

You Apply A Constant Force $\mathbf{F} = (68.0\text{N})\hat{i} + (36.0\text{N})\hat{j}$ To A 450 Kg Car As The Car Travels 44.0 M In A Direction

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This scenario presents a compelling opportunity to analyze the dynamics of a car subjected to a constant force as it moves over a specified distance. By examining the forces involved, the resulting acceleration, and the work done on the vehicle, we can gain insights into fundamental physics principles such as Newton's second law, work-energy theorem, and vector analysis. This article delves into the step-by-step calculations and conceptual understanding necessary to fully comprehend the motion of the car under the given conditions.

Understanding the Problem Setup

Given Data

- Force applied: $\mathbf{F} = (68.0\text{N})\hat{i} + (36.0\text{N})\hat{j}$
- Mass of the car: $m = 450\text{kg}$
- Distance traveled: $d = 44.0\text{m}$

Objective

Determine various physical quantities such as:

- The acceleration of the car
- The work done by the applied force
- The final velocity of the car after traveling 44.0 meters
- The direction of the car's motion relative to the force

Analyzing the Force and Its Components

Vector Nature of the Force

The applied force is expressed in vector form, indicating both magnitude and direction in a two-dimensional plane:

$$\mathbf{F} = 68.0\hat{i} + 36.0\hat{j}$$

This represents a force vector with components along the x-axis ($\hat{i}$) and y-axis ($\hat{j}$).

Magnitude of the Force

To understand the overall influence of the force, compute its magnitude:

$$|\vec{F}| = \sqrt{(68.0)^2 + (36.0)^2} \quad \text{N}$$
$$|\vec{F}| = \sqrt{4624 + 1296} = \sqrt{5920} \approx 77.0, \text{ N}$$

Direction of the Force

The direction can be characterized by the angle θ relative to the x-axis:

$$\theta = \arctan \left(\frac{F_y}{F_x} \right) = \arctan \left(\frac{36.0}{68.0} \right)$$
$$\theta \approx \arctan(0.5294) \approx 28.0^\circ$$

This indicates the force points approximately 28 degrees above the x-axis.

Calculating the Car's Acceleration

Applying Newton's Second Law

Newton's second law states:

$$\vec{F}_{\text{net}} = m \vec{a}$$

Given the force is constant and assuming no other forces (like friction or air resistance) are acting, the acceleration vector is:

$$\vec{a} = \frac{\vec{F}}{m}$$

Calculating each component:

$$a_x = \frac{F_x}{m} = \frac{68.0, \text{ N}}{450, \text{ kg}} \approx 0.151, \text{ m/s}^2$$
$$a_y = \frac{F_y}{m} = \frac{36.0, \text{ N}}{450, \text{ kg}} \approx 0.080, \text{ m/s}^2$$

The acceleration vector:

$$\vec{a} \approx 0.151, \hat{i} + 0.080, \hat{j}, \text{ m/s}^2$$

Magnitude and Direction of Acceleration

- Magnitude:

$$|\vec{a}| = \sqrt{(0.151)^2 + (0.080)^2} \approx \sqrt{0.0228 + 0.0064} \approx \sqrt{0.0292} \approx 0.171, \text{ m/s}^2$$

\]

- Direction:

$$\phi = \arctan \left(\frac{a_y}{a_x} \right) \approx \arctan \left(\frac{0.080}{0.151} \right) \approx 28.0^\circ$$

\]

Note that the acceleration points in the same direction as the force.

Determining the Velocity After Traveling 44.0 M

Using Work-Energy Principle

The work done by the force on the car is converted into kinetic energy, assuming no losses:

\[

$$W = \Delta KE = \frac{1}{2} m v^2 - 0$$

\]

since initial velocity $(v_0 = 0)$ (assuming the car starts from rest).

Calculating Work Done

Work done by a constant force over a displacement is:

\[

$$W = \vec{F} \cdot \vec{d}$$

\]

where $(\vec{d})$ is the displacement vector of magnitude 44.0 m in some direction.

Important: Since the direction of $(\vec{d})$ is not specified, we assume that the displacement vector is aligned with the direction of the force for simplicity, or that the component of the force along the direction of displacement dominates.

If the car travels in the direction of the force vector, then:

\[

$$W = |\vec{F}| \times d \times \cos \theta$$

\]

but here, since the force and displacement are in the same direction (or the problem implies movement along the force), we can simplify by projecting the force onto the displacement.

Assuming the displacement is along the direction of the force:

\[

$$W = |\vec{F}| \times d = 77.0, \text{ N} \times 44.0, \text{ m} = 3388, \text{ J}$$

\]

Alternatively, if the displacement is along the x-axis or y-axis, or at an angle, the calculation adjusts accordingly. For simplicity, we proceed with the assumption that the force vector and displacement are aligned.

Calculating Final Velocity

Using the work-energy theorem:

$$\frac{1}{2} m v^2 = W$$
$$v = \sqrt{\frac{2 W}{m}} = \sqrt{\frac{2 \times 3388 \text{ J}}{450 \text{ kg}}}$$
$$v \approx \sqrt{\frac{6776}{450}} \approx \sqrt{15.05} \approx 3.88 \text{ m/s}$$

Thus, the car's speed after traveling 44 meters is approximately 3.88 m/s.

Additional Considerations and Real-World Implications

Effect of Friction and External Forces

In real scenarios, forces such as friction, air resistance, and rolling resistance oppose the applied force, reducing the net acceleration and the final velocity. If such forces are present, the net force would be:

$$\vec{F}_{\text{net}} = \vec{F} - \vec{F}_{\text{resistance}}$$

and the calculations for acceleration and work would need to incorporate these opposing forces.

Energy Efficiency and Power

The power exerted by the force can be calculated as:

$$P = \vec{F} \cdot \vec{v}$$

which depends on the velocity at any instant. Initially, with zero velocity, the power is zero, but as the car accelerates, the power increases.

Implication for Vehicle Dynamics

Understanding how the applied force influences acceleration and velocity is crucial for vehicle design, safety analysis, and fuel efficiency optimization. The ability to predict final velocity after a certain distance helps in planning acceleration profiles and understanding limits under given forces.

Summary of Key Results

- The acceleration of the car is approximately 0.171 m/s^2 with components along x and y axes.
- The magnitude of the acceleration vector points roughly 28° above the x-axis.
- The work done by the applied force over 44 meters is approximately 3388 Joules.
- The final velocity of the car after traveling 44 meters, assuming no other resistances, is approximately 3.88 m/s .
- The force and displacement are assumed to be aligned for simplicity; real-world scenarios may require more detailed vector analysis based on actual direction of movement.

Conclusion

Applying a constant force to a mass involves fundamental principles of physics that govern motion and energy transfer. By dissecting the force into components, calculating acceleration, work, and final velocity, we gain a comprehensive understanding of the dynamics involved. This analysis not only illuminates the behavior of vehicles under external forces but also underscores the importance of vector analysis

Frequently Asked Questions

What are the components of the constant force applied to the car?

The force components are 68.0 N in the x-direction and 36.0 N in the y-direction.

How can we calculate the work done by the force on the car over 44.0 meters?

Work done is calculated using $W = F \cdot d \cdot \cos\theta$, where θ is the angle between the force and displacement. Alternatively, since the force and displacement are known, we can find the component of the force in the direction of motion and multiply by the distance.

What is the magnitude of the applied force?

The magnitude of the force is $\sqrt{(68.0^2 + 36.0^2)} \approx 77.4 \text{ N}$.

How do we determine the acceleration of the car under this constant force?

Using Newton's second law, acceleration $a = F_{\text{total}} / m$. First, find the total force magnitude, then divide by the car's mass of 450 kg .

What is the acceleration of the car given the applied force?

The total force magnitude is approximately 77.4 N , so acceleration $a \approx 77.4 \text{ N} / 450 \text{ kg} \approx 0.172 \text{ m/s}^2$.

How long does it take for the car to travel 44.0 meters under this acceleration?

Using the kinematic equation $d = (1/2) a t^2$, solve for t : $t = \sqrt{2d / a}$. Substituting $d=44.0$ m and $a \approx 0.172$ m/s² gives $t \approx 23.0$ seconds.

What is the final velocity of the car after traveling 44.0 meters?

Using $v = at$, the final velocity $v \approx 0.172$ m/s² $\times$ 23.0 s $\approx$ 3.96 m/s.

How much work is done by the force on the car during the 44.0 meters of travel?

Work done $W = F_{\text{component}} \times d$ in the direction of motion. First, find the component of the force in the direction of displacement, then multiply by 44.0 m to find the work.

Is the applied force in the same direction as the car's displacement?

The force components are in x and y directions, but the displacement is along a certain direction (not specified). To determine the work and motion, the angle between the force and displacement must be considered. If the displacement is along the same direction as the force's resultant component, the work is maximized.

Additional Resources

In-Depth Analysis of Applying a Constant Force to a Car Traveling a Specified Distance

Introduction

Understanding the dynamics of forces acting on vehicles is fundamental in physics, especially when analyzing motion in real-world scenarios. In this review, we focus on a specific case: applying a constant force $\vec{F} = (68.0\text{ N})\hat{i} + (36.0\text{ N})\hat{j}$ to a car with a mass of 450 kg as it travels 44.0 meters in a certain direction. This analysis covers the physics principles involved, calculations to determine the car's acceleration, velocity, and work done, as well as implications for real-world applications such as automotive engineering and safety.

Understanding the Force Vector

Components of the Force

The force vector $\vec{F}$ has two components:

- Horizontal component, $(F_x = 68.0\text{ N})$: This acts along the x-axis, influencing the forward or backward motion depending on the direction.
- Vertical component, $(F_y = 36.0\text{ N})$: Typically, vertical forces are associated with gravity and normal force, but a force with a vertical component can suggest an inclined plane or lifting force. In this context, the presence of a vertical component indicates the force is not purely horizontal, and the direction of the resultant movement depends on the overall force direction.

Resultant Force Magnitude and Direction

Calculating the magnitude of the total force:

$$|\vec{F}| = \sqrt{F_x^2 + F_y^2} = \sqrt{(68.0)^2 + (36.0)^2} \approx \sqrt{4624 + 1296} = \sqrt{5920} \approx 77.0\text{ N}$$

And the angle (θ) relative to the x-axis:

$$\theta = \arctan\left(\frac{F_y}{F_x}\right) = \arctan\left(\frac{36.0}{68.0}\right) \approx \arctan(0.529) \approx 28^\circ$$

This indicates the force is directed approximately 28° above the x-axis, providing a combined effect of pushing the car forward and slightly upward.

Analysis of the Car's Motion

Given Data Summary

- Mass of the car, $(m = 450\text{ kg})$
- Displacement, $(d = 44.0\text{ m})$
- Force, $\vec{F} = (68.0\text{ N})\hat{i} + (36.0\text{ N})\hat{j}$

Assumptions and Simplifications

- The force remains constant throughout the displacement.
- External resistive forces such as friction and air resistance are not explicitly considered unless specified.
- The motion occurs along a plane, and the force's vertical component affects the net vertical forces but does not do work if the vertical displacement is negligible or zero. However, since the displacement is specified as 44.0 m in a certain direction, the effective work depends on the component of the force in that direction.

Calculating the Work Done on the Car

Work-Energy Principle

The work done by a force on an object is given by:

$$W = \vec{F} \cdot \vec{d}$$

where $\vec{d}$ is the displacement vector.

Determining Displacement Direction

Since the problem states the car travels 44.0 meters in a certain direction, and the force vector makes an angle of approximately 28° with the x-axis, we need to know whether the displacement is aligned with the force or at some angle.

Case 1: Displacement in the direction of the force (most straightforward assumption).

Case 2: Displacement along a different direction.

For simplicity, assume the car moves along the direction of $\vec{F}$, i.e., along the line at 28° to the x-axis.

Calculating the Work

- The displacement vector:

$$\vec{d} = 44.0 \text{ m} \times \hat{u}$$

where $\hat{u}$ is the unit vector in the direction of motion.

- The work:

$$W = |\vec{F}| |\vec{d}| \cos \phi$$

where ϕ is the angle between force and displacement directions.

If displacement is along force direction:

$$\phi = 0^\circ \Rightarrow \cos \phi = 1$$

then,

$$W = 77.0 \, \text{N} \times 44.0 \, \text{m} = 3388 \, \text{J}$$

This means approximately 3388 Joules of work are done on the car by the applied force during the 44.0-meter travel, assuming the force and displacement are aligned.

Determining the Car's Acceleration and Final Velocity

Net Force Components and Acceleration

- The net force in the x-direction:

$$F_x = 68.0 \, \text{N}$$

- The net force in the y-direction:

$$F_y = 36.0 \, \text{N}$$

- The net acceleration vector:

$$\vec{a} = \frac{\vec{F}}{m}$$

\]

which has components:

\[

$$a_x = \frac{68.0\,\text{N}}{450\,\text{kg}} \approx 0.151\,\text{m/s}^2$$

\]

\[

$$a_y = \frac{36.0\,\text{N}}{450\,\text{kg}} \approx 0.080\,\text{m/s}^2$$

\]

The magnitude of acceleration:

\[

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(0.151)^2 + (0.080)^2} \approx \sqrt{0.0228 + 0.0064} \approx \sqrt{0.0292} \approx 0.171\,\text{m/s}^2$$

\]

and the direction:

\[

$$\theta_a = \arctan\left(\frac{a_y}{a_x}\right) \approx \arctan\left(\frac{0.080}{0.151}\right) \approx 28^\circ$$

\]

which matches the force angle, confirming that acceleration points in the same direction as the applied force.

Calculating Final Velocity

Using kinematic equations:

\[

$$v_f^2 = v_i^2 + 2ad$$

\]

Assuming the car starts from rest ($v_i=0$):

\[

$$v_f = \sqrt{2ad}$$

\]

Plugging in the values:

\[

$$v_f = \sqrt{2 \times 0.171\,\text{m/s}^2 \times 44.0\,\text{m}} \approx \sqrt{15.05} \approx 3.88\,\text{m/s}$$

\]

Thus, after traveling 44 meters under this force, the car reaches a final speed of approximately 3.88 m/s.

Implications of Vertical Force Component

While the vertical component of the force (36.0 N) does not directly contribute to motion in the horizontal direction, it can influence other factors:

- Normal Force Adjustment: If the force acts upward, it reduces the normal force exerted by the road on the car, which could decrease friction if the road surface and tire conditions allow.
- Potential Vertical Displacement: If the vertical component causes a lift or descent, it could affect the contact between tires and the road, impacting traction.
- Energy Considerations: Since vertical force does not do work in the horizontal direction, its primary effect is on the normal force and possibly on the contact mechanics.

In real-world applications, such vertical forces could originate from aerodynamic lift, road inclines, or aerodynamic elements designed to modify vertical forces for performance.

Energy Considerations and Power

Work-Energy Relationship

The work done by the force is converted into kinetic energy:

$$W = \Delta KE = \frac{1}{2} m v_f^2$$

Calculating the kinetic energy:

$$KE = \frac{1}{2} \times 450\text{ kg} \times (3.88\text{ m/s})^2 \approx 225 \times 15.05 \approx 3388\text{ J}$$

which confirms the earlier work calculation matches the energy required to reach the final speed.

Average Power