Compare The Wavelength Of A 1.0-MeV Gamma-ray Photon With That Of A Neutron Having The Same Kinetic Energy.

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Understanding the fundamental properties of particles, especially their wavelengths, is crucial in fields like quantum physics, nuclear physics, and materials science. When examining particles such as gamma-ray photons and neutrons with identical kinetic energies, their differing natures—one being a massless electromagnetic wave and the other a massive particle—lead to significant differences in their wavelengths. This article provides a comprehensive comparison of the wavelengths of a 1.0-MeV gamma-ray photon and a neutron with the same kinetic energy, explaining the underlying physics, calculations, and implications.

Introduction to Particle Wavelengths and De Broglie's Hypothesis

The concept of particle wavelength stems from De Broglie's hypothesis, which states that every particle exhibits wave-like properties. The wavelength (λ) of a particle is inversely related to its momentum (p), given by the de Broglie relation:

$$
\lambda = \frac{h}{p}
$$

where h is Planck's constant (6.626×10^{-34} Js).

For massless particles like photons, energy and momentum are directly related, whereas for massive particles like neutrons, the total energy includes rest mass energy and kinetic energy, affecting their momentum and, consequently, their wavelength.

Key Concepts and Definitions

Gamma-ray Photons

- Electromagnetic radiation with high energy, typically above 100 keV.
- Massless particles that always travel at the speed of light ($c = 3 \times 10^{8}$ m/s).

- Energy relates to frequency (f) via $E = hf$.
- Wavelength is inversely proportional to energy:

$$
\lambda = \frac{c}{f} = \frac{hc}{E}
$$

Neutrons

- Neutral subatomic particles with a rest mass of approximately 1.675×10^{-27} kg.
- Have both rest energy ($E_0 = mc^2$) and kinetic energy (K).
- Travel at speeds less than c, depending on their kinetic energy.
- Their de Broglie wavelength depends on their total momentum, which incorporates their kinetic and rest mass energy.

Calculating the Wavelength of a 1.0-MeV Gamma-ray Photon

Since gamma-ray photons are massless, their energy directly relates to their wavelength:

$$
\lambda_{\text{photon}} = \frac{hc}{E}
$$

where:

- $h = 6.626 \times 10^{-34}$ Js
- $c = 3 \times 10^{8}$ m/s
- $E = 1.0$ MeV = $1.0 \times 10^{6} \times 1.602 \times 10^{-19}$ J = 1.602×10^{-13} J

Calculating the wavelength:

$$
\lambda_{\text{photon}} = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{1.602 \times 10^{-13}}
= \frac{1.9878 \times 10^{-25}}{1.602 \times 10^{-13}}
\approx 1.24 \times 10^{-12}\ \text{meters}
$$

This wavelength (~1.24 picometers) is characteristic of high-energy gamma rays, comparable to atomic scales.

Calculating the Wavelength of a Neutron with 1.0-MeV Kinetic Energy

Unlike photons, neutrons are massive particles, and their de Broglie wavelength depends on their total momentum, which includes relativistic effects at high energies.

Step 1: Determine Total Energy and Momentum

Total energy:

$$
E_{\text{total}} = E_0 + K
$$

where:

- Rest energy:

$$
E_0 = mc^2 = 1.675 \times 10^{-27} \text{ kg} \times (3 \times 10^{8} \text{ m/s})^2 = 1.505 \times 10^{-10} \text{ J}
$$

- Kinetic energy:

$$
K = 1.0 \text{ MeV} = 1.602 \times 10^{-13} \text{ J}
$$

Total energy:

$$
E_{\text{total}} = 1.505 \times 10^{-10} + 1.602 \times 10^{-13} \approx 1.5066 \times 10^{-10} \text{ J}
$$

Since $K \gg E_0$ is not true here, but the kinetic energy is small compared to rest energy, a non-relativistic approximation suffices; however, for accuracy at 1 MeV, relativistic formulas are preferable.

Step 2: Find Relativistic Momentum

The relativistic momentum:

$$
p = \frac{\sqrt{E_{\text{total}}^2 - (mc^2)^2}}{c}
$$

\]

Calculate:

$$
p = \frac{\sqrt{(1.5066 \times 10^{-10})^2 - (1.505 \times 10^{-10})^2}}{3 \times 10^{8}}
$$

Numerator:

$$
\sqrt{(2.270 \times 10^{-20}) - (2.265 \times 10^{-20})} = \sqrt{5 \times 10^{-23}} = 7.07 \times 10^{-12}
$$

Thus:

$$
p = \frac{7.07 \times 10^{-12}}{3 \times 10^{8}} \approx 2.36 \times 10^{-20} \text{ kg·m/s}
$$

Step 3: Calculate the Wavelength

Using de Broglie’s relation:

$$
\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34}}{2.36 \times 10^{-20}} \approx 2.81 \times 10^{-14} \text{ meters}
$$

This wavelength (~0.028 femtometers) is extremely small, reflecting the particle’s high momentum at 1 MeV kinetic energy.

Comparison of the Wavelengths

Aspect	Gamma-ray Photon	Neutron (1.0 MeV)
Wavelength	approximately 1.24 pm	approximately 0.028 fm
Nature	Massless electromagnetic wave	Massive particle with rest mass
Speed	c (speed of light)	Less than c, depends on energy
Relation to Energy	Directly proportional ($\lambda = hc/E$)	Inversely proportional to momentum ($\lambda = h/p$)

Key observations:

- The gamma-ray photon has a significantly longer wavelength than the neutron with the same kinetic energy.
- The difference stems from the massless nature of photons versus the massive nature of neutrons.
- The neutron’s small wavelength indicates high spatial localization, relevant in neutron scattering experiments.

Implications of Wavelength Differences

Understanding these differences is essential in various applications:

- Gamma-ray interaction: Longer wavelength (~picometers) influences how gamma rays interact with matter, including photoelectric absorption, Compton scattering, and pair production.
- Neutron scattering: The extremely short wavelength (~femtometers) allows neutrons to probe atomic nuclei, making neutron scattering a powerful tool in material science.
- Quantum mechanics: The wave properties impact how particles behave in quantum systems, affecting diffraction, interference, and tunneling phenomena.

Additional Considerations

- Relativistic effects: At energies approaching or exceeding rest mass energy, relativistic formulas must be used for accurate calculations.
- Non-relativistic approximation: For low kinetic energies, classical relations suffice, but at 1 MeV, relativistic formulas are more accurate.
- Particle interactions: The different wavelengths influence how particles interact with potential barriers, fields, and other particles.

Conclusion

In summary, a 1.0-MeV gamma-ray photon has a wavelength of approximately 1.24 picometers, which is characteristic of electromagnetic radiation at high energies. In contrast, a neutron with the same kinetic energy has a much shorter wavelength—on the order of 0.028 femtometers—due to its mass and the resulting relativistic momentum. This stark difference highlights the profound impact of particle mass and energy on their wave-like properties and emphasizes the importance of wavelength considerations in nuclear physics, quantum mechanics, and advanced material research.

Understanding these fundamental differences enhances our ability to interpret experimental results, design detectors, and develop technologies based on particle interactions at atomic and subatomic scales.

Frequently Asked Questions

How does the wavelength of a 1.0-MeV gamma-ray photon compare to that of a neutron with the same kinetic energy?

The gamma-ray photon has a much shorter wavelength than the neutron because photons are massless and have higher momentum for the same energy, resulting in a smaller de Broglie wavelength compared to the massive neutron.

Why does a gamma-ray photon have a different wavelength than a neutron at the same kinetic energy?

Because gamma-ray photons are massless particles traveling at the speed of light, their wavelength is determined solely by their energy, whereas neutrons have mass, affecting their momentum and resulting in a different wavelength despite having the same kinetic energy.

What is the approximate wavelength of a 1.0-MeV gamma-ray photon?

The wavelength of a 1.0-MeV gamma-ray photon is approximately 1.24 x 10^-12 meters (picometers), calculated using the relation $\lambda = hc/E$.

How do you calculate the wavelength of a neutron with 1.0 MeV kinetic energy?

You use the de Broglie wavelength formula, $\lambda = h/p$, where p is the momentum derived from the neutron's kinetic energy: $p = \sqrt{(2mE)}$. Plugging in the neutron mass and energy gives an approximate wavelength of about 0.027 nanometers.

Which has a longer wavelength, the 1.0-MeV gamma-ray photon or the neutron with the same energy?

The neutron has a longer wavelength (~0.027 nm) compared to the gamma-ray photon (~0.00124 nm) because of its mass, which results in lower momentum and thus a larger de Broglie wavelength at the same kinetic energy.

Additional Resources

Compare The Wavelength Of A 1.0-MeV Gamma-ray Photon With That Of A Neutron Having The Same Kinetic Energy

Understanding the fundamental properties of particles at high energies is a cornerstone of modern physics. Among the intriguing comparisons is analyzing the wavelength of a high-energy gamma-ray photon versus that of a neutron possessing the same kinetic energy. This comparison not only elucidates the wave-particle duality but also highlights the profound differences arising from their intrinsic nature—massless versus massive particles—and how quantum mechanics governs their behavior. This article delves into the detailed calculation of their wavelengths, explores the implications of their differences, and discusses the broader context in which these particles operate.

Introduction to Particle Wavelengths and Their Significance

The concept of wavelength is fundamental in quantum mechanics, bridging the wave and particle aspects of matter and radiation. The de Broglie wavelength, λ, is defined as:

$$
\lambda = \frac{h}{p}
$$

where:
- h is Planck’s constant ($6.626 \times 10^{-34} \, \text{Js}$),
- p is the momentum of the particle.

For photons, which are massless, the wavelength relates directly to their energy via the speed of light, c. For massive particles, such as neutrons, the wavelength depends on their momentum derived from their kinetic energy. Comparing these two types of particles under the same kinetic energy provides insight into their wave-like behavior, measurement techniques, and their roles in various applications, from medical imaging to fundamental physics experiments.

Calculating the Wavelength of a 1.0-MeV Gamma-ray Photon

Energy and Momentum Relation for Photons

Gamma-ray photons are electromagnetic radiation with extremely high energy—here, 1.0 MeV (million electron volts). Since photons are massless, their entire energy is related to their momentum:

$$
E = pc

\]

where:
- E is the photon energy,
- p is the photon momentum,
- c is the speed of light ($3.0 \times 10^8\, \text{m/s}$).

The wavelength is given as:

$$
\lambda = \frac{h c}{E}
$$

Numerical Calculation

Given:
- $E = 1.0\, \text{MeV} = 1.0 \times 10^6 \, \text{eV}$,
- $1\, \text{eV} = 1.602 \times 10^{-19}\, \text{J}$.

Convert energy:

$$
E = 1.0 \times 10^6 \times 1.602 \times 10^{-19} = 1.602 \times 10^{-13}\, \text{J}
$$

Calculate wavelength:

$$
\lambda = \frac{h c}{E} = \frac{(6.626 \times 10^{-34})(3.0 \times 10^8)}{1.602 \times 10^{-13}}
$$

$$
\lambda \approx \frac{1.9878 \times 10^{-25}}{1.602 \times 10^{-13}} \approx 1.24 \times 10^{-12} \, \text{m}
$$

Result: The wavelength of a 1.0-MeV gamma-ray photon is approximately 1.24 picometers (pm).

Calculating the Wavelength of a Neutron with 1.0-MeV Kinetic Energy

Mass and Energy Considerations for Neutrons

Unlike photons, neutrons are massive particles with rest mass m_n:

$$
m_n = 1.675 \times 10^{-27}\, \text{kg}
$$

The kinetic energy of the neutron is:

$$
K = \frac{1}{2} m v^2
$$

and the relativistic relation between energy and momentum must be considered, especially at high energies like 1 MeV, where relativistic effects become significant.

The total energy:

$$
E_{total} = K + m c^2
$$

where the rest energy:

$$
m c^2 = (1.675 \times 10^{-27})(3.0 \times 10^8)^2 \approx 1.505 \times 10^{-10}\, \text{J} \approx 938\, \text{MeV}
$$

Since $K = 1\, \text{MeV}$, which is small compared to the rest energy, the neutron is still non-relativistic to moderately relativistic. The relativistic momentum:

$$
p = \frac{\sqrt{E_{total}^2 - (m c^2)^2}}{c}
$$

but for practical purposes, at 1 MeV kinetic energy, non-relativistic approximation suffices with relativistic correction. To be precise, use:

$$
p c = \sqrt{E_{total}^2 - (m c^2)^2}
$$

where:

$$
E_{total} = m c^2 + K = 938\, \text{MeV} + 1\, \text{MeV} = 939\, \text{MeV}
$$

Calculating:

$$
p c = \sqrt{(939)^2 - (938)^2} \, \text{MeV} = \sqrt{(882,321) - (879,844)} \, \text{MeV} \approx \sqrt{2,477} \approx 49.77\, \text{MeV}
$$

Then, the momentum:

$$
p = \frac{49.77\, \text{MeV}}{c}
$$

Calculate the wavelength:

$$
\lambda = \frac{h}{p}
$$

Express h in appropriate units:

$$
h c = 197.3\, \text{MeV} \cdot \text{fm}
$$

So,

$$
\lambda = \frac{h c}{p c} = \frac{197.3\, \text{MeV} \cdot \text{fm}}{49.77\, \text{MeV}} \approx 3.96\, \text{fm}
$$

or approximately 3.96 femtometers (fm).

Result: The de Broglie wavelength of a neutron with 1.0-MeV kinetic energy is approximately 4 femtometers.

Comparison of the Two Wavelengths

Particle Type	Kinetic Energy	Wavelength	Approximate Value	Notes
Gamma-ray photon	1.0 MeV	1.24 pm	1.24×10^{-12} meters	Massless, directly related to energy
Neutron	1.0 MeV	4 fm	4×10^{-15} meters	Massive, wave depends on momentum

Key observations:

- The gamma-ray photon has an extremely short wavelength (~1.24 pm), comparable to atomic dimensions, which makes gamma rays capable of probing nuclear structures.
- The neutron's wavelength (~4 fm) is on the order of nuclear sizes, enabling neutrons to be used in neutron scattering experiments to investigate atomic nuclei.

Implications and Applications

Wave-Particle Duality and Measurement Techniques

- Due to their short wavelength, gamma rays are instrumental in high-resolution imaging, nuclear spectroscopy, and medical applications like radiotherapy.
- Neutron wavelengths in the femtometer range are essential in neutron scattering experiments, providing insights into the structure of materials at the atomic and nuclear levels.

Differences in Behavior and Usage

- Photons, being massless, always travel at the speed of light and have wavelengths inversely proportional to their energy.
- Neutrons, being massive, can have their wavelength tuned by adjusting their kinetic energy, making them versatile probes in materials science and nuclear physics.

Pros and Cons

Gamma-ray Photons:
- Pros:
- High energy allows deep penetration.
- Suitable for imaging and cancer therapy.
- Cons:
- Difficult to control and focus due to their high frequency.
- Can cause radiation damage.

Neutrons:
- Pros:
- Sensitive to light elements and magnetic structures.
- Non-destructive probing of materials.
- Cons:
- Require nuclear reactors or spallation sources.
- Difficult to produce and handle.

Conclusion

Comparing the wavelengths of a 1.0-MeV gamma-ray photon and a neutron with the same kinetic energy underscores the profound differences dictated by their intrinsic properties. The photon's wavelength (~1.24 pm) is at the scale of atomic and subatomic structures, enabling applications in high-energy physics and medical imaging. The neutron's wavelength (~4 fm) aligns with the size of atomic nuclei, making it invaluable in nuclear physics research. This comparison exemplifies how the wave nature of particles varies dramatically with their mass and energy, shaping their roles across scientific disciplines. Understanding these differences enhances our capacity to harness particles for technological advancement, fundamental research, and practical applications in diverse fields.

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