sample size is small ($n < 30$)

In most practical situations where the population variance is unknown, the one-sample t-test is employed.

Calculating the Test Statistic

Step 1: Compute Sample Mean ($\bar{x}$)

$$
\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i
$$

Step 2: Compute Sample Standard Deviation (s)

$$
s = \sqrt{\frac{1}{n - 1} \sum_{i=1}^n (x_i - \bar{x})^2}
$$

Step 3: Formulate the Test Statistic

For the one-sample t-test, the test statistic t is:

$$
t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}
$$

where:

- $\bar{x}$ = sample mean
- μ_0 = hypothesized population mean (here, 10)
- s = sample standard deviation
- n = sample size

This statistic follows a Student's t-distribution with $n - 1$ degrees of freedom under the null hypothesis.

Performing the Calculation: Step-by-Step Example

Let's assume the following sample observations:

Observation	Value
1	12
2	9
3	11
4	10
5	13
6	8
7	12
8	9

9	10
10	11

Sample size (n): 10

Step 1: Calculating the Sample Mean ($\bar{x}$)

$$
\bar{x} = \frac{12 + 9 + 11 + 10 + 13 + 8 + 12 + 9 + 10 + 11}{10}
$$

$$
\bar{x} = \frac{105}{10} = 10.5
$$

Step 2: Calculating the Sample Standard Deviation (s)

First, compute each deviation:

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
12	1.5	2.25
9	-1.5	2.25
11	0.5	0.25
10	-0.5	0.25
13	2.5	6.25
8	-2.5	6.25
12	1.5	2.25
9	-1.5	2.25
10	-0.5	0.25
11	0.5	0.25

Sum of squared deviations:

$$
\sum (x_i - \bar{x})^2 = 2.25 + 2.25 + 0.25 + 0.25 + 6.25 + 6.25 + 2.25 + 2.25 + 0.25 + 0.25 = 23.75
$$

Sample variance:

$$
s^2 = \frac{23.75}{n - 1} = \frac{23.75}{9} \approx 2.64
$$

Sample standard deviation:

$$
s = \sqrt{2.64} \approx 1.62
$$

Step 3: Calculating the t-Statistic

$$
t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}} = \frac{10.5 - 10}{1.62 / \sqrt{10}} = \frac{0.5}{1.62 / 3.16} \approx \frac{0.5}{0.512} \approx 0.977
$$

Interpreting the Results

Step 1: Determine the Critical Value

The critical value depends on:

- Significance level (α), commonly 0.05
- Type of test (two-tailed or one-tailed)
- Degrees of freedom ($df = n - 1 = 9$)

Using a t-distribution table or calculator for $\alpha = 0.05$:

- Two-tailed test: critical value $t_{0.025, 9} \approx \pm 2.262$
- One-tailed test: critical value $t_{0.05, 9} \approx 1.833$

Step 2: Decision Rule

- If the calculated t exceeds the critical value (in absolute value for two-tailed), reject H_0.
- Otherwise, fail to reject H_0.

In our case:

$$
t \approx 0.977 < 2.262 \quad \text{(for two-tailed)}
$$

Step 3: Conclusion

Since the calculated t-value (approximately 0.977) is less than the critical value (2.262), we fail to reject the null hypothesis at the 5%

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