Consider The System Of Equations Dx = X(4 X 5y) Dy = Y(1 4x), Taking (x, Y) > 0. Recall That A Nullcline

Consider The System Of Equations Dx = X(4 X 5y) Dy = Y(1 4x), Taking (x, Y) > 0. Recall That A Nullcline

Understanding the behavior of dynamical systems described by differential equations is fundamental in various scientific fields, from physics and biology to economics and engineering. The system given:

\[
\begin{cases}
\frac{dx}{dt} = x(4x + 5y) \\
\frac{dy}{dt} = y(1 + 4x)
\end{cases}
\]

with the condition \((x, y) > 0\), presents an intriguing case for analysis. It involves nonlinear interactions between the variables \(x\) and \(y\), and understanding nullclines, equilibrium points, stability, and phase portraits are essential steps in deciphering its dynamics. This comprehensive article aims to explore this system thoroughly, guiding you through the process of analyzing nonlinear differential equations with an emphasis on nullclines, equilibria, and stability.

Understanding the System of Equations

Formulation of the System

The given system:

\[
\begin{cases}
\frac{dx}{dt} = x(4x + 5y) \\
\frac{dy}{dt} = y(1 + 4x)
\end{cases}
\]

describes the evolution over time of two variables, \(x(t)\) and \(y(t)\), which are strictly positive in the domain of interest (\(x, y > 0\)). The equations are nonlinear due to the products involving \(x\) and \(y\).

This system could model biological interactions (e.g., predator-prey), chemical reactions, or economic models where the growth rate of each variable depends on both variables' current state.

Nullclines and Their Significance

Definition of Nullclines

Nullclines are curves in the phase plane where the derivative of one variable is zero. They partition the phase space into regions where the variables increase or decrease.

- x-nullcline: The set of points where $\frac{dx}{dt} = 0$,

- y-nullcline: The set of points where $\frac{dy}{dt} = 0$.

Understanding nullclines helps identify equilibrium points and the qualitative behavior of trajectories.

Calculating Nullclines for the Given System

1. x-nullcline:

$$
\frac{dx}{dt} = x(4x + 5y) = 0
$$

Since $x > 0$, the only way for the product to be zero is when:

$$
4x + 5y = 0
$$

But because $x, y > 0$, $4x + 5y > 0$, so the nullcline here is at the boundary $x = 0$, which we exclude from the domain of interest.

Note: For $x > 0$, the only nullcline occurs at the boundary $x = 0$, which is outside the domain ($x > 0$). Therefore, within the domain, $\frac{dx}{dt} > 0$ when $4x + 5y > 0$.

2. y-nullcline:

$$
\frac{dy}{dt} = y(1 + 4x) = 0
$$

Again, with $y > 0$, the nullcline occurs when:

$$
1 + 4x = 0 \Rightarrow x = -\frac{1}{4}
$$

Since $x > 0$, this nullcline is outside the domain of interest. Within the domain ($x > 0$), $\frac{dy}{dt} > 0$.

Implication: Within the positive quadrant, both derivatives are positive for all $x, y > 0$, indicating that solutions tend to increase in both variables, but this needs to be validated with a detailed phase plane analysis.

Equilibrium Points and Their Identification

Finding Equilibria

Equilibrium points occur where both derivatives are zero simultaneously:

$$
\begin{cases}
x(4x + 5y) = 0 \\
y(1 + 4x) = 0
\end{cases}
$$

Given the positivity constraints ($x, y > 0$), the only possible solutions occur when:

- $x = 0$ or $4x + 5y = 0$, but since $x > 0$ and $y > 0$, these are outside the domain, so these are not equilibrium candidates within the domain.

- $y = 0$ or $1 + 4x= 0$, but again, $y > 0$, and $x > 0$, so these are not valid within the domain.

However, in the context of the positive quadrant, the only equilibrium points occur where the derivatives are zero due to the factors:

- At $x=0$:

$$
x=0 \Rightarrow \text{derivative } \frac{dx}{dt} = 0
$$

- At $y=0$:

$$
y=0 \Rightarrow \text{derivative } \frac{dy}{dt} = 0
$$

But within the domain $(x, y) > 0$, no points satisfy both conditions simultaneously unless considering boundary points.

Therefore, the interior equilibrium occurs where:

$$
4x + 5y = 0, \quad 1 + 4x= 0
$$

which is outside the domain. Hence, no positive interior equilibrium exists.

Summary: The system has no equilibrium points in the strictly positive quadrant; only boundary points at $x=0$ or $y=0$ are possible.

Stability Analysis of Boundary Equilibria

Analyzing Behavior at $x=0$ and $y=0$

- At $x=0$:

$$
\frac{dx}{dt} = 0, \quad \frac{dy}{dt} = y(1 + 0) = y
$$

For $y > 0$, $\frac{dy}{dt} > 0$, so solutions move away from $x=0$ into the positive x region, indicating that $x=0$ is not stable.

- At $y=0$:

$$
\frac{dy}{dt} = 0, \quad \frac{dx}{dt} = x(4x + 0) = 4x^2
$$

For $x > 0$, $\frac{dx}{dt} > 0$, solutions move away from $y=0$, indicating instability in this boundary as well.

Phase Plane Analysis and Trajectory Behavior

Qualitative Behavior in the Positive Quadrant

Given the derivatives:

$$
\frac{dx}{dt} = x(4x + 5y)
$$

$$
\frac{dy}{dt} = y(1 + 4x)
$$

and that for $x, y > 0$:

- Both derivatives are positive, suggesting that solutions tend to increase in both x and y.

- Since there are no interior equilibrium points, trajectories tend to move away from the axes towards infinity or possibly approach other attractors, depending on the system's parameters.

Is there a stable equilibrium or limit cycle?

- The absence of interior equilibria suggests no fixed points in the positive domain.

- The behavior at boundaries indicates solutions diverge away from axes, hinting at unbounded growth or possibly approaching infinity.

- To confirm, numerical simulations or phase portrait plotting can illustrate the trajectory behavior.

Transformations and Further Analysis

Change of Variables or Rescaling

To analyze the system more effectively, consider transformations such as:

- Scaling variables to simplify coefficients.
- Using phase plane methods to visualize trajectories.

Potential for Limit Cycles or Other Dynamics

While the current analysis suggests unbounded growth, certain nonlinear systems exhibit limit cycles. To check for such phenomena:

- Use Poincaré-Bendixson Theorem (though limited here due to unbounded solutions).
- Apply numerical simulations to observe long-term behavior.

Conclusion and Key Takeaways

- The system has no equilibrium points within the strictly positive quadrant, only boundary points at axes.
- Nullclines are primarily at the axes, with the derivatives indicating solutions tend to move away from the axes.
- Both derivatives are positive for all \(x, y > 0\), indicating solutions grow unboundedly in the positive quadrant.
- The behavior suggests solutions diverge to infinity unless constrained by additional factors or modified system parameters.
- Understanding nullclines, equilibrium points, and stability provides insight into the qualitative behavior of nonlinear dynamical systems.

Further Reading

Frequently Asked Questions

What is the system of equations given in the problem?

The system is Dx = X(4 × 5y) and Dy = Y(1 4x), with (x, y) > 0.

How do you interpret the nullclines in this system?

Nullclines are the curves where either Dx = 0 or Dy = 0, indicating points where the respective variable's rate of change is zero.

What are the nullclines for Dx = 0 in the given system?

Since Dx = X(4 × 5y), the nullcline Dx = 0 occurs when either X = 0 or 4 × 5y = 0, which simplifies to X = 0 or y = 0.

What are the nullclines for Dy = 0 in this system?

Given Dy = Y(1 4x), the nullcline Dy = 0 occurs when either Y = 0 or 1 4x = 0, which simplifies to Y = 0 or x = 0.

Based on the nullclines, where are the equilibrium points located?

The equilibrium points are at the intersections of the nullclines, specifically at (x, y) = (0, 0) in the positive quadrant, but since (x, y) > 0, the relevant equilibrium points are where the nullclines intersect in the positive domain.

How does the positivity condition (x, y) > 0 influence the nullcline analysis?

Since x and y are greater than zero, the nullclines at x=0 or y=0 are outside the domain of interest, focusing the analysis on nullclines where both

variables are positive.

What is the significance of a nullcline in understanding the dynamics of this system?

Nullclines help identify potential equilibrium points and provide insight into the system's phase portrait by indicating where variables do not change.

Are there any other critical points or nullclines to consider in this system?

Given the nullclines at x=0 and y=0, within the positive domain, no other nullclines exist unless the equations are further analyzed for internal nullclines or bifurcations.

How can the stability of the equilibrium points be assessed in this system?

Stability can be analyzed by computing the Jacobian matrix at the equilibrium points and examining the eigenvalues to determine if the points are stable or unstable.

Additional Resources

Consider The System Of Equations Dx = X(4 X 5y) Dy = Y(1 4x), Taking (x, y) > 0. Recall That A Nullcline

Understanding the behavior of nonlinear dynamical systems often hinges on analyzing their nullclines—curves in the phase plane where the derivatives of the system vanish. In this guide, we delve into the system:

$$
\begin{cases}
\frac{dx}{dt} = x(4x - 5y) \\
\frac{dy}{dt} = y(1 - 4x)
\end{cases}
$$

with the assumption that $x > 0$ and $y > 0$. We will explore how the nullclines shape the dynamics, identify equilibrium points, analyze their stability, and interpret the phase portrait. By the end, you'll have a comprehensive understanding of this system's qualitative behavior.

Understanding Nullclines: The Foundations of Phase

Plane Analysis

Nullclines are curves in the phase space where either $\frac{dx}{dt} = 0$ or $\frac{dy}{dt} = 0$. They partition the plane into regions where the respective variables increase or decrease, providing critical insight into the system's trajectories.

- x-nullcline: the set of points where $\frac{dx}{dt} = 0$.
- y-nullcline: the set of points where $\frac{dy}{dt} = 0$.

By analyzing these nullclines, we can find equilibrium points (intersections) and understand the local behavior of solutions near these points.

Deriving Nullclines for the Given System

1. x-nullcline

Set $\frac{dx}{dt} = 0$:

$$
x(4x - 5y) = 0
$$

Since $x > 0$, the factor $x \neq 0$, so:

$$
4x - 5y = 0 \quad \Rightarrow \quad y = \frac{4}{5}x
$$

This is the x-nullcline, a straight line through the origin with slope $\frac{4}{5}$.

2. y-nullcline

Set $\frac{dy}{dt} = 0$:

$$
y(1 - 4x) = 0
$$

Given $y > 0$, then:

$$
1 - 4x = 0 \quad \Rightarrow \quad x = \frac{1}{4}
$$

This is the y-nullcline, a vertical line at $x = \frac{1}{4}$.

Equilibrium Points: Where Nullclines Intersect

Equilibrium solutions occur at intersections of nullclines:

- From the x-nullcline: $y = \frac{4}{5}x$
- From the y-nullcline: $x = \frac{1}{4}$

Substitute $x = \frac{1}{4}$ into the x-nullcline:

$$
y = \frac{4}{5} \times \frac{1}{4} = \frac{4}{5} \times \frac{1}{4} = \frac{1}{5}
$$

Therefore, the only positive equilibrium point is:

$$
\boxed{
E = \left(\frac{1}{4}, \frac{1}{5} \right)
}
$$

This is the interior equilibrium of the system in the positive quadrant.

Phase Plane Analysis: Dynamics Around Equilibrium

Sign of $\frac{dx}{dt}$

- Above the x-nullcline $y = \frac{4}{5}x$:

\[

$$
4x - 5y < 0 \quad \Rightarrow \quad \frac{dx}{dt} = x(4x - 5y) < 0
$$

- Below the x-nullcline:

$$
4x - 5y > 0 \quad \Rightarrow \quad \frac{dx}{dt} > 0
$$

Since $x > 0$, the sign depends on $4x - 5y$.

Sign of $\frac{dy}{dt}$

- To the right of the y-nullcline $x = \frac{1}{4}$:

$$
1 - 4x < 0 \quad \Rightarrow \quad \frac{dy}{dt} < 0
$$

- To the left of that line:

$$
1 - 4x > 0 \quad \Rightarrow \quad \frac{dy}{dt} > 0
$$

Stability Analysis of the Equilibrium

To determine the nature (stability) of the equilibrium at $E = \left(\frac{1}{4}, \frac{1}{5}\right)$, we compute the Jacobian matrix:

$$
J =
\begin{bmatrix}
\frac{\partial}{\partial x} \left(x(4x - 5y) \right) &
\frac{\partial}{\partial y} \left(x(4x - 5y) \right) \\
\frac{\partial}{\partial x} \left(y(1 - 4x) \right) &
\frac{\partial}{\partial y} \left(y(1 - 4x) \right)
\end{bmatrix}
$$

Computing partial derivatives:

- $\frac{\partial}{\partial x} \left(x(4x - 5y) \right) = (4x - 5y) + x \times 4 = 4x - 5y + 4x = 8x - 5y$
- $\frac{\partial}{\partial y} \left(x(4x - 5y) \right) = x \times (-5) = -5x$
- $\frac{\partial}{\partial x} \left(y(1 - 4x) \right) = y \times (-4) = -4y$
- $\frac{\partial}{\partial y} \left(y(1 - 4x) \right) = (1 - 4x) + y \times 0 = 1 - 4x$

Evaluating at the equilibrium point:

$$
x = \frac{1}{4}, \quad y = \frac{1}{5}
$$

- $J_{11} = 8 \times \frac{1}{4} - 5 \times \frac{1}{5} = 2 - 1 = 1$
- $J_{12} = -5 \times \frac{1}{4} = -\frac{5}{4}$
- $J_{21} = -4 \times \frac{1}{5} = -\frac{4}{5}$
- $J_{22} = 1 - 4 \times \frac{1}{4} = 1 - 1 = 0$

Jacobian matrix:

$$
J =
\begin{bmatrix}
1 & -\frac{5}{4} \\
-\frac{4}{5} & 0
\end{bmatrix}
$$

Eigenvalues:

Solve $\det(J - \lambda I) = 0$:

$$
\det
\begin{bmatrix}
1 - \lambda & -\frac{5}{4} \\
-\frac{4}{5} & -\lambda
\end{bmatrix}
= (1 - \lambda)(-\lambda) - \left(-\frac{5}{4}\right)\left(-\frac{4}{5}\right) = 0
$$

Simplify:

$$
-(1 - \lambda)\lambda - \left(\frac{5}{4} \times \frac{4}{5}\right) = 0
$$

Note that:

$$
\frac{5}{4} \times \frac{4}{5} = 1
$$

So:

$$
-(1 - \lambda)\lambda - 1 = 0
$$

Expanding:

$$
-\lambda + \lambda^2 - 1 = 0
$$

Rearranged:

$$
\lambda^2 - \lambda - 1 = 0
$$

Solve with quadratic formula:

$$
\lambda = \frac{1 \pm \sqrt{1 + 4}}{2} = \frac{1 \pm \sqrt{5}}{2}
$$

- $\lambda_1 = \frac{1 + \sqrt{5}}{2} > 0$
- $\lambda_2 = \frac{1 - \sqrt{5}}{2} < 0$

Since eigenvalues are real and of opposite signs, the equilibrium point E is a saddle point—unstable in the phase plane.

Phase Portrait and Trajectory Behavior

Based on the nullclines and stability analysis:

- The system has a single interior equilibrium at $\left(\frac{1}{4}, \frac{1}{5}\right)$, which is a saddle.
- Nullcline \(y = \frac{4

[Consider The System Of Equations Dx X 4 X 5y Dy Y 1 4x Taking X Y Gt 0 Recall That A Nullcline]()

Consider The System Of Equations Dx X 4 X 5y Dy Y 1 4x Taking X Y Gt 0 Recall That A Nullcline

Related Articles

- [Multithreading An Interactive Program Will Increase Responsiveness To The User By : Group Of Answer Choices]
- [In The Circuit Shown In Fig 7.54 The Transistor Has A Beta Of 200. What Is The Dc Voltage At The Collector?]
- [The Radius Of Each Semicircle In This Composite Figure Is _____.1 In.2 In.4 In.8 In.will Mark As Brainliest]

[Back to Home]