Which Graph Represents The Function F (x) = StartFraction 2 Over X Minus 1 EndFraction + 4?On A Coordinate

Which Graph Represents The Function F (x) = $\frac{2}{x - 1}$ + 4? On A Coordinate

Understanding how to interpret and analyze the graph of the function $F(x) = \frac{2}{x - 1} + 4$ is essential for students and professionals working with rational functions. This article provides a comprehensive guide to identifying the correct graph representation of this function by analyzing its key features, asymptotes, transformations, and behavior on a coordinate plane.

Understanding the Basic Structure of the Function

Before diving into the graphical analysis, it is important to understand the basic structure of the given function:

$$F(x) = \frac{2}{x - 1} + 4$$

This is a rational function composed of a reciprocal component $\frac{2}{x - 1}$ and a constant shift of +4. To interpret its graph properly, we should analyze its components separately.

Analyzing the Parent Function

The core of this function is $\frac{1}{x}$, which is known as the parent rational function. The parent function has the following characteristics:

- It has two branches: one in the first quadrant (positive x and y) and another in the third quadrant (negative x and y).
- It has a vertical asymptote at $x=0$, where the function is undefined.
- It has a horizontal asymptote at $y=0$, as x approaches infinity or negative infinity.

Transformations Applied to the Parent Function

The given function incorporates several transformations:

1. Horizontal Shift: The denominator is $x - 1$, indicating a shift of the graph to the right by 1 unit. The vertical asymptote moves from $x=0$ to $x=1$.

2. Vertical Reflection and Stretch: The numerator is 2, which affects the steepness of the branches. Since 2 is positive, the graph maintains the same orientation as the parent function but scaled vertically.

3. Vertical Shift: The entire graph is shifted upward by 4 units due to the +4 outside the fraction.

Putting these together, the transformations are:

- Horizontal shift: right by 1 unit, resulting in a vertical asymptote at $x=1$.
- Vertical shift: upward by 4 units, shifting all points accordingly.
- No reflection since the numerator is positive.

Key Features of the Graph

To correctly identify the graph, focus on the following features:

Vertical Asymptote

- Located where the denominator equals zero: $x - 1 = 0 \Rightarrow x=1$.
- The graph approaches infinity or negative infinity as x approaches 1 from the left and right.

Horizontal Asymptote

- As $x \to \pm \infty$, the $\frac{2}{x - 1}$ term approaches 0.
- The entire function approaches the horizontal line $y=4$.

Y-Intercept

- Find the point where $x=0$:

$$
F(0) = \frac{2}{0 - 1} + 4 = \frac{2}{-1} + 4 = -2 + 4 = 2
$$

- So, the graph passes through $(0, 2)$.

X-Intercept

- Find where $F(x) = 0$:

$$
0 = \frac{2}{x - 1} + 4
$$
$$
\frac{2}{x - 1} = -4
$$
$$
2 = -4(x - 1)
$$
$$
2 = -4x + 4
$$
$$
-4x = 2 - 4 = -2
$$
$$
x = \frac{-2}{-4} = \frac{1}{2}
$$
- The graph passes through $\left(\frac{1}{2}, 0\right)$.

Behavior Near Asymptotes and at Infinity

Understanding how the function behaves near asymptotes and at the extremes of the domain will help in distinguishing the correct graph:

- As $x \to 1^-$, the denominator approaches zero from the negative side, so $\frac{2}{x - 1}$ tends to $-\infty$, making $F(x) \to -\infty$.

- As $x \to 1^+$, $\frac{2}{x - 1} \to +\infty$, so $F(x) \to +\infty$.

- As $x \to \pm \infty$, the function approaches $y=4$. The graph will get closer to the horizontal asymptote but never touch it.

Summary of key points:

Feature	Value / Behavior

Vertical asymptote	$x=1$
Horizontal asymptote	$y=4$
Y-intercept	$(0, 2)$
X-intercept	$\left(\frac{1}{2}, 0\right)$
Behavior near asymptote	$x \to 1^-$: $F(x) \to -\infty$; $x \to 1^+$: $F(x) \to +\infty$

Plotting the Function Step-by-Step

To visualize the graph, follow these steps:

1. Plot the asymptotes:
- Draw a vertical dashed line at $x=1$.
- Draw a horizontal dashed line at $y=4$.

2. Mark key points:
- $(0, 2)$, which is below the asymptote.
- $\left(\frac{1}{2}, 0\right)$, on the x-axis.

3. Determine the behavior on each side of the asymptote:
- For $x < 1$, as $x \to 1^-$, $F(x) \to -\infty$, and as $x \to -\infty$, $F(x) \to 4$ from below.
- For $x > 1$, as $x \to 1^+$, $F(x) \to +\infty$, and as $x \to +\infty$, $F(x) \to 4$ from below.

4. Sketch the branches:
- On the left side of the asymptote, the graph starts near $-\infty$ when approaching $x=1$ and approaches $y=4$ as $x \to -\infty$.
- On the right side, the graph starts near $+\infty$ close to $x=1$ and approaches $y=4$ as $x \to +\infty$.

Note: The graph will be continuous in each branch, approaching asymptotes but never crossing them.

Identifying the Correct Graph Among Options

When presented with multiple graph options, use the following checklist to identify the correct one:

- Vertical asymptote at $x=1$: Is there a dashed or unbroken vertical line at $x=1$?
- Horizontal asymptote at $y=4$: Does the graph approach $y=4$ as $x \to \pm \infty$?
- Intercepts:
- Does the graph pass through $(0, 2)$?
- Does it cross the x-axis at $\left(\frac{1}{2}, 0\right)$?

- Behavior near asymptotes:
- On the left of $x=1$, does the graph tend to $-\infty$?
- On the right of $x=1$, does it tend to $+\infty$?

Matching these features with the options will help you select the correct graph.

Common Mistakes and Pitfalls

While analyzing the graph, be cautious of:

- Misplacing the vertical asymptote: Ensure it's at $x=1$, not elsewhere.
- Incorrect horizontal asymptote: Remember that horizontal asymptotes for rational functions are determined by degrees of numerator and denominator; in this case, as $x \to \pm \infty$, $F(x) \to 4$.
- Overlooking the shifts: The upward shift of +4 moves the horizontal asymptote from $y=0$ to $y=4$.
- Confusing intercepts: Confirm the calculations for x- and y-intercepts.

Conclusion

The graph of $F(x) = \frac{2}{x - 1} + 4$ features a vertical asymptote at $x=1$, a horizontal asymptote at $y=4$, and passes through points $(0, 2)$ and \(\left(\frac{1

Frequently Asked Questions

What type of graph best represents the function F(x) = 2/(x - 1) + 4?

It is a hyperbola with a vertical asymptote at x = 1 and a horizontal asymptote at y = 4.

Where are the asymptotes located for the graph of F(x) = 2/(x - 1) + 4?

The vertical asymptote is at x = 1, and the horizontal asymptote is at y = 4.

How does shifting the basic hyperbola y = 1/x affect its graph in the function F(x) = 2/(x - 1) + 4?

The graph shifts the basic hyperbola one unit to the right (due to x - 1) and four units up (due to + 4).

What is the domain and range of $F(x) = 2/(x - 1) + 4$?

The domain is all real numbers except $x = 1$; the range is all real numbers except $y = 4$.

How does the coefficient 2 in $F(x) = 2/(x - 1) + 4$ affect the graph?

It affects the steepness of the hyperbola; a larger absolute value makes the branches steeper.

Is the function $F(x) = 2/(x - 1) + 4$ even, odd, or neither?

It is neither even nor odd because it does not satisfy the symmetry conditions for either.

What are the key features to identify the graph of $F(x) = 2/(x - 1) + 4$?

Vertical asymptote at $x=1$, horizontal asymptote at $y=4$, and hyperbolic shape approaching these asymptotes.

How would the graph change if the function was $F(x) = -2/(x - 1) + 4$?

The hyperbola would be reflected across the x-axis, flipping the branches vertically.

Why does the graph of $F(x) = 2/(x - 1) + 4$ have a vertical asymptote at $x=1$?

Because the denominator $x - 1$ becomes zero at $x=1$, causing the function to approach infinity or negative infinity.

What is the behavior of the graph of $F(x) = 2/(x - 1) + 4$ as x approaches the asymptote at $x=1$?

As x approaches 1 from the left, the function approaches negative infinity; from the right, it approaches positive infinity.

Additional Resources

Which Graph Represents The Function F (x) = \(\frac{2}{x - 1}\) + 4? On A Coordinate Plane

Understanding the graphical representation of functions is fundamental in mathematics, especially for visual learners and those seeking to interpret real-world data. Today, we delve into the analysis of a specific rational function, \(F(x) = \frac{2}{x - 1} + 4 \), exploring how its algebraic form translates into a visual graph on the coordinate plane. The goal is to identify which graph among multiple options correctly depicts this function’s behavior and features.

This exploration involves examining key aspects such as domain, asymptotes, intercepts, and the general shape of the graph, providing clarity on how algebraic expressions manifest visually. Whether you're a student preparing for exams or a teacher preparing lesson materials, understanding this function graph will enhance your grasp of rational functions and their graphical characteristics.

Understanding the Function's Algebraic Structure

Before visualizing the graph, it's essential to analyze the function algebraically. The given function is:

$$F(x) = \frac{2}{x - 1} + 4$$

This is a rational function, composed of a rational component $\frac{2}{x - 1}$ shifted vertically by 4 units. To interpret it visually, we need to identify its key features:

- Vertical Asymptote: The value of x that makes the denominator zero, causing the function to approach infinity or negative infinity.
- Horizontal (or Oblique) Asymptote: The line the function approaches as x approaches infinity or negative infinity.
- Intercepts: Points where the graph crosses the axes.

Let's analyze each component in detail.

Analyzing the Key Features of the Function

1. Domain and Vertical Asymptote

The domain of $F(x)$ excludes any value that makes the denominator zero:

$$
x - 1 = 0 \Rightarrow x = 1
$$

Hence, the function is undefined at $x = 1$. At this point, the function exhibits a vertical asymptote—a line that the graph approaches but never touches or crosses (except possibly in the limit).

Vertical asymptote: $x = 1$

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2. Horizontal Asymptote

As $x \to \pm \infty$, the term $\frac{2}{x - 1}$ approaches zero because the denominator grows without bound. Therefore, the entire function approaches:

$$
F(x) \to 0 + 4 = 4
$$

Horizontal asymptote: $y = 4$

This means that, far to the left and right, the graph will get closer to $y=4$, but not necessarily touch it.

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3. Intercepts

- Y-intercept: Set $x=0$:

$$
F(0) = \frac{2}{0 - 1} + 4 = \frac{2}{-1} + 4 = -2 + 4 = 2
$$

So, the graph crosses the y-axis at $(0, 2)$.

- X-intercept: Set $F(x) = 0$:

$$
0 = \frac{2}{x - 1} + 4
$$

Subtract 4 from both sides:

$$
-4 = \frac{2}{x - 1}
$$

Multiply both sides by $x - 1$:

$$
-4 (x - 1) = 2
$$

Distribute:

$$
-4x + 4 = 2
$$

Subtract 4 from both sides:

$$
-4x = -2
$$

Divide both sides by -4:

$$
x = \frac{-2}{-4} = \frac{1}{2}
$$

Thus, the graph crosses the x-axis at $(\frac{1}{2}, 0)$.

Visualizing the Graph: Key Features Summarized

Feature	Description	Coordinates / Line
Vertical asymptote	Line $x = 1$	Vertical dashed line at $x=1$
Horizontal asymptote	Line $y=4$	Horizontal dashed line at $y=4$
Y-intercept	Where the graph crosses y-axis	$(0, 2)$
X-intercept	Where the graph crosses x-axis	$\left(\frac{1}{2}, 0\right)$

The Graph's General Shape and Behavior

Given the features, the graph of $F(x)$ will display the following behaviors:

- For $x < 1$: As $x \to 1^-$, $F(x)$ approaches $-\infty$. When x is very negative, the term $\frac{2}{x - 1}$ approaches zero from the negative side, so $F(x) \to 4^-$. The graph in this region approaches the horizontal asymptote from below and goes downward toward the vertical asymptote at $x=1$.

- For $x > 1$: When $x \to 1^+$, $F(x) \to +\infty$. As $x \to +\infty$, $F(x) \to 4^+$, approaching the asymptote from above.

- The graph will be hyperbolic in shape, with two branches: one on the left of the vertical asymptote and one on the right.

Recognizing the Correct Graph: Key Visual Clues

To identify the correct graph among multiple options, look for:

1. Vertical asymptote at $x=1$: A dashed or marked line indicating the asymptote at $x=1$.
2. Horizontal asymptote at $y=4$: A line that the graph approaches but does not touch, near $y=4$.
3. Intercepts:
- Y-intercept at $(0, 2)$.
- X-intercept at $\left(\frac{1}{2}, 0\right)$.
4. End behavior:
- As $x \to -\infty$, $F(x) \to 4^-$.
- As $x \to +\infty$, $F(x) \to 4^+$.
5. Branches:
- For $x<1$, the graph descends from near $-\infty$ at the asymptote and approaches $y=4$ from below.
- For $x>1$, the graph rises from $-\infty$ at the asymptote and approaches $y=4$ from above.

Confirming the Correct Graph: Practical Approach

Suppose you are presented with four different graphs. To identify the one that accurately portrays $F(x) = \frac{2}{x-1} + 4$:

- Step 1: Locate the vertical asymptote at $x=1$. Confirm that only one graph has this feature.
- Step 2: Check the horizontal asymptote at $y=4$. The graph should get closer to this line as x moves away from the asymptote.
- Step 3: Verify the intercepts:
- The y-intercept should be at $(0, 2)$.
- The x-intercept should be at $\left(\frac{1}{2}, 0\right)$.
- Step 4: Observe the end behavior on both sides of the vertical asymptote.
- Step 5: Confirm the shape of the branches: one approaching from below on the left, one from above on the right.

If these features align precisely, that is the graph representing the function.

Additional Considerations: Transformations and Shifts

The function $F(x) = \frac{2}{x - 1} + 4$ can be viewed as a parent function $f(x) = \frac{1}{x}$, shifted:

- Rightward shift: 1 unit (due to $x - 1$)
- Upward shift: 4 units (adding 4)

Understanding these transformations helps in quickly visualizing the graph without plotting numerous points.

Conclusion: Which Graph Best Represents $F(x) = \frac{2}{x - 1} + 4$?

In summary, the correct graph will display:

- A vertical asymptote at $x=1$,
- A horizontal asymptote at $y=4$,
- Intercepts at $(0, 2)$ and $\left(\frac{1}{2}, 0\right)$,
- Behavior approaching the asymptotes as described.

Careful observation of these features ensures accurate identification. This rational function exemplifies how algebraic transformations—shifts and scaling—manifest visually, illustrating the deep connection between algebra and geometry. Mastery of these concepts enables students and educators alike to interpret and analyze rational functions effectively, fostering a stronger understanding of their behavior across the coordinate plane.

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