Find A Polynomial F(x) Of Degree 4 That Has The Following Zeros. 0, 7, -1, -9 Leave Your Answer In Factored

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When working with polynomials in algebra, one common task is to find a polynomial function that has specific zeros or roots. This process involves understanding the relationship between zeros and factors of the polynomial. In this article, we will explore how to find a degree 4 polynomial $F(x)$ that has the zeros at 0, 7, -1, and -9, and how to leave the answer in factored form. This comprehensive guide will help you understand the steps involved, the significance of each zero, and how to construct the polynomial systematically.

Understanding Zeros and Factors of a Polynomial

Before diving into the construction of the polynomial, it's essential to understand the fundamental connection between the zeros of a polynomial and its factors.

What Are Zeros of a Polynomial?

- The zeros (or roots) of a polynomial are the values of x for which $F(x) = 0$.
- For example, if $x = a$ is a zero of the polynomial, then plugging a into the polynomial will yield zero, i.e., $F(a) = 0$.

How Do Zeros Relate to Factors?

- The Fundamental Theorem of Algebra states that every polynomial can be factored into linear factors (over the complex numbers).
- Specifically, if a is a zero of the polynomial, then $(x - a)$ is a factor of the polynomial.
- Conversely, the factors of the polynomial can be expressed as $(x - a)$, where a is a zero.

Implication for Polynomial Construction

- Given zeros at 0, 7, -1, and -9, the corresponding factors are:

- $(x - 0) = x$
- $(x - 7)$
- $(x + 1)$
- $(x + 9)$

Constructing the Polynomial from Zeros

Given the zeros, constructing the polynomial involves creating the product of the factors corresponding to each zero.

Step 1: Write the Factors

Based on the zeros provided:

$$
F(x) = k \times x \times (x - 7) \times (x + 1) \times (x + 9)
$$

where k is a non-zero constant that determines the leading coefficient of the polynomial.

Step 2: Determine the Leading Coefficient k

- If not specified, the simplest approach is to assume $k = 1$, resulting in a monic polynomial.
- A monic polynomial has a leading coefficient of 1, which simplifies the expression.
- Unless there's a specific requirement, setting $k = 1$ is standard practice for constructing the polynomial from zeros.

Step 3: Expand or Leave in Factored Form

- The problem asks to leave the answer in factored form, so we will write the polynomial as a product of factors.
- However, understanding how to expand the polynomial can be useful for other applications, but is not necessary here.

Final Polynomial in Factored Form

Combining all the factors with $k=1$, the polynomial becomes:

$$
F(x) = x \times (x - 7) \times (x + 1) \times (x + 9)
$$

This is the polynomial in factored form, with degree 4, as required.

Optional: Expanding the Polynomial

Although the problem requests the answer in factored form, it's instructive to see how the polynomial looks when fully expanded.

Step 1: Expand $(x - 7)(x + 1)$

Using FOIL (First, Outer, Inner, Last):

$$
(x - 7)(x + 1) = x \times x + x \times 1 - 7 \times x - 7 \times 1 = x^2 + x - 7x - 7 = x^2 - 6x - 7
$$

Step 2: Expand $(x + 9)(x^2 - 6x - 7)$

Applying distributive property:

$$
(x + 9)(x^2 - 6x - 7) = x \times (x^2 - 6x - 7) + 9 \times (x^2 - 6x - 7)
$$

Calculating each:

$$
x \times x^2 = x^3
$$
$$
x \times (-6x) = -6x^2
$$
$$
x \times (-7) = -7x

$$
9 \times x^2 = 9x^2
$$

$$
9 \times (-6x) = -54x
$$

$$
9 \times (-7) = -63
$$

Adding all these terms:

$$
x^3 - 6x^2 - 7x + 9x^2 - 54x - 63 = x^3 + (-6x^2 + 9x^2) + (-7x - 54x) - 63
$$

$$
= x^3 + 3x^2 - 61x - 63
$$

Step 3: Multiply by x (the zero at 0)

Finally, multiply this result by x:

$$
F(x) = x \times (x^3 + 3x^2 - 61x - 63) = x^4 + 3x^3 - 61x^2 - 63x
$$

This is the expanded form, though, again, the problem specifies leaving it in factored form.

Summary and Key Takeaways

- The zeros of a polynomial correspond directly to its factors.
- For zeros at 0, 7, -1, and -9, the factors are x, $(x - 7)$, $(x + 1)$, and $(x + 9)$.
- Assuming a leading coefficient of 1 (monic polynomial), the degree 4 polynomial is:

$$
\boxed{
F(x) = x(x - 7)(x + 1)(x + 9)
}
$$

- The polynomial is in factored form, as required.
- To expand, you can use FOIL or distributive methods, which can be useful for understanding the polynomial's behavior or for specific calculations.

Additional Considerations

What if the zeros have multiplicities?

- If any zero is repeated, its corresponding factor appears multiple times.
- For example, a zero at 7 with multiplicity 2 would lead to the factor $(x - 7)^2$.
- In such cases, the degree increases accordingly.

Complex Zeros and Non-Real Roots

- The zeros given are real; if complex zeros were involved, factors would include complex conjugates.
- The process of constructing the polynomial remains similar, but factors may be quadratic with no real roots.

Impact of the Leading Coefficient

- Choosing $k \neq 1$ scales the entire polynomial.
- For example, $k=2$ would produce a polynomial with leading coefficient 2.

Conclusion

Finding a degree 4 polynomial with specific zeros is a straightforward process rooted in understanding the fundamental relationship between zeros and factors. By expressing each zero as a linear factor and multiplying them together, we can construct the polynomial in factored form. This method is essential for solving polynomial equations, analyzing polynomial graphs, and understanding the roots' influence on polynomial behavior. Remember, unless specified otherwise, assuming a leading coefficient of 1 simplifies the process and provides a clean, standard polynomial in factored form.

Keywords for SEO Optimization:

- Polynomial with given zeros
- Degree 4 polynomial
- Factored form of polynomial

- Constructing polynomials from zeros
- Zeros of polynomial
- Polynomial factors
- How to find a polynomial from roots
- Algebra polynomial construction
- Polynomial zeros and factors
- Factored polynomial form

Frequently Asked Questions

How do you find a degree 4 polynomial with zeros at 0, 7, -1, and -9?

To find such a polynomial, you multiply the factors corresponding to each zero: $(x - 0)$, $(x - 7)$, $(x + 1)$, and $(x + 9)$. The polynomial is then $F(x) = k \cdot x \cdot (x - 7) \cdot (x + 1) \cdot (x + 9)$, where k is any non-zero constant.

What is the factored form of a degree 4 polynomial with zeros 0, 7, -1, and -9?

The factored form is $F(x) = k \cdot x \cdot (x - 7) \cdot (x + 1) \cdot (x + 9)$, with k being a constant (usually 1 for simplicity).

How do I determine the polynomial's leading coefficient when finding it from zeros?

Typically, if not specified, you can set the leading coefficient k to 1 for the simplest polynomial. Adjust k if needed for specific conditions such as a particular leading coefficient.

Can the polynomial be written explicitly in expanded form?

Yes, by expanding the factored form: $F(x) = x(x - 7)(x + 1)(x + 9)$. First expand the pairs $(x - 7)(x + 1)$ and $(x + 9)$, then multiply all factors together to get the expanded polynomial.

Why is it sufficient to leave the polynomial in factored form?

Leaving the polynomial in factored form clearly shows the zeros and their multiplicities, which is often the goal in such problems. It also makes it easier to analyze roots and their properties.

What if the zeros have multiplicities? How does that change the polynomial?

If zeros have multiplicities greater than one, their corresponding factors are raised to higher powers. For example, a zero at 7 with multiplicity 2 would contribute (x - 7)^2 to the factored form.

How do I verify the zeros of the polynomial once I have it in factored form?

To verify, substitute each zero into the polynomial. If the polynomial equals zero at those points, the zeros are confirmed. For example, plugging in x = 0, 7, -1, and -9 should yield zero.

Is it necessary to include the constant k in the polynomial, and what value should it have?

Including the constant k allows flexibility; often, k=1 is used for simplicity unless specified otherwise. The zeros are unaffected by k, but the overall shape and scale of the polynomial depend on it.

Additional Resources

Find A Polynomial F(x) Of Degree 4 That Has The Following Zeros: 0, 7, -1, -9

When tasked with finding a polynomial \(F(x)\) of degree 4 that has the following zeros: 0, 7, -1, -9, it's essential to understand how to translate these roots into a factored polynomial form. This process involves leveraging the fundamental theorem of algebra, which tells us that a polynomial's zeros correspond directly to its factors. In this guide, we'll walk through the steps carefully, exploring how to construct the polynomial from its zeros and presenting the final answer in factored form.

Understanding the Basics: Zeros and Factors

Before diving into the solution, let's clarify some foundational concepts:

- Zeros of a polynomial: Values of \(x\) where the polynomial evaluates to zero.
- Factors of a polynomial: Expressions that multiply together to give the polynomial, typically of the form \((x - r)\), where \(r\) is a root.
- Degree of the polynomial: The highest power of \(x\) in the polynomial; in this case, degree 4.

Since the zeros are given as 0, 7, -1, and -9, these roots directly translate to factors:

- \(x = 0 \Rightarrow x\) (since \(x - 0 = x\))
- \(x = 7 \Rightarrow x - 7\)

- $x = -1 \Rightarrow x + 1$
- $x = -9 \Rightarrow x + 9$

Step 1: Construct the Basic Polynomial from the Zeros

The general form of the polynomial, based on the roots, is:

$$ F(x) = a \times (x)(x - 7)(x + 1)(x + 9) $$

Here, a is a non-zero constant that can be any real number. Without additional information (such as a specific point the polynomial passes through), the simplest choice is to set $a = 1$.

Note: The polynomial's degree is 4 because it has four roots, and including the multiplicities, this polynomial will be degree 4.

Step 2: Write the Polynomial in Factored Form

Using the roots, the factored form of $F(x)$ is:

$$ \boxed{
F(x) = x \times (x - 7) \times (x + 1) \times (x + 9)
} $$

This is the polynomial in factored form, which directly relates to the zeros.

Step 3: Expand the Factors (Optional)

While the problem asks to leave the answer in factored form, understanding how to expand the polynomial can be instructive. Let's briefly expand to see the polynomial in standard form:

1. Multiply the quadratic pairs:

- $(x + 1)(x + 9) = x^2 + 10x + 9$
- $(x)(x - 7) = x^2 - 7x$

2. Multiply these two quadratics:

$$ (x^2 + 10x + 9) \times (x^2 - 7x) $$

Using distributive property:

$$
\begin{aligned}

& x^2 \times x^2 = x^4 \\
& x^2 \times (-7x) = -7x^3 \\
& 10x \times x^2 = 10x^3 \\
& 10x \times (-7x) = -70x^2 \\
& 9 \times x^2 = 9x^2 \\
& 9 \times (-7x) = -63x
\end{aligned}
\]

Adding the like terms:

\[
x^4 + (-7x^3 + 10x^3) + (-70x^2 + 9x^2) + (-63x)
\]

Simplifies to:

\[
x^4 + 3x^3 - 61x^2 - 63x
\]

3. Multiply by the remaining factor \(x\):

\[
F(x) = x \times (x^4 + 3x^3 - 61x^2 - 63x) = x^5 + 3x^4 - 61x^3 - 63x^2
\]

This is the expanded form, but as noted, the factored form is often more useful for analysis and is the answer we aim to leave as per the instructions.

Step 4: Final Answer in Factored Form

The polynomial \(F(x)\) of degree 4 with zeros at 0, 7, -1, and -9 in factored form is:

\[
\boxed{
F(x) = x \times (x - 7) \times (x + 1) \times (x + 9)
}
\]

Additional Considerations

- Leading coefficient \(a\): If the problem specifies a particular value for the leading coefficient, you can multiply the entire polynomial by that constant. In most cases, setting \(a=1\) suffices.
- Multiplicity of roots: If any root has a multiplicity greater than 1, the corresponding factor would be raised to that power. Here, all roots are simple, so each factor appears once.

- Graphical interpretation: The zeros indicate where the polynomial crosses or touches the x-axis at 0, 7, -1, and -9.

Summary

To wrap up, finding a degree 4 polynomial with specified zeros involves:

- Translating zeros into linear factors: $(x - r)$ for each root r.
- Multiplying these factors together to get the polynomial in factored form.
- Recognizing that the leading coefficient can be chosen as 1 unless otherwise specified.
- Understanding that the zeros directly determine the factors, making the process straightforward once roots are known.

In conclusion, the polynomial that satisfies the given zeros, left in factored form, is:

$$
\boxed{
F(x) = x(x - 7)(x + 1)(x + 9)
}
$$

This form clearly displays the roots and provides a complete, concise answer to the problem.

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