Consider The Parametric Curve Given By The Equations X(t)=t2 13t40 Y(t)=t2 13t 1 How Many Units Of Distance

Consider The Parametric Curve Given By The Equations X(t)=t2 13t40 Y(t)=t2 13t 1 How Many Units Of Distance is a compelling question that invites us to explore the fascinating world of parametric equations and the geometry of curves. Understanding how to analyze such curves not only deepens our appreciation for mathematical concepts but also provides practical tools for fields like physics, engineering, and computer graphics. In this article, we will delve into the detailed process of analyzing the given parametric equations, calculating the length of the curve, and understanding the implications of these calculations.

Understanding Parametric Curves

What Are Parametric Equations?

Parametric equations describe a curve in the plane using a parameter, often denoted as t. Instead of expressing y as a function of x or vice versa, both x and y are expressed independently as functions of t:

- X(t): the x-coordinate as a function of t
- Y(t): the y-coordinate as a function of t

This approach allows for the representation of more complex curves, including loops, spirals, and other intricate shapes that are difficult to express with a single function y=f(x).

Benefits of Using Parametric Equations

- Flexibility in describing complex shapes.
- Easier computation of derivatives and arc lengths.
- Enhanced visualization of motion along a path, especially in physics simulations.

Analyzing the Given Parametric Equations

The equations provided are:

```
X(t) = t^2 - 13t + 40
Y(t) = t^2 - 13t + 1
```

(Note: Assuming the original equations intended to use standard notation: t^2 instead of t2, and proper subtraction signs.)

Step 1: Simplify and Understand the Equations

Both functions are quadratic in t:
- X(t) = t^2 - 13t + 40
- Y(t) = t^2 - 13t + 1

The key observation here is that both x and y are quadratic functions with similar structure, differing only in the constant term.

Step 2: Find the Relationship Between X(t) and Y(t)

To analyze the curve, it helps to eliminate the parameter t and find a direct relation between x and y.

Subtract Y(t) from X(t):

```
X(t) - Y(t) = (t^2 - 13t + 40) - (t^2 - 13t + 1) = 39
```

This simplifies to:

```
X(t) - Y(t) = 39
```

which indicates that for all t, the points (X(t), Y(t)) lie on the straight line:

```
X - Y = 39
```

This is a crucial insight: the parametric curve described by the equations is a straight line in the xy-plane, specifically, the line x - y = 39.

Implication: Since the equations define a linear relation, the curve is a line segment or an infinite line depending on the domain of t.

Determining the Domain of t and Length of the Curve

Step 3: Find the Domain of t

The original equations do not specify the domain of t. Typically, for parametric curves, the domain might be all real numbers or a specific interval.

Assuming the domain is all real numbers, the points (X(t), Y(t)) trace out the entire line x - y = 39, extending infinitely in both directions.

Step 4: Calculating the Units of Distance (Arc Length)

The key question: How many units of distance are covered by the curve?

Given that the curve is a straight line, the arc length over a finite interval $[t_1, t_2]$ is simply the length of the segment connecting the corresponding points.

Formula for Arc Length:
For a parametric curve with functions X(t) and Y(t), the length L over t in $[t_1, t_2]$ is:

```
L = ∫ from t₁ to t₂ √[(dX/dt)² + (dY/dt)²] dt
```

Step 5: Compute the Derivatives dX/dt and dY/dt
- dX/dt = derivative of t^2 - 13t + 40:

```
dX/dt = 2t - 13
```

- dY/dt = derivative of t^2 - 13t + 1:

```
dY/dt = 2t - 13
```

Notice that both derivatives are identical, which aligns with the earlier observation that the difference between x and y is constant.

Step 6: Simplify the Integrand
Calculate:

```
(dX/dt)² + (dY/dt)² = (2t - 13)² + (2t - 13)² = 2  (2t - 13)²
```

Thus:

```
√[(dX/dt)² + (dY/dt)²] = √[2  (2t - 13)²] = √2  |2t - 13|
```

Step 7: Set Up the Arc Length Integral
The length over an interval $[t_1, t_2]$ is:

$$L = \int \text{ from } t_1 \text{ to } t_2 \ \sqrt{2} \ \ |2t - 13| \ dt$$

Step 8: Determine the Limits of Integration
Without specific bounds, if we assume t varies over an interval $[t_1, t_2]$, the total length depends on these bounds.

Suppose we want to find the length between two points corresponding to t-values t_1 and t_2.

Example: Calculating the Length Between Two Specific t-values

Let's choose $t_1 = 0$ and $t_2 = 13$ for illustration.

Calculate the length:

$$L = \sqrt{2} \ \ \int_0^{13} |2t - 13| \ dt$$

Note that $|2t - 13|$ changes at $t = 6.5$, where $2t - 13 = 0$.

Split the integral:

$$L = \sqrt{2} \ \ [\int_0^{6.5} (13 - 2t) \ dt + \int_{6.5}^{13} (2t - 13) \ dt]$$

Compute each integral:

1. For t in [0, 6.5]:

$$\int (13 - 2t) \ dt = 13t - t^2$$

Evaluated from 0 to 6.5:

$$(13 \ \ 6.5 - (6.5)^2) - (0) = (84.5 - 42.25) = 42.25$$

2. For t in [6.5, 13]:

$$\int (2t - 13) \ dt = t^2 - 13t$$

Evaluated from 6.5 to 13:

$$(13^2 - 1313) - (6.5)^2 - 136.5) = (169 - 169) - (42.25 - 84.5) = 0 - (-42.25) = 42.25$$

Adding both parts:

$$\text{Total integral} = 42.25 + 42.25 = 84.5$$

Multiplying by $\sqrt{2}$:

$$L = \sqrt{2} \quad 84.5 \approx 1.4142 \quad 84.5 \approx 119.7 \text{ units}$$

Result: The length of the curve between $t=0$ and $t=13$ is approximately 119.7 units.

Implications and Practical Applications

Understanding the Line Equation

Since the parametric equations describe a line $x - y = 39$, the total length of this line segment over any finite interval of t depends on the bounds chosen for t. If t spans from t_1 to t_2, the length can be computed directly from the derivatives as shown.

Real-World Applications

- Physics: Modeling motion along a straight path where position depends quadratically on time.
- Engineering: Designing trajectories or paths with precise length measurements.
- Computer Graphics: Rendering curves and calculating distances along paths for animations or simulations.
- Mathematics Education: Demonstrating how derivatives relate to arc length and the importance of understanding parametric equations.

Extensions and Further Exploration

- Analyzing more complex curves: For non-linear or curved parametric equations, similar methods apply but may involve more intricate calculus.
- Parameter domain selection: By choosing specific t intervals, one can model particular segments of the curve.
- Visualization: Plotting the parametric equations helps visualize the line and understand the geometric context.

Conclusion

In conclusion, the given parametric equations define a straight

Frequently Asked Questions

What is the parametric form of the curve given in the problem?

The parametric equations are $X(t) = t^2 + 13t + 40$ and $Y(t) = t^2 + 13t + 1$.

How do you interpret the given parametric equations for the curve?

They define the coordinates (X, Y) of points on the curve as functions of the parameter t, allowing us to analyze its shape and properties.

What is the main goal in calculating the total distance traveled along the curve?

The goal is to find the arc length of the curve over a specified interval of t by integrating the speed (the magnitude of the derivative) over that interval.

How do you compute the differential arc length for the parametric curve?

The differential arc length ds is given by $ds = \sqrt{(dX/dt)^2 + (dY/dt)^2}\, dt$, which involves calculating derivatives of X(t) and Y(t).

What steps are involved in calculating the total distance for the curve given X(t) and Y(t)?

First, find dX/dt and dY/dt, then compute the integrand $\sqrt{(dX/dt)^2 + (dY/dt)^2}$, and finally integrate this expression over the desired t-interval.

What is the significance of understanding the total units of distance along a parametric curve?

It helps in applications like physics and engineering to determine the actual length traveled along a path, which is essential for motion analysis and design considerations.

Additional Resources

Parametric Curves: Exploring the Distance Along a Famous Mathematical Path

---

Introduction to Parametric Curves and Their Significance

In the world of mathematics and geometric analysis, parametric curves serve as a fundamental tool for representing complex shapes and paths. Unlike the simple Cartesian equations $y = f(x)$, parametric equations describe a curve through a pair (or more) of functions that depend on a parameter, typically denoted as t. This approach offers remarkable flexibility, allowing us to model intricate paths such as spirals, loops, or even physical trajectories in space.

Today, we delve into a specific parametric curve defined by the functions:

$$
\boxed{
X(t) = t^2 + 13t + 40 \\
Y(t) = t^2 + 13t + 1
}
$$

Our goal: determine the length of the curve over a specified interval of the parameter t. This is a classic problem in calculus, with applications spanning physics, engineering, computer graphics, and more.

Understanding the Parametric Equations

The Curves' Form and Behavior

Let's analyze the given functions:

$$
X(t) = t^2 + 13t + 40
$$
$$
Y(t) = t^2 + 13t + 1
$$

At first glance, both are quadratic in t, sharing the same quadratic and linear terms but differing by a constant: 40 vs. 1. This indicates the curve is a kind of "shifted" parabola in the coordinate plane.

Key Observations:

- Both $X(t)$ and $Y(t)$ are quadratic with the same coefficients for t^2 and t.
- The difference between $X(t)$ and $Y(t)$ is constant:

$$
X(t) - Y(t) = (t^2 + 13t + 40) - (t^2 + 13t + 1) = 39
$$

- This implies the entire curve lies along a line where the x-coordinate exceeds the y-coordinate by 39 units at every point.

Insight: The parametric equations define a set of points where $X(t)$ is always 39 units greater than $Y(t)$. The curve is essentially a straight line in the parametric sense, but because both $X(t)$ and $Y(t)$ are quadratic functions of t, the path traced out is more complex than a simple linear segment.

Calculating the Distance Along the Curve

The Fundamental Formula

The length L of a parametric curve between $t = a$ and $t = b$ is given by the integral:

$$
L = \int_a^b \sqrt{\left(\frac{dX}{dt}\right)^2 + \left(\frac{dY}{dt}\right)^2} \, dt
$$

This formula computes the accumulated length by integrating the infinitesimal distances along the path.

Step 1: Find Derivatives $\frac{dX}{dt}$ and $\frac{dY}{dt}$

$$
\frac{dX}{dt} = 2t + 13
$$
$$
\frac{dY}{dt} = 2t + 13
$$

Remarkably, both derivatives are identical for all t. This reinforces the earlier observation that the difference between $X(t)$ and $Y(t)$ is constant, and the derivatives are the same.

Step 2: Simplify the Integrand

The integrand becomes:

$$
\sqrt{\left(2t + 13\right)^2 + \left(2t + 13\right)^2} = \sqrt{2 \times (2t + 13)^2} = \sqrt{2} \times |2t + 13|
$$

Since absolute value is involved, the integral's value depends on the range a to b, but in general, the length formula simplifies to:

$$
L = \int_a^b \sqrt{2} \times |2t + 13| \, dt
$$

Choosing the Interval for t

To compute a meaningful length, we must specify the interval $[a, b]$. In the absence of explicit bounds, common choices include:

- From $t = 0$ to $t = 1$
- From $t = -\infty$ to $t = \infty$ (which leads to an infinite curve length)
- A finite interval, e.g., $t \in [0, 10]$

Let's assume we're interested in the interval $t \in [0, 5]$, a typical choice to examine a segment of the curve.

Computing the Arc Length for $t \in [0, 5]$

Step 1: Set up the integral

$$
L = \sqrt{2} \int_0^5 |2t + 13| \, dt
$$

Step 2: Determine the sign of $2t + 13$ over the interval

- For $t \geq 0$:

$$
2t + 13 \geq 13 > 0
$$

- Therefore, the absolute value simplifies:

$$
|2t + 13| = 2t + 13
$$

Step 3: Integrate

$$
L = \sqrt{2} \int_0^5 (2t + 13) \, dt
$$

$$
L = \sqrt{2} \left[t^2 + 13t \right]_0^5
$$

Calculating:

$$
t^2 + 13t \bigg|_0^5 = (25 + 65) - (0 + 0) = 90
$$

Thus:

$$
L = \sqrt{2} \times 90 = 90 \sqrt{2}
$$

Result:
The length of the curve from $t=0$ to $t=5$ is $90 \sqrt{2}$ units.

Extending to Other Intervals: Infinite or Finite

The previous calculation applies to the interval where $2t + 13$ remains positive. For other intervals, especially those crossing the point where $2t + 13 = 0$, the absolute value must be handled carefully.

Critical point:

$$
2t + 13 = 0 \Rightarrow t = -6.5
$$

- For $t < -6.5$, $2t + 13 < 0$, so:

$$
|2t + 13| = -(2t + 13) = -2t - 13
$$

- For $t > -6.5$, $2t + 13 > 0$, so:

$$
|2t + 13| = 2t + 13
$$

Implication: When computing the length over intervals crossing $t = -6.5$, split the integral accordingly.

Visualizing the Curve

While the algebraic approach provides the length analytically, visual intuition helps. Plotting the parametric equations over a range of t:

- The path is a straight line in the xy-plane, since $X(t) - Y(t) = 39$ is constant.
- Both $X(t)$ and $Y(t)$ are quadratic functions, but because their derivatives are identical, the parametric curve is a straight line segment in the plane, with $x(t)$ and $y(t)$ changing quadratically with t.

In essence:

$$
Y(t) = X(t) - 39
$$

which is a line with slope 0 in the xy-plane, shifted by 39 units in x.

However, the parametric form with quadratic functions suggests that the path traced out is a quadratic function of t, but the derivatives' equality indicates the curve is linear in the parameter t.

Practical Applications and Broader Context

Understanding how to compute the length of such parametric curves isn't just an academic exercise. It has real-world applications:

- Physics: Calculating the distance traveled by an object moving along a path parameterized by time.
- Engineering: Designing trajectories for robotic arms or vehicles.
- Computer Graphics: Rendering curves and paths accurately.
- Mathematical Modeling: Analyzing complex paths in space or on surfaces.

Moreover, the approach exemplified above—finding derivatives, simplifying the integrand, and integrating over a specified interval—is fundamental to many areas of calculus and differential geometry.

Final Remarks and Takeaways

- Parametric equations offer a flexible way to describe curves, especially when the path's shape isn't easily expressed as a single function $y=f(x)$.
- The arc length calculation hinges on derivatives and integral calculus, requiring careful handling of absolute values and interval bounds.
- In the specific case examined, the derivatives' equality simplifies the integral significantly.
-

[Consider The Parametric Curve Given By The Equations X T T2 13t40 Y T T2 13t 1 How Many Units Of Distance]()

Consider The Parametric Curve Given By The Equations X T T2 13t40 Y T T2 13t 1 How Many Units Of Distance

Related Articles

- [The Future Value Of $200 Received Today And Deposited At 8 Percent For Three Years Is Approximately (Choose]()
- [Describe Two Trends You Can Find In The Graph Below. Use Calculations And Numbers To Support Your Thinking.]()
- [The Construction Manager For ABC Construction Must Decide Whether To Build Single Family Homes, Apartments,]()

[Back to Home]()