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Understanding how to find the area of a specific region bounded by curves is a fundamental aspect of calculus and analytical geometry. When dealing with polar equations, such as $r = 8 \sin \theta$, the process involves integrating over the relevant angular intervals. This article aims to guide you through the step-by-step procedure to find the area of the region that lies inside one curve and outside another, specifically focusing on the curve $r = 8 \sin \theta$. Whether you're a student preparing for exams or a math enthusiast looking to deepen your understanding, this comprehensive guide will clarify the concepts and methods involved.

Understanding the Curves in Polar Coordinates

Before delving into calculations, it's essential to understand the nature of the curves involved, especially their shapes, intersections, and key features.

Polar Coordinates and Their Significance

- Polar coordinates describe a point in the plane using a radius r and an angle θ.
- The equation $r = f(\theta)$ defines a curve by specifying the distance from the origin as a function of the angle.

The Curve $r = 8 \sin \theta$

- This is a classic cardioid-like curve (specifically a limacon with an inner loop).
- Key features:
- When $\theta = 0$, $r = 0$.
- When $\theta = \pi/2$, $r = 8$.
- The maximum value of r is 8, occurring at $\theta = \pi/2$.
- The curve is symmetric with respect to the vertical axis.
- Graphing this curve reveals a shape that is symmetric about the vertical axis, crossing the origin at $\theta = 0$ and $\theta = \pi$.

Intersections and Key Points of the Curves

To find the region that lies inside one curve and outside another, identifying their intersection points is crucial.

Determining the Intersection Points

- Suppose the second curve is given by $r = g(\theta)$, and in this case, it might be a different curve.
- The problem states to find the area inside the first curve $r = 8 \sin \theta$ and outside the second curve. For demonstration, let's assume the second curve is a circle or a related curve.

Note: Since only $r = 8 \sin \theta$ is specified, the "second curve" needs to be defined or clarified. For illustration, suppose the second curve is the circle $r = 4$, a common example.

Example: Finding the Intersection of $r = 8 \sin \theta$ and $r = 4$

- Set $8 \sin \theta = 4$.
- Solving for θ:

$$
\sin \theta = \frac{4}{8} = \frac{1}{2}
$$

$$
\theta = \arcsin \frac{1}{2} = \frac{\pi}{6} \quad \text{or} \quad \frac{5\pi}{6}
$$

- These are the angular points where the curves intersect.

Implication: The region of interest is bounded between $\theta = \pi/6$ and $\theta = 5\pi/6$ for the given example.

Formulating the Area Calculation

Once intersection points are identified, the next step involves setting up the integral to compute the desired area.

Area in Polar Coordinates

- The area A enclosed between two curves $r = f(\theta)$ and $r = g(\theta)$ over $\theta \in [\alpha, \beta]$ is given by:

$$
A = \frac{1}{2} \int_\alpha^\beta \left[f(\theta)^2 - g(\theta)^2 \right] d\theta
$$

- This formula accounts for the difference in areas swept out by the two curves.

Applying to Our Scenario

- To find the area inside $r = 8 \sin \theta$ and outside the second curve $r = g(\theta)$, determine which curve is outer and which is inner over each interval.
- The integral limits are determined by the intersection points.

Example process:

1. Determine the relevant intervals:
- From the intersection points, establish θ intervals where $r = 8 \sin \theta$ is outside $g(\theta)$.
2. Set up the integral:
- For each interval, the area is:

$$
A_i = \frac{1}{2} \int_{\theta_{i, start}}^{\theta_{i, end}} \left[(r_{outer}(\theta))^2 - (r_{inner}(\theta))^2 \right] d\theta
$$

3. Sum the areas:
- Total area is the sum over all relevant intervals.

Step-by-Step Calculation with a Specific Example

Let's proceed with a concrete example: find the area that lies inside $r = 8 \sin \theta$ and outside $r = 4$.

1. Find the intersection points

- Set $8 \sin \theta = 4$.
- $\sin \theta = 0.5$.
- Solutions:

$$
\theta = \frac{\pi}{6} \quad \text{and} \quad \frac{5\pi}{6}
$$

2. Determine the region

- For $\theta \in [\pi/6, 5\pi/6]$:
- $r = 8 \sin \theta$ is outside $r = 4$ (since $8 \sin \theta \geq 4$ here).
- Outside the circle $r=4$, but inside the cardioid $r=8 \sin \theta$.

3. Set up the integral for the area

- The area inside the cardioid and outside the circle:
$$
A = \frac{1}{2} \int_{\pi/6}^{5\pi/6} \left[(8 \sin \theta)^2 - 4^2 \right] d\theta
$$
- Simplify:
$$
A = \frac{1}{2} \int_{\pi/6}^{5\pi/6} \left[64 \sin^2 \theta - 16 \right] d\theta
$$

4. Compute the integral

- Break into two parts:
$$
A = \frac{1}{2} \left[64 \int_{\pi/6}^{5\pi/6} \sin^2 \theta \, d\theta - 16 \int_{\pi/6}^{5\pi/6} d\theta \right]
$$

- Recall:
$$
\int \sin^2 \theta \, d\theta = \frac{\theta}{2} - \frac{\sin 2\theta}{4} + C
$$

- Compute each integral:

1. $\int_{\pi/6}^{5\pi/6} \sin^2 \theta \, d\theta$:

$$
\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4} \right]_{\pi/6}^{5\pi/6}
$$

- At $\theta = 5\pi/6$:
$$
\frac{5\pi/6}{2} - \frac{\sin (2 \times 5\pi/6)}{4} = \frac{5\pi/12} - \frac{\sin (5\pi/3)}{4}
$$
$$
\sin (5\pi/3) = - \frac{\sqrt{3}}{2}
$$
$$

\Rightarrow \frac{5\pi/12} - \frac{-\sqrt{3}/2}{4} = \frac{5\pi/12} + \frac{\sqrt{3}/2}{4} =
\frac{5\pi/12} + \frac{\sqrt{3}}{8}
\]

- At \(\theta = \pi/6 \):
\[
\frac{\pi/6}{2} - \frac{\sin (2 \times \pi/6)}{4} = \frac{\pi/12} - \frac{\sin (\pi/3)}{4}
\]
\[
\sin (\pi/3) = \frac{\sqrt{3}}{2}
\]
\[
\Rightarrow \frac{\pi/12}

Frequently Asked Questions

What is the region bounded by the curves $r = 8 \sin \theta$ and the x-axis?

The region is the area inside the cardioid $r = 8 \sin \theta$ that lies above the x-axis, corresponding to θ from 0 to π.

How do you find the points of intersection between $r = 8 \sin \theta$ and the x-axis?

Set $r = 0$: $8 \sin \theta = 0$, which implies $\sin \theta = 0$, so $\theta = 0$ and $\theta = \pi$.

What is the formula for the area enclosed by a polar curve $r = f(\theta)$ between $\theta = a$ and $\theta = b$?

The area is given by $(1/2) \int_a^b [f(\theta)]^2 \, d\theta$.

How do you determine the region that lies inside $r = 8 \sin \theta$ and outside another curve?

Identify the curves and their intersection points, then set up the integral for the area where one curve's radius is greater than the other's, within the intersection bounds.

What is the relevant range of θ to find the area inside $r = 8 \sin \theta$ and outside the x-axis?

From $\theta = 0$ to $\theta = \pi$, because $r \geq 0$ in this range, and the curve is above the x-axis.

How do you compute the area of the region inside r = 8 sin θ and outside the x-axis?

Calculate the area enclosed by the cardioid from $\theta = 0$ to π using the integral $(1/2) \int_0^\pi (8 \sin \theta)^2 d\theta$.

What is the value of the area enclosed by r = 8 sin θ from θ = 0 to π?

The area is $(1/2) \int_0^\pi 64 \sin^2 \theta \, d\theta$, which evaluates to 32π.

How do you evaluate the integral for the area of the cardioid r = 8 sin θ?

Use the identity $\sin^2 \theta = (1 - \cos 2\theta)/2$ to simplify the integral, then integrate term-by-term.

What is the final area of the region inside r = 8 sin θ and outside the x-axis?

The area is 32π square units.

Can this method be applied to find the area between two polar curves?

Yes, by finding their intersection points and integrating the difference of their squared radii over the intersection interval.

Additional Resources

Find The Area Of The Region That Lies Inside The First Curve And Outside The Second Curve is a classic problem in the study of polar coordinates and curve analysis. This type of problem is not only fundamental in understanding how to calculate areas bounded by curves but also showcases the elegance and power of polar equations in describing complex shapes such as limacons, cardioids, and roses. Whether you're a student working through calculus problems or a professional analyzing geometric regions, mastering the approach to such area calculations is essential.

In this comprehensive guide, we'll explore how to find the area of the region that lies inside the curve defined by $r = 8 \sin(\theta)$ and outside another curve, which, in typical problem contexts, might be $r = 0$ (the pole) or another polar curve. For clarity and completeness, we'll assume the problem involves calculating the area inside the curve $r = 8 \sin(\theta)$ but outside a specified inner boundary. We'll walk through the process step-by-step, covering key concepts, methods, and examples to ensure you can confidently approach similar problems.

Understanding the Curves: The Polar Equation $r = 8 \sin(\theta)$

Before diving into calculations, it's crucial to understand the nature of the given curve:

The Curve $r = 8 \sin(\theta)$

- Type of curve: This is a lemniscate-like or cardioid-shaped curve, often called a limacon with a certain parameter.
- Shape characteristics:
- When $\theta = 0$, $r = 0$.
- When $\theta = \pi/2$, $r = 8$.
- The maximum radius (r_{max}) occurs at $\theta = \pi/2$, $r = 8$.
- The curve passes through the pole (origin) at $\theta = 0$ and $\theta = \pi$, because $\sin(0) = 0$ and $\sin(\pi) = 0$.
- Symmetry: The curve is symmetric about the y-axis because sine is symmetric about $\theta = \pi/2$.

Visualizing the Curve

The graph of $r = 8 \sin(\theta)$ is a circle-like shape that extends from the pole up to $r = 8$ at $\theta = \pi/2$, then back down to the pole at $\theta = \pi$, and symmetrically on the other side.

Clarifying the Region: Inside First Curve & Outside Second Curve

The core of the problem involves identifying the region bounded by the given curve $r = 8 \sin(\theta)$ and another boundary, which could be:

- The pole (origin), i.e., $r = 0$.
- Another curve, such as r = some function of θ.
- A specific angular boundary, i.e., between two angles.

For simplicity, and as is common in such problems, we'll analyze the area inside the curve $r = 8 \sin(\theta)$ and outside the pole $r = 0$ (which is trivial), or between two curves where needed.

Step-by-Step Approach to Find the Area

1. Identify the Bounds of θ

To compute the area enclosed by a polar curve, the key is to determine the intervals of θ over which the region exists.

- For $r = 8 \sin(\theta)$, the curve is traced as θ goes from 0 to π.
- The curve is symmetric about the vertical axis, so calculations from 0 to $\pi/2$ can be doubled if necessary.

2. Set Up the Area Integral

The general formula for the area A enclosed by a polar curve $r(\theta)$ between $\theta = a$ and $\theta = b$ is:

$$A = \frac{1}{2} \int_{a}^{b} [r(\theta)]^2 d\theta$$

For our curve, r = 8 sin(θ), the area from θ = 0 to θ = π is:

$$ A = \frac{1}{2} \int_{0}^{π} (8 \sin θ)^2 dθ $$

which simplifies to:

$$ A = \frac{1}{2} \int_{0}^{π} 64 \sin^2 θ dθ $$
$$ A = 32 \int_{0}^{π} \sin^2 θ dθ $$

3. Computing the Integral

To evaluate $\int \sin^2 θ dθ$, recall the power-reduction formula:

$$
\sin^2 θ = \frac{1 - \cos 2θ}{2}
$$

Thus,

$$
A = 32 \int_{0}^{π} \frac{1 - \cos 2θ}{2} dθ
$$
$$
A = 16 \int_{0}^{π} (1 - \cos 2θ) dθ
$$

Now, integrate term-by-term:

$$
A = 16 \left[\int_{0}^{π} 1 dθ - \int_{0}^{π} \cos 2θ dθ \right]
$$

$$
A = 16 \left[θ - \frac{\sin 2θ}{2} \right]_0^{π}
$$

Evaluating:

$$
A = 16 \left[π - \frac{\sin 2π}{2} - (0 - \frac{\sin 0}{2}) \right]
$$

Since $\sin 0 = 0$ and $\sin 2π = 0$:

$$
A = 16 \times π
$$

Therefore, the total area enclosed by r = 8 sin(θ) from 0 to π is:

$$ \boxed{\textbf{Area} = 16π} $$

Extending the Analysis: Find the Area Inside and Outside Specific Curves

Suppose, for example, the problem asks for the area inside r = 8 sin(θ) and outside r = 4. This involves:

- Finding the points of intersection between the curves.
- Determining the angular bounds where one curve is outside the other.
- Using the difference of the squared radii for the area calculation.

1. Find Intersection Points

Set:

$$
8 \sin \theta = 4
$$
$$
\sin \theta = \frac{1}{2}
$$

which yields:

$$
\theta = \frac{\pi}{6} \quad \text{and} \quad \theta = \frac{5\pi}{6}
$$

These are the points where the curves intersect.

2. Determine the Region of Interest

Between θ = π/6 and θ = 5π/6:

- The curve r = 8 sin(θ) is outside r = 4 when r = 8 sin(θ) > 4.
- The curve r = 4 is a circle of radius 4, constant for all θ.

3. Set Up the Integral for the Area

The area A of the region inside r = 8 sin(θ) and outside r = 4 from θ = π/6 to θ = 5π/6 is:

$$
A = \frac{1}{2} \int_{\pi/6}^{5\pi/6} [(8 \sin \theta)^2 - 4^2] d\theta
$$

Simplify:

$$
A = \frac{1}{2} \int_{\pi/6}^{5\pi/6} (64 \sin^2 \theta - 16) d\theta
$$

Express \(\sin^2 θ\) using the power-reduction formula:

\[
A = \frac{1}{2} \int_{π/6}^{5π/6} \left[64 \times \frac{1 - \cos 2θ}{2} - 16 \right] dθ
\]

\[
A = \frac{1}{2} \int_{π/6}^{5π/6} \left(32 (1 - \cos 2θ) - 16 \right) dθ
\]

\[
A = \frac{1}{2} \int_{π/6}^{5π/6} \left(32 - 32 \cos 2θ - 16 \right) dθ
\]

\[
A = \frac{1}{2} \int_{π/6}^{5π/6} (16 - 32 \cos 2θ) dθ
\]

Now, integrate term-by-term:

\[
A = \frac{1}{2} \left[16θ - 16 \sin 2θ \right]_{π/6}^{5π/6}
\]

Evaluate at the bounds:

At θ = 5π/6:

\[
16 \times \frac{5π}{6} - 16 \sin \left(2 \times \frac{5π}{6} \right) = \frac{80π}{6} - 16 \sin \left(\frac{10π}{6} \right)
\]

\[
= \frac{40π}{3} - 16 \sin \left(\frac{5π}{3} \right)
\]

Since \(\sin(5π/3) = -\frac{\sqrt{3}}{2}\),

\[
= \frac{40π}{3} - 16 \left(-\frac{\sqrt{3}}{2} \right

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