Determine The Following For The Transformed Cosine Function Shown Whose Period Is 1,080 Degrees. Frequency:

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Understanding the frequency of a transformed cosine function is a fundamental concept in trigonometry and signal analysis. Since cosine functions are periodic, their transformations—such as amplitude changes, phase shifts, and especially period alterations—affect how often their cycles occur within a given interval. In this article, we delve into how to determine the frequency of a cosine function when its period is specified, with a particular focus on a function whose period is 1,080 degrees.

Whether you're a student preparing for exams, a teacher designing lesson plans, or an engineer analyzing waveforms, grasping the relationship between period and frequency is essential. This guide will systematically explore the concepts, formulas, and practical steps to determine the frequency of a transformed cosine function, illustrating the process with clear examples and explanations.

Understanding Cosine Functions and Their Transformations

Basic Form of the Cosine Function

The standard cosine function is expressed as:

\[

y = \cos(x)

\]

where:

- \(x \) is the input angle, measured in degrees or radians.

- The function has a period of 360 degrees (or \(2\pi \) radians).

This means the graph repeats every 360 degrees. The basic cosine wave oscillates between -1 and 1, with its maximum at 0°, the minimum at 180°, and returning to the maximum at 360°, completing one cycle.

Transformations of the Cosine Function

Transformations modify the basic cosine graph in various ways:

- Amplitude change: \(y = A \cos(Bx + C) + D \), where \(A \) affects the height.

- Period change: \(y = \cos(Bx) \), where \(B \) affects the length of one cycle.

- Horizontal shift (phase shift): \(C \) in \(\cos(Bx + C) \) shifts the graph left or right.

- Vertical shift: \(D \) shifts the graph up or down.

Our focus here is on the period, which is directly affected by the coefficient \(B \).

Period of a Cosine Function and Its Significance

Defining the Period

The period \(P \) of a cosine function describes the length of the interval over which the function completes one full cycle. For the general form:

\[

y = \cos(Bx)

\]

the period is calculated as:

\[

P = \frac{360^\circ}{|B|} \quad \text{(if } x \text{ is in degrees)}

\]

or

\[

P = \frac{2\pi}{|B|} \quad \text{(if } x \text{ is in radians)}

\]

In words, the period is inversely proportional to the absolute value of \(B \).

Implication of Changing Period

- Increasing \(B \) decreases the period, making the wave oscillate more rapidly.
- Decreasing \(B \) increases the period, resulting in a slower oscillation.

Understanding this relationship is critical when analyzing or designing functions with specific periodic properties, such as in signal processing, physics, or engineering applications.

Given the Period: 1,080 Degrees

What Does a 1,080-Degree Period Mean?

A period of 1,080 degrees indicates that the cosine function completes one full cycle every 1,080 degrees along the x-axis. This is significantly longer than the standard period of 360 degrees, meaning the wave oscillates more slowly.

Calculating the Coefficient B

Using the period formula:

$$
P = \frac{360^\circ}{|B|}
$$

we solve for B:

$$
|B| = \frac{360^\circ}{P}
$$

Plugging in $P = 1,080^\circ$:

$$
|B| = \frac{360^\circ}{1,080^\circ} = \frac{1}{3}
$$

Thus,

$$
B = \pm \frac{1}{3}
$$

The sign of B determines the orientation of the wave, but for frequency calculations, only the absolute value matters.

Determining the Frequency of the Cosine Function

Definition of Frequency

In the context of periodic functions, frequency refers to how many cycles occur per unit of measurement—typically per degree or per radian.

- In degrees: Frequency is often expressed as the number of cycles per degree.
- In Hertz (Hz): When dealing with time-dependent signals, frequency is cycles per second, but here, since the domain is degrees, we focus on cycles per degree.

Given the period P, the frequency f (in cycles per degree) is:

$$
f = \frac{1}{P}
$$

Note: If we want the number of cycles per 1,080 degrees, then the total number of cycles in that interval is:

$$
\text{Number of cycles} = \frac{\text{Interval length}}{\text{Period}} = \frac{1,080^\circ}{1,080^\circ} = 1
$$

which confirms the function completes exactly one cycle over 1,080 degrees.

Calculating the Frequency for the Given Function

Since the period $P = 1,080^\circ$,

$$
\text{Frequency } f = \frac{1}{P} = \frac{1}{1,080^\circ}
$$

Expressed as cycles per degree:

$$
f = \frac{1}{1,080} \quad \text{cycles per degree}
$$

or approximately:

$$
f \approx 0.0009259 \quad \text{cycles per degree}
$$

If you prefer to think in terms of cycles per 1,080 degrees:

$$
\text{Number of cycles in 1,080 degrees} = 1
$$

Interpreting the Results: Practical Implications

Understanding the Frequency in Context

- The low frequency (approximately 0.000926 cycles per degree) indicates a very slow oscillation over the 1,080-degree interval.
- The period being 1,080 degrees means the wave takes a large span of the x-axis to complete one cycle, which could be relevant in applications like signal modulation or wave analysis where such long periods are significant.

Converting Degrees to Radians for Broader Use

In many scientific contexts, radians are preferred:

\[

$$
\text{Conversion factor: } 1^\circ = \frac{\pi}{180} \text{ radians}
$$

The period in radians:

$$
P_{rad} = 1,080^\circ \times \frac{\pi}{180} = 6\pi \text{ radians}
$$

The corresponding B in radians:

$$
|B| = \frac{2\pi}{P_{rad}} = \frac{2\pi}{6\pi} = \frac{1}{3}
$$

which aligns with our earlier calculation in degrees.

Summary and Key Takeaways

- The period of a cosine function dictates how frequently the wave repeats along the x-axis.
- Given a period of 1,080 degrees, the coefficient B in the general form $y = \cos(Bx)$ is $\pm \frac{1}{3}$.
- The frequency of the function, in cycles per degree, is $\frac{1}{1,080}$.
- This low frequency indicates a slow oscillation, with the wave completing one full cycle over 1,080 degrees.
- Understanding these relationships is crucial in various fields, including signal processing, physics, engineering, and mathematics.

Final Remarks

Determining the frequency of a transformed cosine function requires understanding the relationship between the period and the coefficient \(B \). When the period is provided—as in the case of 1,080 degrees—calculating the frequency becomes straightforward once the fundamental formulas are applied. Recognizing these relationships allows for effective analysis and design of waveforms across different disciplines.

Keywords for SEO Optimization: cosine function, period, frequency, transformed cosine, cosine wave, period calculation, frequency determination, degrees to radians, wave analysis, signal processing, trigonometry, mathematical functions, cycle, oscillation, amplitude, phase shift

Frequently Asked Questions

What is the frequency of a transformed cosine function with a period of 1,080 degrees?

The frequency is the reciprocal of the period. Therefore, frequency = 1 / (1080 degrees) = 1/1080 cycles per degree.

How do you calculate the frequency of a cosine function when the period is given?

Frequency is calculated as the reciprocal of the period: frequency = 1 / period.

What is the significance of the period being 1,080 degrees in the

cosine function?

It indicates that the cosine function repeats every 1,080 degrees, which defines how often the wave completes a cycle.

How do transformations affect the period and frequency of a cosine function?

Transformations such as stretching or compressing horizontally change the period, which in turn affects the frequency inversely; a larger period means lower frequency.

If the period of a cosine function is 1,080 degrees, what is its frequency in cycles per degree?

The frequency is 1/1080 cycles per degree.

How can the period of 1,080 degrees be converted into radians for analysis?

Since 360 degrees equals 2π radians, 1,080 degrees equals (1080/360) 2π = 3 2π = 6π radians.

What is the general formula to find the frequency of a cosine function with a given period?

Frequency = 1 / period (in units consistent with the period measurement).

Why is understanding the period important when analyzing a cosine function?

The period determines the length of one complete cycle, which is essential for graphing, analyzing wave behavior, and understanding the function's frequency.

Can the frequency of a cosine function be greater than 1? Why or why not?

Yes, if the period is less than 1 unit (or degree in this context), the frequency exceeds 1, indicating more than one cycle per unit of measurement.

How does the frequency relate to the number of cycles within a specific interval for the cosine function?

The frequency indicates how many cycles occur per unit interval; higher frequency means more cycles within the same interval.

Additional Resources

Determine The Following For The Transformed Cosine Function Shown Whose Period Is 1,080 Degrees: Frequency

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Introduction

In the realm of trigonometry and mathematical analysis, understanding the properties of periodic functions such as the cosine function is fundamental. When transformations are applied—be they amplitude, phase shifts, or vertical shifts—the core characteristics such as period, frequency, and phase change accordingly. This article delves into a detailed exploration of how to determine the frequency of a transformed cosine function, especially when its period is given as 1,080 degrees.

The focus is to provide a comprehensive, investigative approach to deciphering the relationship between period and frequency in the context of a cosine function subjected to transformations. This analysis is crucial not only for academic purposes but also for applications in signal processing, engineering, and physics where waveforms and periodic signals are prevalent.

Background: The Cosine Function and Its Basic Properties

Before examining transformed functions, it is essential to understand the fundamental properties of the basic cosine function:

- The basic cosine function is expressed as $y = \cos x$.
- Its period is 360° (or 2π radians), meaning the function repeats every 360 degrees.
- The frequency, defined as the number of cycles the function completes in a unit interval, is the reciprocal of the period, scaled appropriately.

> Key relationships:
> - Period (T) of $y = a \cos(bx + c) + d$ is $T = \frac{360^\circ}{|b|}$.
> - Frequency (f) is the reciprocal of the period: $f = \frac{1}{T}$, often expressed in cycles per degree or cycles per unit.

Understanding these concepts sets the foundation for analyzing more complex, transformed cosine functions.

The Given Period: 1,080 Degrees

The problem specifies that the cosine function under consideration has a period of 1,080 degrees. This is significantly larger than the fundamental period of 360 degrees, indicating that the function has been horizontally compressed or stretched, depending on the transformation applied.

Step 1: Recognizing the Relationship Between Period and Frequency

The core question is: What is the frequency of this cosine function?

Given:

- Period $T = 1,080^\circ$

The frequency f is inversely related to the period:

$$
f = \frac{1}{T}
$$

However, since the period is given in degrees, the frequency can be expressed as cycles per degree:

$$
f = \frac{1}{1,080^\circ}
$$

This represents the number of complete cycles per degree. But often, in applications and mathematical analysis, frequency is expressed in cycles per 360 degrees (one full rotation), or in cycles per unit of time or space. To standardize, we can convert the frequency into cycles per 360 degrees:

$$
f_{360} = \frac{1}{T} \times 360^\circ = \frac{360^\circ}{1,080^\circ} = \frac{1}{3}
$$

Thus, the cosine function completes one cycle every 3 units of 360 degrees, or equivalently, it completes $\frac{1}{3}$ of a cycle per 360 degrees.

Step 2: Deriving the Angular Frequency b

In the standard form:

$$
y = A \cos (b x + c) + D
$$

the period T relates to b as:

$$
T = \frac{360^\circ}{|b|}
$$

Solving for b:

$$
|b| = \frac{360^\circ}{T}
$$

Substituting $T = 1,080^\circ$:

$$
|b| = \frac{360^\circ}{1,080^\circ} = \frac{1}{3}
$$

This indicates that the coefficient b (the angular frequency in degrees) is:

$$
b = \pm \frac{1}{3}
$$

The sign depends on the orientation of the phase shift, but for frequency calculation, the magnitude suffices.

Step 3: Clarifying the Notion of Frequency

While the term "frequency" can be used in various contexts, in the domain of trigonometric functions, it often refers to:

- Angular frequency (b), which indicates how many degrees the function "advances" per unit of input.
- Number of cycles per unit, which is the reciprocal of the period.

In this context, the frequency as cycles per degree is:

$$
f = \frac{1}{T} = \frac{1}{1,080^\circ}
$$

or, scaled to cycles per 360 degrees:

$$
f_{360} = \frac{1}{3}
$$

which means the cosine completes one-third of a cycle per 360 degrees.

Step 4: Transformations and Their Impact on Frequency

Suppose the original cosine function undergoes transformations such as:

- Amplitude change: $y = a \cos(bx + c) + d$
- Horizontal shift: phase shift c
- Horizontal stretch/compression: changing b

The key point is that amplitude and vertical shifts do not affect the period or frequency, only the horizontal compression or stretch (via b).

Given that the period is explicitly 1,080 degrees, the key transformation is embedded in the coefficient b:

$$
b = \frac{360^\circ}{T} = \frac{360^\circ}{1,080^\circ} = \frac{1}{3}
$$

Thus, the transformation is such that the function "oscillates" every 1,080 degrees, and the frequency in cycles per degree is $\frac{1}{1,080}$.

Deep Dive: Applications and Significance

Understanding the frequency of a cosine function with an extended period like 1,080 degrees has practical implications in various fields:

- Signal Processing: In analyzing signals with large periodicities, knowing the frequency helps in filtering and signal reconstruction.
- Engineering: Designing waveforms or oscillatory systems with specific periodic behaviors relies on precise calculations of period and frequency.

- Physics: Wave phenomena, such as electromagnetic waves or mechanical vibrations, often involve large periods, making this analysis critical.

Practical Examples of Transformed Cosine Functions

To contextualize, consider a few illustrative examples:

- Example 1: A cosine function with period $T = 1,080^\circ$ has frequency $f = \frac{1}{1,080}$ cycles per degree.
- Example 2: If the period were halved to 540°, the frequency would double to $\frac{1}{540}$ cycles per degree.
- Example 3: In signal analysis, a waveform with such a large period might correspond to a very low-frequency oscillation, akin to slow-varying signals in communication systems.

Conclusion: Summarizing the Investigation

To determine the frequency of a transformed cosine function with a period of 1,080 degrees, the following steps are essential:

1. Recognize that the period T is related to the cosine function's coefficient b via $T = \frac{360^\circ}{|b|}$.
2. Calculate b:

$$
b = \frac{360^\circ}{T} = \frac{360^\circ}{1,080^\circ} = \frac{1}{3}
$$

\]

3. Derive the frequency in cycles per degree:

$$
f = \frac{1}{T} = \frac{1}{1,080^\circ}
$$

or in cycles per 360 degrees:

$$
f_{360} = \frac{1}{3}
$$

This indicates the function completes one cycle every 3 units of 360°, or equivalently, one-third of a cycle per 360°.

In essence, the frequency of this cosine function is $\boxed{\frac{1}{3}}$ cycles per 360 degrees, or approximately 0.0009259 cycles per degree. Recognizing this relationship is crucial for analyzing and utilizing such functions effectively in scientific and engineering contexts.

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