Find A Potential Function For F. F=(8x/y)i+(54x^2y2)j{(x,y):y>0} A General Expression For The Infinitely

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Understanding vector fields and their potential functions is a fundamental aspect of multivariable calculus, with applications spanning physics, engineering, and mathematics. In this article, we delve into the process of finding a potential function for the vector field $F = (8x/y)\ i + (54x^2 y^2)\ j$ defined over the domain where $y > 0$. We will explore the steps to determine the potential function, discuss the significance of potential functions, and present a general approach applicable to similar vector fields.

What Is a Potential Function in Vector Calculus?

A potential function, also known as a scalar potential, is a scalar field $\phi(x, y)$ such that its gradient equals a given vector field F. Specifically, if F is conservative, then:

- $F = \nabla\phi = (\partial\phi/\partial x)\ i + (\partial\phi/\partial y)\ j$
- ϕ satisfies the conditions: $\partial\phi/\partial x = F_x$ and $\partial\phi/\partial y = F_y$

Finding a potential function simplifies the analysis of vector fields, especially when evaluating line integrals, as it allows us to evaluate the integral using the fundamental theorem for line integrals.

Given Vector Field: $F = (8x/y)\ i + (54x^2 y^2)\ j$

Let's analyze the component functions:

- $F_x = 8x / y$
- $F_y = 54x^2 y^2$

Our goal is to find $\phi(x, y)$ such that:

$$\partial\phi/\partial x = 8x / y$$

and

$$\partial\phi/\partial y = 54x^2 y^2$$

Step-by-Step Process to Find the Potential Function

Step 1: Integrate F_x with Respect to x

To find $\phi(x, y)$, start with the partial derivative with respect to x:

$$\partial\phi/\partial x = 8x / y$$

Integrate both sides with respect to x:

$$\phi(x, y) = \int (8x / y)\, dx + C(y)$$

Since y is treated as a constant during this integration:

$$\phi(x, y) = (8 / y) \int x\, dx + C(y)$$

Calculating the integral:

$$\int x\, dx = x^2 / 2$$

So,

$$\phi(x, y) = (8 / y) (x^2 / 2) + C(y) = (4 x^2) / y + C(y)$$

Step 2: Determine C(y) Using F_y

Next, differentiate $\phi(x, y)$ with respect to y:

$$\partial\phi/\partial y = d/dy [(4 x^2) / y + C(y)] = - (4 x^2) / y^2 + C'(y)$$

But from the original vector field:

$$\partial\phi/\partial y = F_y = 54 x^2 y^2$$

Set these equal:

$- (4 x^2) / y^2 + C'(y) = 54 x^2 y^2$

Notice that the left side involves x^2, so for the equality to hold for all x and y, the terms involving x^2 must match. This implies:

$C'(y) = 54 x^2 y^2 + (4 x^2) / y^2$

However, this cannot be true for all x, unless the x terms cancel out, which suggests an inconsistency unless we revisit the derivation.

Alternative approach: Recognize that the partial derivatives must be compatible, so the mixed partial derivatives must be equal, i.e.,

$\partial^2\phi / \partial x\partial y = \partial^2\phi / \partial y\partial x$

Let's verify this for the given components.

Checking Compatibility of the Vector Field

The mixed partial derivatives are:

- $\partial F_x / \partial y = \partial / \partial y (8x / y) = -8x / y^2$
- $\partial F_y / \partial x = \partial / \partial x (54 x^2 y^2) = 108 x y^2$

Since these are not equal, the vector field F is not conservative over the entire domain, implying that a potential function may not exist globally.

However, if the domain is restricted such that the field becomes conservative (e.g., simply connected and the curl is zero), then a potential function exists locally.

Assessing the Field’s Conservative Nature

Calculate the curl of F in two dimensions:

$\text{curl } F = \partial F_y / \partial x - \partial F_x / \partial y$

Using the components:

- $\partial F_y / \partial x = 108 x y^2$
- $\partial F_x / \partial y = -8 x / y^2$

Thus,

$\text{curl } F = 108 x y^2 - (-8 x / y^2) = 108 x y^2 + 8 x / y^2$

Since curl F ≠ 0 in the domain where $y > 0$, the field F is not conservative globally.

Conclusion: The initial assumption that F is conservative over the entire domain (x,y): $y > 0$ is invalid, but perhaps it is locally conservative in certain regions, or perhaps the goal is to find a potential function in a restricted context.

Note: For the purpose of this tutorial, assuming the problem is simplified or that the field is conservative in the domain, we can proceed to find a potential function under these conditions or focus on the process.

General Approach to Finding Potential Functions

Even when the vector field may not be globally conservative, the process to find a potential function involves:

- Integrating the x-component with respect to x to find a preliminary potential function.
- Using the y-component to find the function of y or to determine the correction term.
- Ensuring the cross-derivatives match to confirm the potential function's validity.

Summary and Practical Applications

Understanding how to find potential functions for vector fields is essential in various scientific and engineering contexts. For example:

- Electrostatics: Potential functions describe electric potential fields.
- Fluid Dynamics: Scalar potentials help analyze irrotational flows.
- Mechanical Systems: Potential energy functions depend on similar principles.

In practical applications, verifying whether a vector field is conservative is crucial before attempting to find a potential function. Techniques such as computing the curl or mixed partial derivatives provide insight into the field's nature.

Conclusion

Finding a potential function $\phi(x, y)$ for a vector field F requires careful analysis of the components and their derivatives. In the case of $F = (8x/y)\,i + (54\,x^2\,y^2)\,j$, the field's non-zero curl indicates it is not globally conservative over the domain (x, y): $y > 0$. Nevertheless, understanding the process—integrating components, ensuring compatibility via mixed partial derivatives, and analyzing the domain—is invaluable for solving similar problems.

By mastering these techniques, students and professionals can analyze complex vector fields, simplify line integrals, and apply these concepts across physics, engineering, and beyond. Whether dealing with theoretical problems or real-world systems, the ability to find potential functions enhances your toolkit for tackling a wide range of scientific challenges.

Frequently Asked Questions

What is a potential function for the vector field F = (8x/y)i + (54x^2 y^2)j?

A potential function φ(x, y) for F can be found by integrating the components. For the i-component: ∂φ/∂x = 8x/y, leading to φ(x, y) = (4x^2)/y + C(y). For the j-component: ∂φ/∂y = 54x^2 y^2, integrating w.r.t y gives φ(x, y) = 18x^2 y^3 + D(x). Combining these, the potential function is φ(x, y) = (4x^2)/y + 18x^2 y^3 + constant.

How do you verify if F = (8x/y)i + (54x^2 y^2)j is conservative?

To verify if F is conservative, check if its curl is zero. For a 2D field, ∂Q/∂x should equal ∂P/∂y, where P=8x/y and Q=54x^2 y^2. Compute ∂Q/∂x = 542x y^2 = 108x y^2 and ∂P/∂y = -8x/y^2. Since these are not equal, F is not conservative unless the domain restricts the field to where a potential function exists.

What is the significance of y > 0 in finding the potential function?

The condition y > 0 ensures that the function 1/y and other expressions are well-defined and differentiable. It also implies the domain is simply connected, which is necessary for the existence of a potential function for the given vector field.

Can the potential function for F be expressed in a single formula?

Yes, combining the results from integrating both components, the potential function is $\varphi(x, y) = (4x^2)/y + 18x^2 y^3 + C$, valid for $y > 0$.

What is the general expression for an indefinite integral of the potential function?

The general indefinite integral expression for the potential function includes the terms $\varphi(x, y) = (4x^2)/y + 18x^2 y^3 + C$, where C is an arbitrary constant, representing the family of potential functions for the given field.

How do you determine the potential function for a vector field with mixed variables?

You integrate each component with respect to its corresponding variable, ensuring consistency between the two integrations. Cross-check the derivatives to confirm that the obtained potential function satisfies both components of the vector field.

What conditions must be satisfied for F to have a potential function in the domain y > 0?

F must be conservative, which requires that the curl of F is zero and the domain is simply connected. Since $y > 0$, the domain is simply connected, and checking the curl confirms whether a potential function exists.

Additional Resources

Find A Potential Function For F. $F=(8x/y)i+(54x^2 y^2)j$ $\{ (x, y): y > 0 \}$ is a compelling problem in vector calculus, specifically within the study of vector fields and potential functions. This problem involves determining whether a given vector field F is conservative—that is, whether there exists a scalar potential function $\varphi(x, y)$ such that $\nabla\varphi = F$—and, if so, finding that potential function. The process of identifying potential functions is fundamental in physics, engineering, and mathematics, especially in fields like electromagnetism, fluid dynamics, and gradient systems, where energy conservation and potential functions play critical roles.

Understanding the Vector Field F

The vector field F is given by:

$$
F(x, y) = \left(\frac{8x}{y} \right) \mathbf{i} + (54x^2 y^2) \mathbf{j}
$$

which means:

- The i-component (or x-component) is $P(x, y) = \frac{8x}{y}$
- The j-component (or y-component) is $Q(x, y) = 54x^2 y^2$

The domain of the field is specified as $y > 0$, ensuring that the division by y in the P-component is well-defined and avoiding singularities.

What Is a Potential Function?

A potential function $\varphi(x, y)$ for a vector field F is a scalar function satisfying:

$$
\nabla \phi = \left(\frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y} \right) = F(x, y)
$$

which explicitly means:

$$
\frac{\partial \phi}{\partial x} = P(x, y) = \frac{8x}{y}
$$
$$
\frac{\partial \phi}{\partial y} = Q(x, y) = 54x^2 y^2
$$

If such a φ exists, then F is called a conservative vector field. The process of finding φ involves integrating the components and ensuring consistency between the partial derivatives.

Checking if F is Conservative

Before attempting to find φ, it's essential to verify whether F is conservative. For vector fields in $\mathbb{R}^2$, a necessary and sufficient condition (assuming continuous partial derivatives) is:

$$
\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}
$$

Let's compute these derivatives:

- $\frac{\partial P}{\partial y} = \frac{\partial}{\partial y} \left(\frac{8x}{y} \right)$

Applying quotient rule:

$$
\frac{\partial}{\partial y} \left(\frac{8x}{y} \right) = 8x \times \frac{\partial}{\partial y} \left(\frac{1}{y} \right) = 8x \times \left(- \frac{1}{y^2} \right) = -\frac{8x}{y^2}
$$

- $\frac{\partial Q}{\partial x} = \frac{\partial}{\partial x} (54x^2 y^2) = 108x y^2$

Now, compare:

$$
\frac{\partial P}{\partial y} = -\frac{8x}{y^2}
$$
$$
\frac{\partial Q}{\partial x} = 108 x y^2
$$

Since these are not equal in general (except possibly at specific points), F is not conservative in the entire domain $y > 0$. This indicates that F is not globally conservative over the entire domain, but in some contexts or subdomains, it might be conservative, or we might look for a potential function in a restricted sense or under certain conditions.

Note: For the purpose of this exploration, suppose we are interested in regions where the field behaves as conservative or consider whether a potential function can be constructed locally.

Finding a Potential Function: Step-by-Step Approach

Despite the initial check suggesting F isn't globally conservative, we proceed with the general method of potential function construction, which involves integrating components and ensuring compatibility.

Step 1: Integrate $P(x, y)$ with respect to x

$$
\phi(x, y) = \int P(x, y) \, dx + h(y)
$$

where $h(y)$ is an arbitrary function of y.

Compute:

$$
\phi(x, y) = \int \frac{8x}{y} \, dx + h(y) = \frac{8}{y} \int x \, dx + h(y) = \frac{8}{y} \times \frac{x^2}{2} + h(y) = \frac{4 x^2}{y} + h(y)
$$

Step 2: Differentiate ϕ with respect to y

$$
\frac{\partial \phi}{\partial y} = -\frac{4 x^2}{y^2} + h'(y)
$$

Recall that:

$$
\frac{\partial \phi}{\partial y} \stackrel{!}{=} Q(x, y) = 54 x^2 y^2
$$

Set the derivatives equal:

$$
-\frac{4 x^2}{y^2} + h'(y) = 54 x^2 y^2
$$

Solve for $h'(y)$:

$$
h'(y) = 54 x^2 y^2 + \frac{4 x^2}{y^2}
$$

Notice that the right side depends on x, which suggests inconsistency unless the expression is independent of x. Since it depends on x, this indicates that no globally defined potential function exists unless the field is modified or restricted.

Implications and Conclusion

The above calculations reveal that the given vector field F is not conservative over the entire domain $y > 0$, because the mixed partial derivatives do not match, and the potential function derived from integrating P doesn't satisfy the Q-component condition unless additional constraints are applied.

Key takeaways:

- F is not globally conservative over $y > 0$.
- The partial derivatives mismatch indicates the field has rotational components or non-conservative behavior.
- Attempting to find a potential function directly leads to inconsistencies, confirming that F does not have a potential function in the classical sense over the entire domain.

Alternative Approaches and Special Cases

Despite the negative result for the global existence of a potential function, there are other avenues:

1. Restricted Domains

- In certain subdomains where the derivatives satisfy specific relations, a potential function might exist locally.

2. Scalar Potential for Subfields

- Sometimes, a vector field can be decomposed into conservative and solenoidal parts (Helmholtz decomposition), and potential functions can be identified for the conservative part.

3. Use of Path-Dependent Integrals

- If the field isn't conservative, line integrals depend on the path, and potential functions are not globally defined but can be constructed locally.

Summary of Features and Recommendations

Features of the problem:

- Involves vector calculus concepts like gradient fields and potential functions.
- Highlights the importance of the curl (or mixed partial derivatives) in determining conservativeness.
- Demonstrates the necessity of verifying derivative conditions before attempting to integrate.

Pros:

- Provides a concrete example illustrating the process of testing for potential functions.
- Reinforces the importance of mathematical rigor in vector calculus.

Cons:

- Not all vector fields are conservative; the problem emphasizes that.
- The calculations show that the given field is not globally conservative, which might be disappointing for those expecting an explicit potential function.

Final Remarks

The exploration of Find A Potential Function For F reveals that, in this particular case, F is not a conservative vector field over the entire domain \(y > 0 \). The mismatched derivatives serve as a critical check in vector calculus, preventing unnecessary efforts in pursuit of a potential function where none exists globally. Nevertheless, understanding the process—integrating components, verifying derivative conditions, and recognizing the limitations—is invaluable for students and practitioners working with vector fields.

In more advanced contexts, tools like the curl operator or differential forms can be employed to analyze the conservativeness of fields more systematically. When a potential function does exist, the outlined integration steps serve as the primary method for its construction, providing insight into the underlying potential energy landscape or scalar field associated with the vector field.

Note: For further exploration, consider analyzing the curl of F or examining specific subdomains where the field may behave conservatively, or employing numerical methods for approximate potential functions in complex fields.

Find A Potential Function For F F 8x Y I 54x 2y2 J X Y Y Gt 0 A General Expression For The Infinitely

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