Write An Equation For The Line That Is Parallel To The Given Line And That Passes Through The Given Point.

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Understanding how to write the equation of a line that is parallel to a given line and passes through a specific point is a fundamental skill in algebra and coordinate geometry. This process involves analyzing the slope of the original line, recognizing the properties of parallel lines, and applying the point-slope form of a line’s equation. Whether you're a student preparing for exams, a teacher designing lesson plans, or someone interested in applied mathematics, mastering this concept is essential for solving many geometric problems.

In this comprehensive guide, we'll explore step-by-step methods to find the equation of a parallel line passing through a given point. We will delve into the underlying principles involving slopes, the different forms of linear equations, and practical examples to solidify your understanding. By the end of this article, you'll be equipped to handle similar problems confidently and accurately.

Understanding the Basics of Line Equations

Before diving into the process, it’s crucial to understand the fundamental forms of linear equations and the role of slope.

Forms of a Line Equation

There are several common forms of linear equations:

- **Slope-Intercept Form:** $y = mx + b$
- **Point-Slope Form:** $y - y_1 = m(x - x_1)$
- **Standard Form:** $Ax + By = C$

Where:
- m = slope of the line
- (x_1, y_1) = a specific point on the line

- A, B, C = constants with A and B not both zero

Understanding these forms is essential because the method to find a parallel line depends on the slope and a point through which the line passes.

What Does It Mean for Lines to Be Parallel?

Two lines are parallel if they have the same slope but different y-intercepts. This means they will never intersect, no matter how far they extend. Formally:

- If line 1 has slope m_1, then line 2 is parallel if $m_2 = m_1$.

Step-by-Step Approach to Write the Equation of a Parallel Line

The process involves three main steps:

1. Identify the slope of the given line
2. Use the point you are given to write the new line's equation in point-slope form
3. Simplify or convert the equation into your preferred form (slope-intercept or standard form)

Let's explore each step in detail.

Step 1: Find the Slope of the Given Line

The first step is to determine the slope (m_1) of the original line. Depending on how the line is presented, this might involve:

1.1. Given the Equation of the Original Line

If the original line's equation is provided, such as:

- $y = 2x + 3$

then the slope is directly read as 2.

If the line is in standard form, like:

- $3x – 4y = 7$

you can rearrange it into slope-intercept form:

- $3x – 4y = 7$
- $–4y = –3x + 7$
- $y = (3/4)x – 7/4$

Now, the slope is $m_1 = 3/4$.

1.2. Given Two Points on the Original Line

If the original line passes through points (x_1, y_1) and (x_2, y_2), calculate the slope using:

$$
m_1 = \frac{y_2 - y_1}{x_2 - x_1}
$$

For example, if points are (1, 2) and (3, 6):

$$
m_1 = \frac{6 - 2}{3 - 1} = \frac{4}{2} = 2
$$

This slope will be used to determine the slope of the parallel line.

Step 2: Use the Slope and the Given Point to Write the Equation

Once you know the slope of the original line, the slope of the parallel line is the same. Now, you need the specific point through which the new line passes.

2.1. Applying the Point-Slope Form

The point-slope form is especially convenient:

$$
y - y_1 = m (x - x_1)
$$

where:
- (x_1, y_1) is the given point
- m = slope of the parallel line (same as the original line)

Example:

Suppose the original line has slope $m_1 = 2$, and the point given is (4, 5). The equation of the line parallel to the original and passing through (4, 5):

$$
y - 5 = 2 (x - 4)
$$

Expanding:

$$
y - 5 = 2x - 8
$$

$$
y = 2x - 8 + 5
$$

$$
y = 2x - 3
$$

This is the slope-intercept form of the new line.

2.2. Alternative: Standard Form

You can also convert the point-slope form into standard form:

$$
y - y_1 = m(x - x_1)
$$

Rearranged to:

$$
mx - y = m x_1 - y_1
$$

or

$$
A x + B y = C

\]

depending on your preference.

Step 3: Simplify and Write the Final Equation

The last step involves expressing the equation in the desired form, whether it's slope-intercept, standard, or another preferred form.

3.1. Converting to Slope-Intercept Form

If you've used point-slope form, you might want to solve for y:

\[
y = m x + b
\]

In the previous example, the equation was:

\[
y = 2x - 3
\]

which is already in slope-intercept form.

3.2. Converting to Standard Form

To convert y = 2x – 3 into standard form:

\[
2x – y = 3
\]

Ensure all terms are on one side and coefficients are integers.

Practical Examples to Reinforce the Concept

Let's work through several examples to illustrate the process.

Example 1: Find the Equation of a Line Parallel to $y = -\frac{1}{2}x + 4$ Passing Through (6, 2)

Step 1: Slope of given line: $m_1 = -\frac{1}{2}$

Step 2: Use point (6, 2):

$$
y - 2 = -\frac{1}{2}(x - 6)
$$

Expand:

$$
y - 2 = -\frac{1}{2}x + 3
$$

Solve for y:

$$
y = -\frac{1}{2}x + 3 + 2 = -\frac{1}{2}x + 5
$$

Final Equation: $y = -\frac{1}{2}x + 5$

Example 2: Original Line is $3x + 4y = 12$, Point (0, 5)

Step 1: Convert to slope-intercept form:

$$
3x + 4y = 12
$$
$$
4y = -3x + 12
$$
$$
y = -\frac{3}{4}x + 3
$$

Step 2: Slope of original line: $m_1 = -\frac{3}{4}$

Use point (0, 5):

$$
y - 5 = -\frac{3}{4}(x - 0) \Rightarrow y - 5 = -\frac{3}{4}x
$$

Add 5:

$$
y = -\frac{3}{4}x + 5
$$

Final Equation: $y = -\frac{3}{4}x + 5$

Special Cases and Additional Tips

While the above method works for most scenarios, some special cases require extra attention.

1. Vertical Lines

- If the original line is vertical, e.g., $x = 7$, then all lines parallel to it are also vertical with the same x-value.
- To pass through a point (x_1, y_1), the line's equation is simply:

$$
x = x_1
$$

Example: Parallel to $x = 7$ passing through (3, 4):

$$
x = 3
$$

2. Horizontal Lines

- If the original line is horizontal, e.g., $y = 4$, then all parallel lines are $y = c$, where c is any constant.
- To pass through

Frequently Asked Questions

How do I find the equation of a line parallel to a given

line that passes through a specific point?

First, determine the slope of the given line. Since parallel lines have the same slope, use that slope and the given point to write the equation in point-slope form: $y - y_1 = m(x - x_1)$. Then, convert to slope-intercept or standard form as needed.

What is the step-by-step process to write an equation for a line parallel to y = 2x + 3 passing through (4, 5)?

Step 1: Identify the slope of the given line, which is 2. Step 2: Use this slope and the point (4, 5) in the point-slope form: $y - 5 = 2(x - 4)$. Step 3: Simplify to get $y - 5 = 2x - 8$, then add 5 to both sides: $y = 2x - 3$.

Can I write the equation of a line parallel to a vertical line such as x = 7 passing through a point?

Yes. A line parallel to a vertical line $x=7$ is also vertical and has the same x-coordinate. If the point is (x_0, y_0), then the parallel line's equation is $x = x_0$.

If the given line is in standard form, like 3x - 4y = 12, how do I find the parallel line passing through a point?

First, rewrite the given line in slope-intercept form to identify its slope: $y = (3/4)x - 3$. Then, use this slope and the point in point-slope form: $y - y_1 = (3/4)(x - x_1)$. Finally, simplify to get the equation of the parallel line.

What should I do if the given point lies on the given line when writing a parallel line equation?

If the point lies on the original line, the parallel line passing through that point is the line itself. Otherwise, use the point and the original line's slope to write the new parallel line's equation.

Additional Resources

Write An Equation For The Line That Is Parallel To The Given Line And That Passes Through The Given Point

Understanding how to write an equation for the line that is parallel to the given line and that passes through the given point is a fundamental skill in algebra and analytic geometry. This process involves recognizing the properties of parallel lines—primarily their equal slopes—and applying algebraic techniques to derive the desired equation. Whether you're working on homework, preparing for a test, or exploring geometric concepts, mastering this skill allows you to analyze and construct lines efficiently and accurately.

In this comprehensive guide, we'll explore the steps involved in translating a word problem into a precise algebraic equation, highlight common pitfalls, and provide practical tips to ensure clarity and correctness. By the end, you'll be equipped with a clear methodology to handle similar problems with confidence.

Understanding the Core Concepts

Before diving into the step-by-step process, it's essential to understand some foundational concepts:

What is a Parallel Line?

Two lines are parallel if they are in the same plane and never intersect, regardless of how far they extend. Geometrically, this means they have the same slope but different y-intercepts.

Slope-Intercept Form of a Line

The most common form of a line's equation in algebra is:

$y = m x + b$

where:
- m is the slope of the line,
- b is the y-intercept (where the line crosses the y-axis).

How to Find the Equation of a Parallel Line

Given a line L_1 with equation $y = m x + b_1$, any line L_2 parallel to L_1 must have the same slope, m. To find L_2, you need:
- The slope (which is the same as L_1),
- A point through which L_2 passes.

Step-by-Step Guide to Write the Equation

Step 1: Identify the Slope of the Given Line

If your given line's equation is provided in slope-intercept form (e.g., $y = 2x + 3$), then:
- The slope (m) is the coefficient of x (here, 2).

If the given line is in standard form ($Ax + By = C$), you need to convert it to slope-intercept form:
- Solve for y: $y = (-A / B) x + (C / B)$,
- Then identify m as $-A / B$.

Example:
Given line: $3x - 4y = 7$
- Solve for y: $-4y = -3x + 7 \rightarrow y = (3/4) x - 7/4$
- Slope, $m = 3/4$

Step 2: Determine the Slope of the Parallel Line

Since parallel lines share the same slope, the slope of the new line L_2 passing through the given point is also m.

Example:
If the original line has slope m = 3/4, then the new line's slope is also 3/4.

Step 3: Use the Point-Slope Form

The point-slope form is especially useful for constructing the line's equation using known point and slope:

$y - y_1 = m (x - x_1)$

where:
- (x_1, y_1) is the point the line passes through,
- m is the slope.

Example:
Suppose the point is (2, 5), and the slope m = 3/4.

Plug into the point-slope form:
$y - 5 = (3/4)(x - 2)$

Step 4: Simplify to Slope-Intercept Form

Expand and simplify to get the line's equation in $y = m x + b$ form:

1. Distribute m:
$y - y_1 = m x - m x_1$

2. Add y_1 to both sides:
$y = m x - m x_1 + y_1$

3. Calculate the constant term:
$b = - m x_1 + y_1$

Continuing the example:
$y - 5 = (3/4)(x - 2)$
=> $y - 5 = (3/4) x - (3/4)(2)$
=> $y - 5 = (3/4) x - (3/2)$

Add 5 to both sides:
$y = (3/4) x - (3/2) + 5$

Express 5 as a fraction with denominator 2:
$5 = 10/2$

So:
$y = (3/4) x - (3/2) + (10/2)$

=> $y = (3/4)x + (7/2)$

Final equation: $y = (3/4)x + (7/2)$

Practical Examples

Let's walk through some real-world problems to reinforce these concepts.

Example 1: Standard Form to Parallel Line Equation

Given: The line $2x - y = 4$, passing through point (1, 3).
Find: The equation of the line parallel to the given line that passes through (1, 3).

Solution:
- Convert to slope-intercept form:
$2x - y = 4 \rightarrow -y = -2x + 4 \rightarrow y = 2x - 4$
- Slope of given line, $m = 2$
- Use point-slope form:
$y - 3 = 2(x - 1)$
- Expand:
$y - 3 = 2x - 2$
- Solve for y:
$y = 2x - 2 + 3 \rightarrow y = 2x + 1$

Answer: The equation of the parallel line is $y = 2x + 1$.

Example 2: Given in Slope-Intercept Form

Given: The line $y = -1/3\, x + 4$, passing through the point (6, 2).
Find: The equation of the line parallel to the given line passing through (6, 2).

Solution:
- Slope of the given line: $m = -1/3$
- Use point-slope form:
$y - 2 = (-1/3)(x - 6)$
- Expand:
$y - 2 = (-1/3)x + 2$
- Add 2 to both sides:
$y = (-1/3)x + 4$

Note: The resulting equation simplifies to the same as the original, confirming that the line passing through (6, 2) with the same slope is $y = (-1/3)x + 4$.

Special Cases and Tips

Vertical Lines

- The slope of a vertical line is undefined.
- Parallel vertical lines have the same x-coordinate for all points.
- To write an equation of a vertical line passing through (x_1, y_1):
$x = x_1$

Example:
Line passing through (3, 5):
Equation: $x = 3$

Horizontal Lines

- The slope of a horizontal line is 0.
- Parallel horizontal lines have the same y-coordinate.
- To write the equation passing through (x_1, y_1):
$y = y_1$

Example:
Line passing through (2, -1):
Equation: $y = -1$

Tips for Success

- Always convert the given line to slope-intercept form to identify the slope easily.
- Carefully identify the point coordinates, ensuring accuracy.
- Use the point-slope form for quick setup, then simplify.
- Be mindful of fractions; find common denominators when necessary.
- For vertical or horizontal lines, remember their special forms.

Summary: The Complete Process in a Nutshell

1. Identify the slope of the given line:
- Convert to $y = m x + b$ if necessary.
2. Use the point-slope form with the given point:
- $y - y_1 = m (x - x_1)$
3. Simplify to slope-intercept form:
- Expand, combine like terms, solve for y.
4. Write the final equation:
- Express in $y = m x + b$ form.

Final Thoughts

The ability to write an equation for the line that is parallel to the given line and that passes through the given point is a cornerstone in algebraic problem-solving and geometric analysis. It combines understanding of slopes, algebraic manipulation, and careful substitution. With practice, you'll develop an intuition for quickly recognizing the properties of lines and translating word problems into precise mathematical expressions. Keep practicing with different formats and contexts to solidify your skills, and you'll find this task becomes second nature.

Remember, mastering these steps not only helps in solving immediate problems but also builds a strong foundation for more advanced topics in mathematics, such as linear programming, calculus, and analytical geometry.

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